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Making Connections in Mathematics Education

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TABLE OF CONTENTS

PREFACE

- v Sarah Nelson, Teachers College, Columbia University
Dyanne Baptiste, Teachers College, Columbia University

ARTICLES

- 1 **Connecting the Tangent Function to Cardinality:
A Method for Introducing Set Theory to High
School Students**
Steve Deihl, East-West School of International Studies
Mara P. Markinson, Queens College,
City University of New York
- 11 **After Presenting Multiple Solution Strategies,
What's Next? Examining the Mathematical
Connections Made by Preservice Teachers**
Indira Gil, Florida International University
Laura Zamudio-Orozco, Florida International University
Barbara King, Florida International University
- 23 **Doctoral Preparation in Mathematics Education:
Time for Research and a Widespread
Professional Conversation**
Robert Reys, University of Missouri
Barbara Reys, University of Missouri
Jeffrey C. Shih, University of Nevada, Las Vegas
- 29 **Quantitative Literacy: Alternative Pathway for
College Developmental Mathematics Students**
Michael George, Borough of Manhattan Community
College, City University of New York
Yevgeniy Milman, Borough of Manhattan Community
College, City University of New York
- 39 **Mentoring Future Mathematics Teachers: Lessons
Learned from Four Mentoring Partnerships**
Angie Hodge, Northern Arizona University
Janice Rech, University of Nebraska at Omaha
Michael Matthews, University of Nebraska at Omaha
Kelly Gomez Johnson, University of Nebraska at Omaha
Paula Jakopovic, University of Nebraska at Omaha

Doctoral Preparation in Mathematics Education — Time for Research and a Widespread Professional Conversation

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ABSTRACT Doctoral programs in mathematics education were established more than a century ago in the United States. From 2010-2014 over 120 different institutions graduated at least one doctorate in mathematics education. There has been limited research reported on the nature of doctoral programs in mathematics education and/or their doctoral graduates. This paper provides a synthesis of research findings related to doctoral preparation in mathematics education that is accompanied by a reflection on the findings and suggestions for future research. The intent of our paper is to provide a rallying call for more widespread and coordinated research on doctoral programs in mathematics education in order to strengthen the quality of doctoral preparation for the next generation of mathematics educators.

KEYWORDS *accreditation, core knowledge, doctoral preparation, mathematics education*

Doctoral programs in mathematics education were established more than a century ago but the majority of programs began to evolve about 60 years ago (Donoghue, 2001). Today, U.S. programs collectively graduate about 130 students each year (Reys & Reys, 2016). These graduates have an important and widespread influence on the field as they do much of the research reported in peer reviewed journals, have major responsibility for preparing the next generation of K-12 teachers, and serve in many leadership roles in professional organizations that represent mathematics educators. Given the influence of graduates of mathematics education doctoral programs and the community of mathematics educators who oversee these programs, it is surprising that ongoing collaboration and/or conversation about the improvement of these programs is rare. This commentary is intended to stimulate more conversation.

About 70 different institutions graduate at least one doctorate in mathematics education annually. However, a graduate of one program is likely to have had very different experiences/preparation than a doctoral student at another institution. Research suggests that even doctoral

programs at peer institutions differ significantly in a range of factors, including the number and type of courses and/or internships required, the nature and quantity of mathematics education courses offered, the extent to which mathematics content is a focus, the number of mathematics education faculty and/or doctoral students in the program, and the length of the program (McIntosh & Crosswhite, 1973; Soonabend, 1981; Reys, Glasgow, Ragan, & Simms, 2001; Reys, Glasgow, Teuscher, & Nevels, 2008). Upon completing a doctorate in mathematics education the graduate chooses among many different career paths, but the majority pursue a career in higher education (Glasgow, 2000).

Do doctoral graduates in mathematics education share a core base of knowledge? If so, what constitutes the core? What characterizes strong doctoral programs in mathematics education? Do certain doctoral programs in mathematics education better serve students with specific career goals (such as preparing them for collegiate teaching of mathematics or to conduct research)? Where are the highly regarded doctoral programs in mathematics education? These questions are rarely addressed or

discussed in the mathematics education community. However, there are data available that sheds light on how current faculty members perceive particularly strong doctoral programs in peer institutions (Reys, et al., 2008; Reys, Reys, Shih, & Safi, 2019).

According to the Carnegie Foundation, one of the purposes of doctoral study is to prepare stewards of the discipline (Golde & Walker, 2006; Reys & Dossey, 2008). In this commentary, we highlight some research from surveys of doctoral graduates in mathematics education and active faculty members in doctoral programs in mathematics education. Our goals are to inform the reader about the current status of doctoral preparation; encourage collaboration, discussion, and regular examination of doctoral programs in mathematics education; and stimulate more research focused on doctoral preparation in mathematics education.

Foundational Research Related to Doctoral Programs

There has been little published research on doctoral preparation in mathematics education (Kilpatrick & Spangler, 2016; Reys, 2017). For example, in a review of five decades of mathematics education research published in the *Journal for Research in Mathematics Education* and *Educational Studies of Mathematics* there was not one citation that mentioned doctoral preparation in mathematics education (Inglis & Foster, 2018). There are several early surveys of doctoral programs available thru ERIC (McIntosh & Crosswhite, 1973; Soonabend, 1981), and then two surveys (Reys et al., 2001; Reys, et al., 2008) that were done in conjunction with national conferences on doctoral programs in mathematics education (Reys & Kilpatrick, 2001; Reys & Dossey, 2008). Since that time there has been a survey of doctoral graduates in mathematