JOURNAL OF
MATHEMATICS
EDUCATION
AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Growth through Reflection in Mathematics Education

TABLE OF CONTENTS

Theory of Professional Competence in Teaching of Mathematics: Development and Explication through Cross-cultural Examination of Teaching Practices in India and the United States

Renu Ahuja
Morgan State University

ABSTRACT This paper describes the theory of professional competence in teaching of mathematics developed through a cross-cultural examination of teaching practices of mathematics teachers recommended as competent by their principals in two selected high-achieving high schools of India and the United States. A detailed study of teacher cases from both of the research settings yielded a rich conceptualization of the relationship between teachers' professional knowledge base and professional competence in the teaching of mathematics. The substantive theory explains the processes of both the development and the display of professional competence and enables predictions of the ways teachers would most likely utilize to meet the performance expectations of their work environments. The study makes a unique contribution to the field of teacher education and views professional competence as a dynamic interplay of various components of teachers' professional knowledge base activated in actual teaching situations in the context of the classroom, school, and wider social culture.

KEYWORDS *grounded theory, cross-cultural, professional competence, mathematics teaching, high school*

Background

The widespread concern about mathematics achievement in the United States has drawn considerable research attention to the quality of mathematics instruction and cross-cultural research in mathematics education. Related to this, mathematics teachers' professional knowledge and competencies for effective instruction are also under scrutiny. Schools' performance and student outcomes are increasingly quantified, measured, and compared on a global scale. Cross-cultural research has the potential to inform policy, practice, and to assist in the development of theory at a level appropriate to regional, national or global priorities (Clarke, Emanuelsson, Jablonka, & Mok, 2006). This paper deals with only a part of the findings of the cross-cultural study of mathematics teachers recommended as competent in selected high-achieving high schools of India and the United

States (US). The overall purpose of the study was to develop a theory of professional competence that characterizes effective mathematics teaching in the two research sites. The paper describes the emergent theory of professional competence in the teaching of mathematics.

The notion of professional competence as a theoretical perspective has been developed in the field of management, industry, and clinical practice, and it is currently being emphasized in higher education. Mathematics teaching and learning is increasingly influenced by globalization, internationalization, and the demands of the corporate world. Fueled by the demands of the changing economy and technical demands of many professions, proficiency in the teaching of mathematics is being emphasized due to the increasing understanding of the influence of teacher competence on students' achievement (Darling-Hammond, 2000; Wenglinsky, 2002). This warrants the development of a sound theo-

retical basis for the notion of professional competence in the teaching of mathematics. The differences and similarities that characterized effective mathematics teaching and learning in the two settings assisted in developing the models of professional competence for each teacher and led to an understanding of how the quality of mathematics teaching and learning is influenced by the context in which this learning occurs. The educational significance of the study lies in the generation of a theory that has potential to enhance the understanding of effective mathematics teaching and learning and has implications for teacher education. To achieve the purpose, this study explored instructional processes, experiences of teachers, their conceptions about mathematics and its teaching, teachers' behaviors during instruction, and contextual factors that characterize effective mathematics teaching in two schools through classroom observations and in-depth semi-structured interviews before and after the observation period.

Methodology and Research Procedures

This study's methodology combined elements of the naturalistic case study with the grounded theory approach to gain an in-depth understanding of each of the teacher participants in the two schools, which led to the characterization of each teacher's practice. Grounded theory method (GT) calls for early data collection, analysis, further theoretical sampling, and category saturation. This process of data collection is "controlled" by the emerging theory (Glaser, 1978, p. 36). A particular strength of grounded theory is that it seeks to explain the process and interactions among the various parts of the model. The complexity of classroom life and the multi-dimensional nature of the transactional processes going on in the classrooms meant that far more was taking place than could be recorded or analyzed. Thus, the theoretical sensitivity of the researcher played an important role in data collection and analysis. As the data analysis progressed, the relevant literature was examined and was used as an additional lens through which to examine the data. The literature, data collection, and data analysis were systematically linked "to generate" theory (Glaser, 1992, p. 16). This study employed the Glaserian version of GT primarily to represent the emergent model, but ideas from other grounded theorists such as Gibson (2007), and Strauss and Corbin (1998) were also incorporated to understand the procedures.

Data Sources and Analysis

Two schools were selected from each of the research settings (criteria and teacher profiles explained elsewhere). "Purposeful sampling," which seeks information-rich cases that can be studied in depth (Bogdan & Biklen, 1992, p. 71), and criterion-based selection (LeCompte & Preissle, 1993) were used in the selection of the teacher participants. In addition to student achievement, this study used recommendations of the principal along with the selection criteria recommended by Palmer, Stough, Burdenski, and Gonzales (2005). The authors recommend that researchers should use the following indicators while selecting the sample of expert teachers:

(a) Teachers should have three to five years of experience in a specific teaching content area and with a particular population of students,

(b) teacher knowledge reflected in relevant certification and degrees that correspond to the field in which these teachers are currently teaching,

(c) recognition as an exemplary teacher by fellow teachers, administrator…based on recent… indicators of teaching effectiveness to include teacher knowledge and skills, and

(d) should be confirmed with documented evidence of teacher impact on student performance (p. 23).

Many scholars agree that the complexity of a teacher's professional knowledge cannot be captured by a single instrument. Kagan (1990) recommends the use of multi-method approaches which not only provide triangulation but also capture the complex and multifaceted aspects of teaching and learning. To achieve the purpose, this study used a concept map activity (Raymond, 1997) in combination with teacher interviews, class observations, and intra-site spreading to capture the complexity and complete picture of mathematics teaching. The classroom observations focused on the teachers' presentation of the topic, questioning skills, answers to students' questions, management of class time, quality and rigor of the mathematics taught, and approaches to dealing with students' difficulties. The analysis of lessons focused on the quality of mathematical content, organization and planning of lessons, patterns of classroom interaction, and the interplay among the following teacher attributes: teachers' content knowledge, conceptions, and instructional practices. It also looked at contextual factors such as school culture and the na-

ture of mathematics as the subject matter context. Data corpus consisted of transcripts from pre and post-observation interviews, video recordings, field notes, and classroom artifacts (algebra, geometry, and calculus). A total of 43 lessons were observed in the classrooms of the nine teacher participants, and a sequence of at least three lessons was videotaped in the classrooms of the five teacher participants who agreed for video recording. The researcher acknowledges that an inductive approach to identifying teacher competencies during instruction may lead to omission of some competencies. Thus, the researcher employed 25 indicators of expertise and competencies related to effective teaching of mathematics determined from the review of extant literature to identify and describe competent mathematics teaching during classroom observations (Ahuja, 2018b). Two more competencies that emerged from the analysis were added to the list. The professional profiles of the participants are presented in Table 1.

Data Analysis

Data analysis in GT includes finding a core category, causal conditions that influence the process, the effect of context and intervening conditions, strategies used by the subjects to carry out the process, and finally the consequences of the behavior for the participants (Creswell, 1998; Glaser, 1978). Audio recordings and video clips of the lessons were listened to and viewed repeatedly, and transcripts were read and reread in an attempt to gain a deeper understanding of recurring themes and concepts. The data were coded relevant to the research questions by hand using the read, theme, re-read, and re-theme model espoused by Glaser and Strauss (1967). Care was taken to constantly return to the data to examine categories and properties in order to ground the emerging hypotheses and generate the theory. New data were coded and compared against previously collected and analyzed data until saturation. Finally, the selective coding resulted in an overarching core category. Through continuous data collection, theoretical sampling, and analysis, a substantive theory was developed to illustrate professional competence in mathematics teaching in the two research contexts.

The GT approach to research assumes that ongoing reading of the literature will be an integral part of data collection, analysis, and writing up of research (Glaser, 2005). The data analysis for this study included comparisons of the relevant literature with emergent themes and findings. A systematic literature review was done during data analysis to find relevant terms used in the literature to capture, examine teacher actions, and describe teaching behaviors observed. Thus, a "dynamic, reflexive, and integrative approach" was used for the literature review (Hussein, Kennedy, & Oliver, 2017, p. 1201; Stebbins, 2001) in order to generate new understanding from the existing literature (Torraco, 2005). Therefore, literature and data were integrated to build a portrait of professional competence that has potential to inform practice in other substantive areas. The researcher's professional experience, preparation in mathematics, and education informed the study and the data at the conceptual level. The bias arising out of her own work experience was checked by the constant comparative analysis, member reflections, triangulation of data sources, and peer debriefing.

The Grounded Theory of Professional Competence in Teaching of Mathematics

According to Glaser (1978), two key questions are asked during grounded theory research: (1) *what is the main concern or Basic Psychological Problem (BPP) of the subjects?* and (2) *what is the Basic Psychological Social Process (BPSP) that continually resolves the concern?* The BPSP refers to

Table 1

Professional Profile of the Participant Teachers

Name (Pseudonym)	Gender	Education	Teaching Exp. (Total, current location, middle, high)
Sumit	M	B.Sc. (Phy, Chem, Math), M.Sc(Math), B.Ed	11, 5, 0, 11
Suman	F	M.Sc. (Math), M.Phil (Math), B.Ed.	12, 10, 0, 12
Komal	F	M.Sc. (Math), Ph.D. (Applied Math)	14, 11, 0, 14
Lena	F	B.S. Accounting, M.S. (Sec. Education), Provisional certificate	5, 3, 1, 4
Sam	M	B.A. (Philosophy), Certified (Sec. Math)	23, 3, 0, 23

the processes such as becoming and personalizing (e.g., becoming a teacher, a nurse, a leader)—in this study: *becoming a professionally competent mathematics teacher.* Through coding, analysis, and reflection a BPSP emerged as the core category. The use of diagrams enabled the researcher to move from the descriptive to the analytical level of analysis. The grounded theory of professional competence in teaching of mathematics and the comprehensive diagram of the substantive model of professional competence following the procedures suggested by Glaser and Strauss (1967) are described in the next section.

The Main Concern

The main concern of the participants was to perform at their maximum potential in order to teach mathematics effectively in the context of the high-achieving status of the schools. The concern may not be clearly articulated by the participants, but the concern was displayed by their classroom behaviors and through their utterances during the interviews. For these teachers, professional competence was teaching in such a way that developed students' mathematical habits of mind, showed good results on assessments, and were well prepared for the next mathematics course.

BPSP: Becoming a Professionally Competent Mathematics Teacher

The purpose of data analysis in GT is to develop a core category that explains and characterizes these processes. The model to be developed is thus a model that explicates the core category. An explication of the core category in this study involved identifying the conditions influencing teachers' engagement with the development and display of professional competence as well as the *contextual factors,* and *context consequences* (the outcomes of the actions and strategies taken during the engagement with BPSP). These elements thus formed the framework of the theory that was developed. The theory enables predictions of the way teachers would most likely engage in order to meet the performance expectations of their work environments. By constantly comparing emergent concepts across the teacher cases and all elements of the data corpus for similarities and differences, themes and categories were linked to the phases of BPSP. *The five phases, six conditions, six contextual factors, and five context consequences* comprise elements of the emergent theory of professional competence (see Figure 1), and are explained in the ensuing section in terms of the emergent themes and subthemes. Some vignettes are also presented from the data.

Core Category: Interplay of cognition, Conception, and the Context

The core category of the study was interplay among conceptions, cognition, and the context. This was determined by identifying and describing existing teacher professional competencies that emerged during the course of data collection and analysis. During analysis, the researcher asked questions to herself and of the data: (1) What teacher competencies were derived from the teachers' content knowledge and pedagogical content knowledge according to their educational level? (2) What teacher competencies were related to their conceptions about teaching and learning of mathematics and the nature of mathematics? (3) Which competencies were specific to the school settings? (4) What competencies did they create in response to performance expectations from the department heads and recent changes in the curriculum or reforms? (5) What competencies did they develop as a result of their experience? Asking questions and making comparisons informed and guided analysis and assisted theorizing. The interplay of conceptions, cognition, and context explains most of the teaching behavior observed. By constantly comparing emergent concepts across the teacher cases and all elements of the data corpus for similarities and differences, themes and categories were linked to the phases of BPSP.

Progression through the Phases of BPSP

Participants' display of professional competence and their reflections on their own practice suggest that there were *five phases* in the development of professional competence: (1) developing a personal epistemology of teaching mathematics based on past and current experiences, (2) seeking knowledge to modify practice and perform at their maximum potential, (3) understanding of contextual factors, (4) maintaining balance between competing priorities, and (5) attaining good balance between emotional and interpersonal intelligence.

Developing a personal epistemology of teaching mathematics

The participants spoke of their experiences of teaching mathematics in ways suggesting a progression through five phases of the teachers' engagement with the development of professional competence. This progression was influenced by their lived experiences: experiences of learning mathematics, experiences of teaching mathematics, and experiences as members of the societal and cultural structures in their countries. These experiences affected their decision-making processes, and led to the
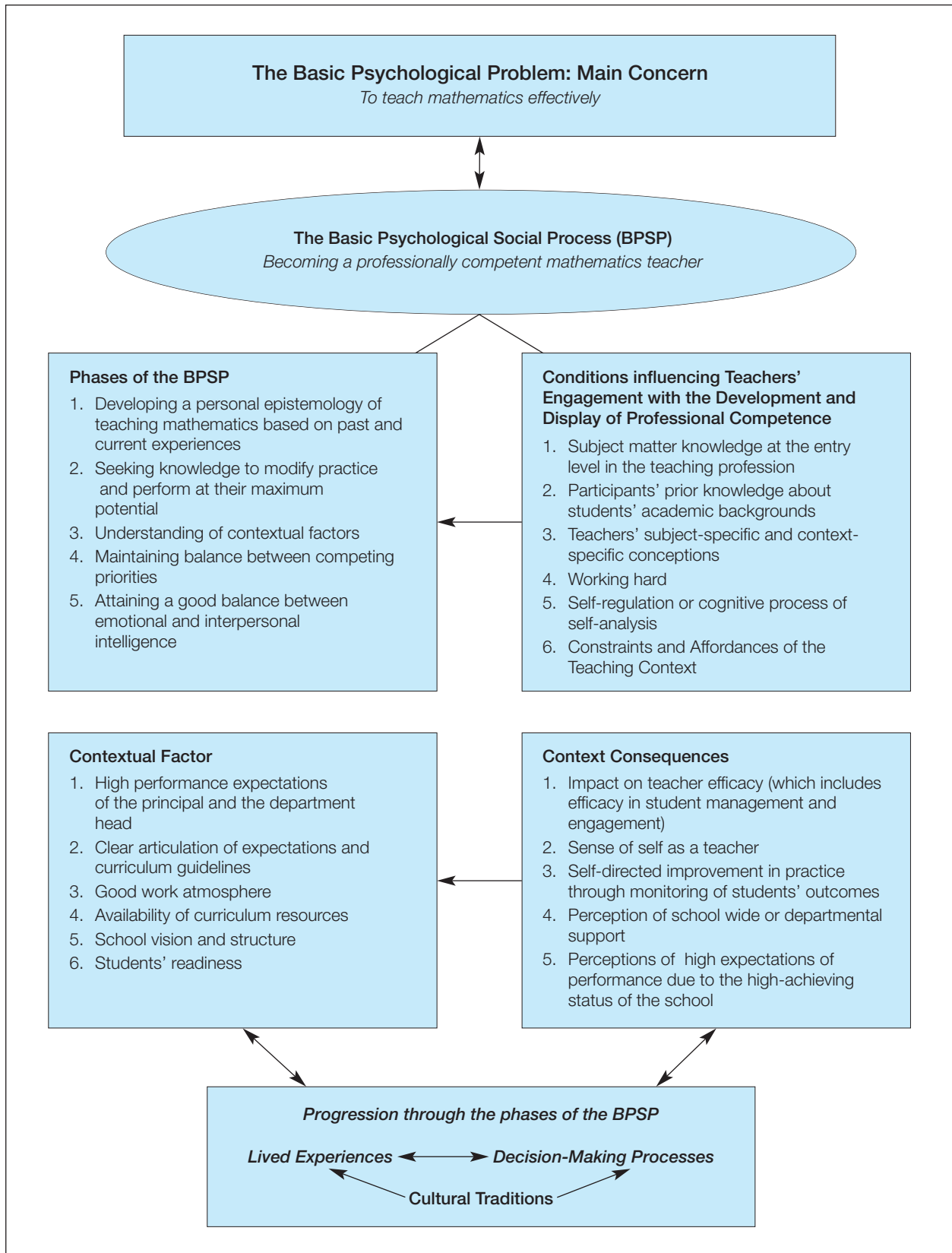
The Basic Psychological Problem: Main Concern
To teach mathematics effectively

The Basic Psychological Social Process (BPSP)
Becoming a professionally competent mathematics teacher

Phases of the BPSP

1. Developing a personal epistemology of teaching mathematics based on past and current experiences
2. Seeking knowledge to modify practice and perform at their maximum potential
3. Understanding of contextual factors
4. Maintaining balance between competing priorities
5. Attaining a good balance between emotional and interpersonal intelligence

Conditions influencing Teachers' Engagement with the Development and Display of Professional Competence

1. Subject matter knowledge at the entry level in the teaching profession
2. Participants' prior knowledge about students' academic backgrounds
3. Teachers' subject-specific and context-specific conceptions
4. Working hard
5. Self-regulation or cognitive process of self-analysis
6. Constraints and Affordances of the Teaching Context

Contextual Factor

1. High performance expectations of the principal and the department head
2. Clear articulation of expectations and curriculum guidelines
3. Good work atmosphere
4. Availability of curriculum resources
5. School vision and structure
6. Students' readiness

Context Consequences

1. Impact on teacher efficacy (which includes efficacy in student management and engagement)
2. Sense of self as a teacher
3. Self-directed improvement in practice through monitoring of students' outcomes
4. Perception of school wide or departmental support
5. Perceptions of high expectations of performance due to the high-achieving status of the school

Progression through the phases of the BPSP

Lived Experiences ⟷ *Decision-Making Processes*

Cultural Traditions

Figure 1. Ahuja Model of Substantive Theory of Professional Competence in Teaching of Mathematics.

development of their personal epistemologies (progression in the development of one's conceptions about knowledge and knowing) of teaching mathematics. Their personal epistemologies of teaching mathematics based on past and current experiences influenced the goals they set for themselves and their students in relation to the performance expectations of their work contexts. Sam (a geometry and AP calculus teacher from the US) during the interview said:

Well, we have a curriculum to cover. Sometimes the curriculum is more or less (pause) laid out to you. By and large, you know what you have to cover for AP courses and exams. For other courses you know what's on the semester exam or on the final exam and you teach accordingly. Sometimes you find yourself doing what is necessary to get that final goal. By experience, you learn to do that.

This suggests that through experience teachers develop heuristics to handle classroom challenges. All the teacher participants reported that from their experience they know some of the concepts students may not grasp, and only through experience did they become familiar with some of the mistakes and misconceptions students have regarding particular topics. Rhodes and Bellamy (1999) assert that for teachers, although a curriculum may be set for them, it inevitably becomes shaped by them to reflect their own belief systems, their thoughts, and feelings about both the content of their instruction and their learners. It is, therefore, very important to achieve a balance between the structure and the flexibility in the teaching and learning of mathematics. Sam, when talking about planning lessons, expressed the following:

Students from one year to the other grasp the topic poorly or easily. I have to add additional examples, worksheets, and explanations depending on their difficulty level so sometimes I am not able to finish what I plan. This is what I mean by being flexible. No pressure of state assessments so can afford to go at a slower pace to clear their doubts. Thus, I go with the flow of the individual class. Thus, I modify my lessons dependent on what students are grasping rather than what's convenient for me.

Sumit (precalculus and calculus teacher from India) also pointed out during the interview that he has been teaching at the school for 11 years, and through his experience he has learned to plan curriculum as per the examination-oriented system. He shared his thoughts:

When you are going to the classroom, there are all sorts of students. You have to make sure that you are not aiming at one category of students… Though normally I try to go as per the average class. Sometime, I feel that students who are too good should not feel neglected. Similarly, students who are weak should not feel that this class is no good to them. I try my best to come down to the level of students and at the same time make sure that I am taking care of good students as well. I try to maintain balance between these three types of students and by experience you learn to handle all this.

The findings suggest that the participants did not demonstrate all the 25 professional competencies in teaching of mathematics, and these competencies were categorized into three broad domains: (a) competencies related to content knowledge and pedagogical content knowledge (a dimension of teachers' cognitions), (b) competencies related to conceptions about nature of mathematics, and teaching and learning of mathematics, and (c) competencies pertaining to the teaching context (see Ahuja, 2018b). These competencies are constituents of teachers' professional knowledge base (Ahuja, 2013). The data provided sufficient evidence that these competencies develop from the interplay of prior school experiences, immediate classroom situations, and social teaching norms at the school.

Seeking knowledge to modify practice and perform at their maximum potential

The teacher participants indicated during the interviews that they modify their lessons based on their past experiences of teaching the topics, and their understanding of mistakes generally made by the students, changes in the curriculum, and types of problems expected on the standardized assessments/examinations. The participants reported during the interviews that they update knowledge by reading extra books, journals, and information available on the internet. Thus, seeking knowledge to modify practice and to perform at their maximum potential represents a phase of the emergent BPSP. It is clear that success in this phase requires an understanding of contextual factors developed through careful analysis of the teaching context. The next section of the paper presents some vignettes from the data to illustrate the emergent themes.

One of the teacher participants from the Indian setting, Sumit (pseudonym), stated that his beliefs about mathematics and its teaching were influenced by the type of students in his classes and by the school culture. Sumit, when asked what influences his teaching practice and what made him stay at the school, remarked that the students' attitudes, his coworkers' attitudes, and the principal's expectations influenced his practice the most. Sumit remarked that he liked working at the school, saying that "A good atmosphere, a good name and fame, and very good level of students; due to which, you have to be the best to be here, and keep proving that you are worth staying here" (Interview transcript). During the interview, he responded that he feels very confident teaching at the 12th-grade level, as evident from the following excerpt:

> Very comfortable. Because the content and the curriculum are the same as it was 10 years ago, they just add some more topics and change the marking scheme. I have been teaching for 11 years. I keep upgrading my knowledge through new books and internet. That is what I do. The only thing I have to do is to arrange the questions as per the difficulty level and that changes from class to class and year to year.

This also suggests Sumit's ability to self-regulate his learning to modify his practice by *seeking knowledge* to keep up with the changing needs of the students and assessments, and the changes in the curriculum. He has engaged himself in the *cognitive process of analysis of self-efficacy*. The researcher defined this construct as the self-awareness of gaps in the knowledge, reflecting about one's own process of problem-solving for better job performance, and achieving means to address this gap in knowledge. This involves reflection on actions (Schon, 1987) and outcomes. Komal, a teacher from India, stated "All students are capable of learning and should be given the opportunity to learn." When asked about the main factors affecting her preparation, she responded:

> Time, resources, data as to where my students are at the start and where I need to get them. Students are learning in new ways all the time, so if I do not change with them, then I will lose them.

This suggests she is engaged in the cognitive process of analysis of self-efficacy and her demonstrated efforts to seek knowledge and means to modify her practice by acknowledgment of the changing needs of the students. Lena, a teacher from the US, shared that her beliefs about mathematics were influenced by the way she was taught mathematics, by the curriculum, and by the current issues presented in mathematics education journals. Komal, when asked what mathematics teaching means to her, stated, "Mathematics teaching to me is an approach to problem-solving in different circumstances. Creating curiosity is very important; what is madam going to do today in class?" She further remarked, "These days students and their parents focus on getting 100% on the exams, and parents' expectations from the school and its status influence how they structure the curriculum and the instruction." Komal's responses suggest the influence of school culture and cultural factors on her competency to enculturate her students mathematically. Sam expressed similar views when asked about the goals for his students. He remarked:

> I think many math teachers and department heads think that we should prepare mathematicians. Whereas, we have to realize that very few or any of the students in a given year major or minor in math or even major in something math intensive such as physics. We have to gear our instruction to the vast majority of the students who will use math tangentially. They might use statistics, psychology or might use geometry in art or engineering. So my vision for my students is that they are able to do everyday math, understand enough math so that they are able to apply math in the area they are willing to pursue; and also socially they feel better about themselves in social situations. They feel comfortable about it. Their parents say all the time when we meet them, "I was never good at math." I don't want them to say that, to their children. Ultimately, they should not feel intimidated with math.

Sam's responses suggested his vision of making his students function socially and mathematically, and his vision seems to be aligned with a sociocultural perspective of teaching mathematics. Similar thoughts were expressed by other participants. Participants' responses suggested their concern for developing students' ability to live competently and confidently in a world that relies heavily on proficiency in mathematics and their *sensitivity* to the teaching context.

Suman, another teacher from India, reported during the interview that she tries to make sure that students are able to understand the concept, and that they develop an appreciation of the subject. To achieve this goal, Suman reported that she makes them feel relaxed in the class. One example of this is that instead of calling on students randomly to answer her questions, she asks them one by one so that every student is able to participate. Suman further explained that she tries to involve students in group discussions to encourage them to discuss different ways of solving the problems and exploring the concepts. From a sociocultural perspective, it is well recognized that fear inhibits learning and learning is more effective when it involves a student's emotions as well as intellect. Suman also remarked that her teaching is influenced by the work culture at the school: "During the last ten years, I have worked with different supervisors; they all have been very organized. The principal's talks have always been very inspiring. She expects us to give 100%." Suman, when asked what mathematics teaching means to her, stated:

My teaching is not influenced by the way, I was taught. It was chalk and talk method followed by drill and practice. Students do not learn that way these days. We never questioned adults. Students have become more vocal these days. You have to connect the content with their lives…I involve them in games and solving puzzles. Our system depends on mental calculations and estimation. They have to be very thorough with basic facts. Sometimes, I have to remark: a vegetable vendor calculates faster than you because he has become habitual. For him, calculation is a habit of mind. You need to focus and take mathematics as a life tool. You have to connect to these kids and respect their individual differences.

Suman revealed her awareness of sociocultural aspects of teaching when she pointed out during the interview that the multifarious nature of Indian society teaches them to respect the differences that exist in the society and the world, and thus making them ready to be a global citizen. According to Suman, a good teacher has to be a global citizen so that he/she can inculcate in his/her students respect for all cultures and teach them to embrace the differences between them. Her awareness about sociocultural aspects of teaching and learning as well as her acknowledgment of the need to give due consideration to the affective domain was also clear from her statement:

If one comes to Indian schools, one sees students of different religions sitting together, studying together, celebrating festivals together, and sharing their problems. They realize that despite their superficial differences, their hopes, dreams, and fears are the same. One also sees teachers coping with large numbers of students with relative ease. A good balance between strict discipline and complete freedom is maintained by the teachers.

This emphasis on sociocultural aspects of learning has become the focus of much research in the context of the achievement gap in mathematics and science between various student populations in the U.S. Researchers (Ladson-Billings, 2001) advocate for inculcating multicultural competence as a part of educators' repertoire in the wake of diverse student populations in the schools.

Understanding of contextual factors
The participants' responses, planning of their instruction, and classroom behaviors suggest that analysis of their teaching context (a dimension of teacher's cognition) is an interplay among cognitive process of analysis of self-efficacy, diagnosing and understanding students' abilities and academic backgrounds, and reflection of their actions on student outcomes. Thus, this competency encompasses developing an understanding and awareness of various contextual factors: (a) understanding of the curriculum; (b) understanding of students' academic and cultural backgrounds (teachers in the Indian setting demonstrated an awareness of students' cultural backgrounds); (c) school culture characterized by predetermined vision and expectations of standards of mathematics teaching and performance; and (d) their reflection on students' outcomes. These teachers added experiential knowledge to their existing knowledge base to improve their context-specific conceptions, and then used this knowledge in decision-making.

Maintaining a balance between competing priorities
Data analysis revealed that, based on their prior experiences (as students, in teacher preparation programs and in prior teaching experiences) and due to the constraints posed by the contextual factors, teachers prioritized their teaching tasks and developed heuristics to analyze their teaching contexts and regulate their teaching. Additionally, principal and department heads as part of the immediate teaching context influenced this balance of competing priorities, leading to the creation of social norms that dictate performance expectations

from the students as well as teachers. It is argued that the extent to which this balance is maintained and nurtured by the teacher determines the patterns of interaction and the mathematics learned in the class. Attaining a good balance between emotional intelligence and interpersonal intelligence plays an important role in this phase as research in Neurology suggests that emotions are central to all judgment and decision making (Demasio, 1994).

Attaining a good balance between emotional and interpersonal intelligence

During the pre- and post-observation interviews, all the teacher participants expressed their satisfaction with their work environments by expressing positive emotions using words and phrases in their statements during. (e.g., "happy ever since," "like working here," "feedback I get from all around," "principal has confidence in my teaching," etc.). Satisfaction with the work environment generates positive emotions thereby affecting teachers' attitudes toward their work. Thus, acquiring emotional intelligence in the context of their teaching is a part of a teacher's cognition in addition to interpersonal intelligence (Gardner, 1993). Responses from the participants suggest that this knowledge base grows with teachers' reflection and observation of students' actions in the class. Some of the behaviors associated with the phases of the BPSP were common across more than one phase suggesting *the interdependence* and *cyclical nature* of these phases.

Conditions influencing Teachers' Engagement with the Development and Display of Professional Competence

Six conditions were determined to influence the progression through the phases of BPSP. These conditions were (1) subject matter knowledge at the entry level in the teaching profession, (2) participants' prior knowledge about students' academic backgrounds (students' abilities and misconceptions about various topics), (3) teachers' subject-specific and context-specific conceptions, (4) self-regulation or cognitive process of analysis of self-efficacy, (5) working hard, and (6) constraints and affordances provided by the teaching contexts. The first two conditions are constituent of teachers' cognition. Teachers with undergraduate majors in mathematics and advanced degrees in mathematics were observed to involve students in mathematically rich classroom dis-

course. The students in the classrooms of these teachers were observed to emulate their teachers. Participants with strong content knowledge provided more powerful conceptual explanations and demonstrated rich subject-specific conceptions. Competence in quality questioning and the ability to orchestrate mathematical discourse was demonstrated by the participants to varying degrees. All the participants asked a mix of higher order and lower order questions during their lessons. An excerpt from a teaching episode from Sam's (see Figure 2a) and Suman's (see Figure 2b) classes given below suggests their competency in quality questioning and scaffolding.

Detailed discussion of vignettes from the lessons observed is beyond the scope of this paper. Teacher participants demonstrated a mix of teacher-centered and student-centered pedagogy tailored to students' needs, and performance expectations of their work environments. Participants' responses revealed good knowledge of their students' abilities and academic needs.

The findings of this study suggest that teachers' knowledge of mathematics affects their *planning* and decision-making and influences their ability to diagnose and handle students' misconceptions and mistakes. These teachers' professed articulation of their goals of successful student outcomes and strategies used to realize those outcomes, perceptions of school-wide support, and high expectations of performance are suggestive of their engagement in the cognitive process of analysis of self-efficacy. Significant differences were noticed in participants' awareness of gaps in their knowledge (a dimension of cognitive process of analysis of self-efficacy). This dimension of teacher cognition was revealed during class observations and likely influenced teachers' handling of students' mistakes and sequential planning of lessons. *Planning* provides teachers with mechanisms to maintain the balance between competing priorities and allows them to focus on students' needs, students' engagement, and classroom management.

Working hard is the process whereby teachers put in extra effort as a way of displaying their satisfaction with their work environment. The process of working hard is suggestive of participants' display of positive emotions that guided their goal-directed behavior of performing to their maximum potential to realize student outcomes. The participants' responses to interview questions indicated the constraints and affordances of the teaching context influencing their planning and instruction.

Excerpts from Sam's Class

Sam, after correcting the problem on the drill, moved to the back of the classroom to check the homework problem based on SAS postulate. A student posted a column proof for the problem. In the proof, statements were correct but reasons for two of the steps were not correct. Sam asked:

Sam: What is segment bisector?

(A Student gave an incomplete definition. Sam wrote the correct definition on the board).

Sam: Ok, what did we not mention when we talk about this definition here?
Student: Midpoint.

Sam: You are close, that's one thing, and we need different reason. What else did we not mention?
Student: Congruent (response came from the same student who posted answer for the problems on the drill and homework during the lessons observed).

Sam: Excellent; that means what? We can't prove segments congruent by definition of segment bisector. You have to add a step before this one.
Student: B is the midpoint.

Sam: You never just say that B is the midpoint. Say B is the midpoint of what?
Student: Segments JL and HC. (Sam writing the step that B is the midpoint of JL & HC on the board)

Sam: Now we have the definition. What else is needed to say that they are congruent?
Student: Midpoint Theorem

Sam: Exactly, so we use the definition of midpoint and then take another step to say that segments are congruent by using the theorem.

An example of lower order questions from Sam's Class:

- **Sam:** Is K the midpoint of GK? (See the screen shot of the clip on the right)

- (A student wrote K as midpoint of GK and MR)
- **Student:** No, only MR.
- **SAM:** K is the midpoint of the segment bisected and not the segment bisecting.

- **Sam:** So far, how many methods do we have for proving triangles congruent?
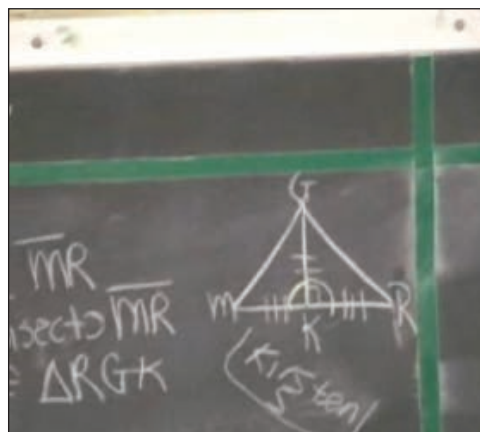- **Students in chorus:** Three
- **Sam:** Name them.

Figure 2a. Excerpt from Sam's geometry class.

Figure 2b. Excerpt from Suman's geometry class

Constraints and affordances of the teaching context

All the participants cited time constraints and excellent teaching expectations from the administration. During the post-observation interview, Lena remarked, "We have to finish the syllabus for the state exam, and some students struggle with the basic mathematics, and they don't show up for the coach class." Lena further stated, "In addition, periods are only 47 minutes long, so group activities are not possible most of the time. But I try to facilitate peer tutoring." She added, "Sometimes, I have to deviate from my beliefs when it is required to follow the district curriculum guidelines or when some students' plans require me to deviate."

Sam also pointed out the time constraints to finish the syllabus and get the students ready for the AP exams. He noted that it is up to the student to come to coach class:

It is not easy during the class to get to the students individually and recognize their difficulties. In my old school, classes were 90 minutes long. Thus, there was ample time to observe students while they were practicing. That was better in a lot of ways. I wish, I could do more, but after 47 minutes, they have to go to the next class.

Sam also shared:

In mathematics we can pretty much say that your ability level is here; you probably will not succeed here. Learning mathematics is more of ability than effort. Unfortunately, when the administration comes to scheduling students, they often ignore such past experiences and recommendations from teachers. They just put students where it is convenient for them or for students. Some students are struggling because of that.

Here, Sam hints at the *constraints and affordances of the teaching context* and the conflict arising out of the interplay between students' academic backgrounds in mathematics and administrative routines.

Teachers in the Indian setting also pointed out the time constraints to finish the prescribed curriculum by the set deadlines to allow for sufficient review and practice for the board examinations. Suman remarked that the emphasis of the mathematics program at the school was to cover more content and challenge students by doing various types of problems expected to be on the exams. Given the short periods, she encouraged students to come before and after school for coaching.

Teachers in this study used their *understanding of the teaching context* (students as context, the subject matter as context, and school structure and culture as context) to shape their interactions with the students. Participants described student readiness and school-wide support as important contextual factors influencing their practice and competence. The influence of *school as context* was seen on teacher's competence (which includes competence in student management and engagement), sense of self as a teacher, self-directed improvement in practice through monitoring of students' outcomes, perception of school-wide or departmental support, and perceptions of high expectations of performance due to the high-achieving status of the school. The influence of *student as context* was seen in teacher participants' professional competence as alertness to students' behaviors and needs during the class. The *constraints and affordances of the teaching context* lead teachers to develop *"coping strategies"* (Hargreaves, 1984, p. 75) *to maintain a balance between competing priorities* (e.g., students' needs, curriculum deadlines, assessment requirements, etc.). As a result of data analysis and reflection, the coping strategies used by the participants were revealed. These strategies were *working hard, seeking knowledge to modify practice, using interpersonal and emotional intelligence* to handle social interactions (with parents, colleagues, students, and administrators) and cognitive conflicts arising in their classroom and work environments, *reflecting* on student outcomes and reflection-in-action and on action (Schon, 1987), and *collaborating* with the department heads and the supervisors.

An Interpretive Summary of the Emergent Themes Relevant to Observed Teaching Practices

In order to resolve the main concern of teaching mathematics effectively to realize successful student outcomes, participants progressed through *five phases*. These phases, along with the conditions # 1-5, represent dimensions of cognition (see Figure 1). The data analysis revealed that teachers' cognitions and conceptions were influenced by *six contextual factors:* high performance expectations of the department head and the principal, clear articulation of expectations and curriculum guidelines, good work atmosphere, availability of curriculum resources, school vision and structure, and students' readiness by and large. The *context consequences* were the outcomes of the actions and strategies used by the teachers during their engagement with the process of developing professional competence. The five consequences were: (1) impact on teacher efficacy (which includes efficacy in student management and engagement), (2) sense of self as a teacher, (3) self-directed improvement in practice through monitoring of students' outcomes, (4) perception of school-wide or departmental support, and (5) perceptions of high expectations of performance due to the high-achieving status of the school. The following section provides an interpretive summary of the emergent themes relevant to observed teaching practices and illuminates the interplay of various components of the professional knowledge base of a competent mathematics teacher as interplay of teachers' conceptions, cognition, and the context in which they enacted their practice.

Teachers' Conceptions

Teachers' conceptions of the nature of mathematics are defined as their "conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline" (Thompson, 1992, p. 132). Research in the field of cognitive neuroscience suggests that cognition is tightly linked to perception and action. Many researchers argue that beliefs are instrumental in defining tasks and selecting the cognitive tools to execute such tasks; hence they play an important role in defining behavior and organizing knowledge (Kagan,

1992; Nespor, 1987). Pintrich (1990) asserts that both "knowledge and beliefs…influence a wide variety of cognitive processes including memory, comprehension, deduction and induction, problem representation, and problem solution" (p. 836). This study provides additional support for the assertion from previous research (Pepin, 1999; Thompson, 1992) that epistemological conceptions about mathematics and its teaching influence the way mathematics is represented in the class, and that teachers' conceptions influence, and are influenced by, teachers' cognitions. For example, Sumit saw mathematics as a hierarchical and logical subject, and emphasized conceptual understanding during his instruction. He purposefully selected relevant examples in a logical sequence. Sumit also demonstrated powerful representations of mathematical concepts indicating strong underpinnings of the content. Thus, professional competence in the teaching of mathematics can be seen as an interplay between teachers' cognition and conceptions.

The data suggested a high degree of consistency observed in professed and enacted conceptions about (a) social norms influencing participants' teaching practice, and (b) nature of mathematics and its teaching for most of the participants. The inconsistency between professed conceptions and actions of some of the teachers seem to arise from the differences in the nature of teachers' cognitions. The factors that might have caused the inconsistency seems to be (a) the depth of conceptual and procedural knowledge of mathematics, (b) ability of the teacher to reflect and analyze the teaching context, (c) the constraints and affordances of the teaching context: students as context (their academic backgrounds and dispositions towards mathematics), (d) teachers' subject-specific and context-specific conceptions, (e) school structure and culture, and (f) teachers' personal epistemology of teaching mathematics developed as a result of their lived experiences shaped by the above factors (Ahuja, 2018a). The findings of this study suggest that teachers enter the teaching profession with some formal knowledge, and then gain knowledge from the teaching contexts based on the school culture and structure. Since teachers' acquire their subject-specific and context-specific conceptions through experience (Calderhead, 1996), the process of acquiring conceptions evolves based on teachers' lived experiences. The culturalist view of cognition emphasizes the construction of knowledge as a cultural process derived from the cultural context of the people. Thus, it is appropriate to conclude that teachers' cognitions and conceptions are informed by cultural traditions.

Since teachers' conceptions are manifested in teacher behaviors and conceptions are culturally determined, it is appropriate to view the effect of culture as a context for professional competence in any cross-cultural investigation of teaching. Once it is recognized that teachers' lived experiences are situated in social settings, it is easy to understand that professional competence is social and developmental, and progression through various phases of development, and display of professional competence is an interplay among teachers' lived experiences, decision-making processes, and cultural traditions existent in teaching contexts. Thus, inter- and intra-cultural differences in the professional competence of teachers can be attributed to the utilization and access to intellectual and cultural resources, and the meanings teachers derive based on their discipline-specific conceptions. Professional competence is intimately related to the personal styles of the teacher. That is, a particular teacher will always bring to their professional practice their own personal characteristics, which arise from and relate to their past and current experiences and cultural traditions in a unique way. Due to the dynamic nature of the culture and evolving lived experiences, it is argued that this process of evolution of teachers' conceptions is also constructive, integrative, and fallible.

Teachers' Cognition
This study suggests that teachers' cognitions are the driving force behind their actions and pedagogical articulation. The data analyses revealed that teachers' cognition is everything that teachers think, experience, and acquire in order to teach effectively. It includes dimensions such as knowledge about the content, planning of lessons, understanding of students, using tacit knowledge and learning from personal experience, updating existing knowledge, asking questions to self, understanding context (student, teaching, and subject), handling cognitive conflict, setting goals, thinking through lesson-specific and/or teaching situation-specific situations, assessing students, interacting with students and school personnel, problem-solving, analyzing teaching tasks and contexts, understanding emotional intelligence (a combination of interpersonal and social intelligence), and communicating effectively. Affective dimensions of cognition include responsibility, patience, professional ethics, and flexibility. For this study, the definition of conceptions accords with that of Thompson (1992), teachers' cognition also includes teachers' intuitions, skills, values, and feelings as well as attitude towards their work. Reflection, a cognitive skill is the ability to give thoughtful consideration to the knowledge used when one spontaneously responds to a situation (Schon,

1987). Reflection can be either reflection-in-action (Schon, 1987) or reflection-after-action with a sole purpose of improving one's practice by highlighting the deficiencies in one's knowledge and skill repertoire.

The process of developing self-awareness of gaps in one's knowledge and making statements about one's own process of problem-solving and achieving means to address this gap in knowledge was revealed during participants' responses during the interviews. This cognitive process was called "analysis of self-efficacy" and was reported as one of the competencies demonstrated by the participants during instruction. This study found that the development of professional competence is characterized by constant engagement in reflection and in responding to challenges of the work context, thereby engaging in the kind of learning that extends one's competence. Thus, professional competence *entails the process of activating* various components of professional knowledge base in a variety of problematic situations. The emergent definition of cognition extends the existent definition of cognition in the literature. The various dimensions of teachers' cognition are experience based and are context dependent. Furthermore, this process is amenable to analysis and modification. The foregoing discussion suggests that due to the cyclical nature of various phases of the process of becoming professionally competent, cognition is a constructive, integrative, and fallible process. The foregoing discussion again suggests that professional competence is an interplay among teachers' conceptions, cognition, and the context.

Teaching Context

Davis and Sumara (1997) view cognition as a process of organizing and reorganizing one's own subjective world of experience which involves revision, reinterpretation of past and present, and projected actions and conceptions. This view underscores the situatedness and context-dependent nature of cognition. This view of cognition in the context of mathematics teaching and learning is referred to as enacted cognition or embodied cognition (Nunez et al., 1999) and is in tandem with the sociocognitive view of mathematics learning. It suggests that both mathematics and competence in its teaching are products shaped by human brains, societies, and culture. Schools, as institutional cultures, aim to create ideal futures for students and teachers (Grossman, Smagorinsky, & Valencia, 1999). Students and teachers aspire to culturally defined futures that motivate their activities, and the ways in which they mediate one another's progress toward those goals (Joseph, Bravmann, Windschitl, Mikel, & Green, 2000). Social teaching norms and immediate classroom situation play key roles in influencing mathematics teaching practice (Raymond, 1997; Yackel & Cobb, 1996). Thus, context is crucial in understanding the experiences of both the students and teachers in mathematics classrooms. McLeod (2001) views teaching as "a social act defined as much by its sociocultural, biographical, and historical contexts as by its structural context" (p.1).

The professional competency: *understanding and the analysis of teaching context* as a constituent of teacher cognition has already been explained in the foregoing discussion. An important common feature of mathematics teaching as observed in the two research settings is the existence of a supportive learning environment characterized by student engagement and cognitively demanding conceptual mathematical tasks. The study found that the contextual factors that facilitated teachers to develop personal mathematical epistemologies and a sense of professional competence were high expectations of the department head and the principal, good work atmosphere, clear articulation of curriculum guidelines, availability of curriculum resources, and high levels of academic press. Thus, *a high level of student engagement* was both a *product of and a stimulus to* the teacher's professional competence. McLaughlin (1993) argued that students help to define the context of teacher work and teachers' perceptions of their students influenced how teachers approached their work. Additionally, a school's mission, organizational structure, policies, procedures, and "patterns of communication" (McLaughlin, 1993, p. 89) also shape teachers' goals of teaching leading to change in their cognitions and conceptions. Hence, the *context* constantly constrains teachers' cognitions and conceptions.

Thus, the core category which appears to account for all of the processes and conditions through which professional competence was displayed is the notion of the *interplay among conceptions, cognition, and the context*. The emergent theory predicts the ways teachers would operate in order to resolve their main concern of professional competence in the teaching of mathematics by activating all the resources available in their professional knowledge base to handle the actual teaching situations which at times can be problematic, and this process is an ongoing pursuit. Having said that, it is appropriate to conclude that professional competence is embedded in practice and is transformed through deliberate goal-directed behavior. This leads to the conclusion that the nature of professional competence is constructive, integrative, and fallible.

Evaluation of GT

The theory is a good and trustworthy theory if it meets the four criteria: fit (validity), work (generality), relevance (understanding), and modifiability (control).

Fit

Since categories are generated directly from the data, the criterion of fit is automatically met (Glaser, 1978). The procedures followed for data analysis were documented. Description and conceptualization of the theory have been explained earlier in this paper. Additionally, to provide a "true description of a given reality" (Janesick, 1998, p. 119) direct quotes from the interviews and class observations along with shots of video clips were presented in the portrayal of mathematics teaching from the two research settings. The compliance with the criterion of fit was ensured by closely following the procedures such as selective coding, theoretical sampling, constant comparison across all the data sources, and memo sorting.

Work

A grounded theory works when it explains the major variations in the way participants respond when dealing with their main concerns. By explaining how the teachers' main concern was resolved and by presenting the interpretation of the emergent theory and its contrast with the extant literature, the researcher has tried to show that it works.

Relevance

"If it fits and works the grounded theory has achieved relevance" (Glaser 1992, p. 15). Relevance is ensuring that the theory "deals with the main concern of the participants" (Glaser, 2001, p. 18). This means that it will offer explanations of the main concern in the substantive area. This study attempted to develop an understanding of how quality in teaching and learning of mathematics is influenced by the context in which this learning occurs. During the data collection and analysis phase, the researcher questioned herself: What is really going on that is important to the participants in the substantive area and in the context of their work that has an impact on their actions.? The BPSP of interplay among conceptions, cognition, and the context has specific relevance to the participants operating within the context of the high-achieving school environment where teacher efforts were supplemented by school-wide efforts. The school culture characterized by set vision and expectations of standards of mathematics teaching and performance, and their reflection on students' outcomes constantly motivated teachers to enhance their professional competence. The framework as envisaged by the theory of pro-

fessional competence enables predictions of the way teachers would most likely engage in order to meet the performance standards of their teaching contexts. Thus, by staying close to the data, avoiding preconceived theories and generating concepts that were germane to the participants' basic psychological problem of teaching mathematics effectively, it was ensured that the grounded theory of professional competence: interplay of conceptions, cognition, and context ensures compliance with the criterion of relevance.

Modifiability

Within the context of this study, the theory was constantly modified through constant comparison across all the elements of data corpus. The technique of constant comparison is both inductive and deductive. Care was taken to return to the data to substantiate the emerging themes and categories, and this process continued until theoretical saturation.

Summary

The Theory of Professional Competence: Interplay of Conceptions, Cognition, and the Context

The substantive theory that emerged from the data suggests that *professional competence in the teaching of mathematics is a complex interplay of teacher's conceptions, cognition, and the context* (see Figure 3). These three components chase each other in a cyclical process through intervening conditional and consequential variables as evident from figures 1 and 3. The study found that pro-
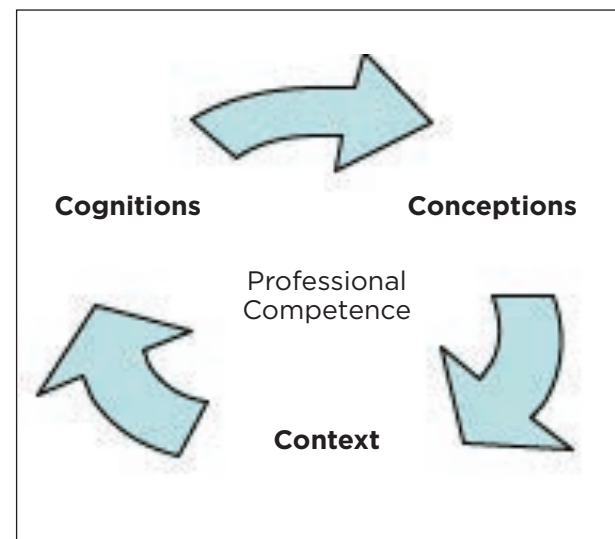

Figure 3. Ahuja Theory of Professional Competence: The Interplay of Conceptions, Cognition, and the Context.

fessional competence can be defined in terms of the competencies and/or qualities, and as a process associated with the individual and the context. An important conclusion that can be drawn from this study is that professional competence is the possession of a set of components of the professional knowledge base and the process of activation of these components in the context of actual teaching situations in response to contextual factors. These components chase each other in a cyclical manner emphasizing the role of cultural, historical, and political influences on the professional competencies of teachers as these influences are deeply rooted in the cultural traditions of countries. The content knowledge combined with experiential knowledge, responsibility, and flexibility emerged as important factors influencing teacher's competence in mathematical discourse, organization of instruction, representation of content, persistence, and the management of class time. The findings of this study suggest that due to the constructive, integrative, and fallible nature of teacher cognition, conceptions, and the context, the nature of professional competence is dynamic, constructive, integrative, and fallible. The idea of professional competence as the result of an interplay among these three components suggests that rather than being a static feature, professional competence is a dynamic one manifesting itself in a variety of behaviors even in the same teacher. It may thus be defined as the possession of a set of components of professional knowledge base and the activation of these components in the context of actual teaching situations.

Limitations

This study is limited to the analysis of mathematics teaching practices in the selected two high-achieving high schools located in Delhi, India, and Maryland in the United States. Limited resources and funding restricted sophisticated analyses of videotaped lessons. The teacher participants selected from the two schools were not representative of the national sample. A small sample with quality interviews and in-depth analysis can produce significant findings (Charmaz, 2014). Findings may be applied in similar contexts. Hence, the goal of this study was not to formulate generalizations, but rather to generate a substantive theory using categories, the relationships between them, and their properties.

Concluding Statement

The author believes that different observers may understand and interpret data differently based on their professional knowledge and experience. Thus, rich portrayals of settings and mathematics teaching (which included direct quotes from the interviews and photos from the videos, examples of teaching episodes, mathematical discourse, student-teacher, student-student interactions) were presented in the detailed description of the findings (discussed elsewhere) to enable the reader to evaluate the authenticity of the findings thereby reducing the effect of researcher bias. In addition, multiple methods of data collection, the constant comparative technique of data analysis, and peer debriefing were employed to ensure trustworthiness of the findings. The teacher participants interpreted their conception of self in the context of the high-achieving status of the school. A learning culture characterized by high performance expectations existed in both the settings where teacher efforts were supplemented by school-wide effort. The knowledge domain of professionally competent mathematics teachers' cognition is vast and is a complex combination of various components of teacher's professional knowledge base (Ahuja, 2013). The teacher's classroom practices can then be seen, in part, as an attempt to find a balance between competing priorities involving curriculum, content, teaching goals, teaching context, social and pedagogical climate, student-teacher relationships, instructional strategies, cognitive conflicts arising out of classroom situations due to misconceptions and prior dispositions held by the students about mathematics, time constraints, and performance. Professional competence lies in responding to these challenges intuitively. The data collected from the participants suggests that competent teachers respond to these challenges spontaneously and this spontaneity develops with experience and is founded on a strong grasp of subject matter knowledge and mathematics-specific pedagogical knowledge.

Teachers' conceptions, cognitions, and the constraints and affordances of the teaching context in the work environments of the teachers are so intertwined that it is almost impossible to separate one from the other while studying and developing the notion of a construct of professional competence in teaching of mathematics, as complex as the discipline of mathematics itself. It is reiterated that the purpose of data analysis in grounded the-

ory research is to develop a core category that explains and characterizes the process in which the participants engage to resolve their main concern. The goal is conceptualization and abstraction rather than description (Glaser, 2001). The theory of professional competence developed because of this study offers a tool to teachers, educators, administrators, and researchers to evaluate and understand the interaction between various parts of the emergent theory. The theory can be tested across various teaching contexts. A point to note here is that certain sub-concepts and subcategories in the theory are not unique to the substantive area of mathematics education.

The findings of this study suggest that teacher's cognitions and conceptions are contextualized. Due to the context-dependent nature of professional competence, holistic assessment (Eraut, 1994) of teachers' competence should be the norm. This holistic assessment should be emphasized during the training of administrators, supervisors, and department heads. Realizing the fact that it is a teacher's responsibility to enculturate his/her students mathematically, it is imperative for other stakeholders in public education to facilitate this process of enculturation by providing a favorable context, climate, and resources. Glaser (1992) states, "the theory provides a conceptual approach to action, changes, and accesses into the substantive area" (p. 15). Due to limited resources, the sample was delimited. The study may be replicated at the elementary and middle school levels. The emergent theory of professional competence has the potential to rise to a formal theory by carrying out comparisons of emergent theories from diverse teaching contexts and other fields.

References

Ahuja, R. (2013). Professional Knowledge Base of a Competent Secondary Mathematics Teacher. *International Journal of Education and Culture, 2*(1), 51–72.

Ahuja, R. (2018a). Conceptions of Effective Mathematics Teaching Across Cultures: Insights from Some Classrooms of India and the United States. *International Journal of Education and Culture, 7*(3–4), 35–52.

Ahuja, R. (2018b). Professional Competencies for Effective Mathematics Instruction: Cross-cultural Findings from Selected Secondary Classrooms of India and the United States. Manuscript under review.

Bogdan, R. C., & Biklen, S. K. (1992). *Qualitative research for education: An introduction to theory and methods.* Boston, MA: Allyn and Bacon.

Calderhead, J. (1996). Teachers: Beliefs and knowledge. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of educational psychology* (pp. 709–725). New York: Macmillan.

Clarke, D.J., Emanuelsson, J., Jablonka, E., & Mok, I.A.C. (Eds.). (2006). *Making connections: Comparing mathematics classrooms around the world.* Rotterdam: Sense Publishers.

Charmaz, K. (2014). *Constructing grounded theory* (2nd ed.). London: Sage.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions.* Thousand Oaks, CA, US: Sage Publications, Inc.

Darling-Hammond, L. (2000). Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives, 8*(1). Retrieved November 20, 2005 from http://epaa.asu.edu/epaa/v8n1/

Davis, B., & Sumara, D. (1997). Cognition, complexity, and teacher education. *Harvard Educational Review, 67*(1), 105–125.

El Hussein, M. T., Kennedy, A., & Oliver, B. (2017). Grounded theory and the conundrum of literature review: Framework for novice researchers. *The Qualitative Report, 22*(4), 1198–1210. Retrieved from http://nsuworks.nova.edu/tqr/vol22/iss4/16

Eraut, M. R. (1994). *Developing professional knowledge and competence.* London: Falmer Press.

Gibson, B. (2007). Accommodating critical theory. In A. Bryant & K. Charmaz (Eds.), *The Sage handbook of grounded theory* (pp. 1–28). London, UK: Sage.

Glaser, B. (1978). *Theoretical sensitivity.* Mill Valley, CA: Sociology Press.

Glaser, B. (1992). *Emergence v Forcing Basics of Grounded Theory Analysis.* Mill Valley, CA: Sociology Press.

Glaser, B. (2001). *The grounded theory perspective: Conceptualization contrasted with description.* Mill Valley, CA: Sociology Press.

Glaser, B. (2005). *The grounded theory perspective III: Theoretical coding.* Mill Valley, CA: Sociology Press.

Glaser, B. G., & Strauss, A. L. (1967). *The discovery of grounded theory: Strategies for qualitative research.* Chicago: Aldine.

Grossman, P. L., Smagorinsky, P., & Valencia, S. (1999). Appropriating tools for teaching English: A theoretical framework for research on learning to teach. *American Journal of Education, 108*(1), 1–29.

Hargreaves, A. (1984). The significance of classroom strategies. In A. Hargreaves & P. Woods (Eds.), *Classrooms and staff rooms: The sociology of teachers and teaching* (pp. 64–85). Milton Keynes: Open University Press.

Hutchinson, S.A. (1993). Grounded theory: The method. In P.L. Munhall & C.O. Boyd (Eds.), *Nursing research: A qualitative perspective* (pp. 180–212). New York: National League for Nursing Press.

Joseph, P., Bravmann, S., Windschitl, M., Mikel, E., & Green, N. (2000). *Cultures of Curriculum.* Mahwah, NJ: Lawrence Erlbaum Associates.

Kagan, D. M. (1990). Ways of evaluating teacher cognition: Inferences concerning the Goldilocks principle. *Review of Educational Research, 60*(3), 419–469.

Ladson-Billings, G. (2001). *Crossing over to Cannan: The journey of new teachers in diverse classrooms.* San Francisco: Jossey-Bass.

LeCompte, M. D., & Preissle, J. (1993). *Ethnography and qualitative design in educational research.* San Diego, CA: Academic Press.

McLaughlin, M. W. (1993). What matters most in teachers' workplace context. In J. W. Little & M. W. McLaughlin (Eds.), *Teachers' work: Individuals, colleagues, and contexts* (pp. 79–103). New York: Teachers College Press.

McLeod, J. H. (2001, November). Teacher's working knowledge: The value of lived experience. Paper presented at the Seventh International Literacy and Education Research Network (LERN) Conference on Learning, RMIT University, Melbourne, 5–9 July 2000. Retrieved April 27, 2007 from http://ultibase.rmit.edu.au/ articles/ nov01/mcleod1.htm.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19,* 317–328.

Nunez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as grounding for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39,* 45–65.

Palmer, D. J., Stough, L. M., Burdenski, T. K. Jr., & Gonzales, M. (2005). Identifying teacher expertise: An examination of researchers' decision making. *Educational Psychologist, 40*(1), 13–25.

Pintrich, P. R. (1990). Implications of psychological research on student learning and college teaching for teacher education. In W.R. Houston (Ed.) *Handbook for research on teacher education.* (pp. 826–857). New York: Macmillan.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practices. *Journal for Research in Mathematics Education, 28*(5), 550–576.

Rhodes, L. K., & Bellamy, G. T. (1999). Choices and consequences in the renewal of teacher education. *Journal of Teacher Education, 50,* 17–26.

Schon, D. A. (1987). Educating the reflective practitioner: Toward a new design for teaching and learning in the professions. San Francisco: Jossey-Bass.

Stebbins, R. A. (2001). *Exploratory research in the social sciences.* Qualitative research methods (Series 48). Thousand Oaks, CA: Sage.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A Synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning,* (pp. 127–146). New York: Macmillan Publishing Company.

Torraco, R. (2005). Writing integrative literature reviews: Guidelines and examples. *Human Resource Development Review, 4*(3), 356–367.

Wenglinsky, H. (2002). How schools matter: The link between teacher classroom practices and student academic performance. *Education Policy Analysis Archives, 10*(12).

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458–477.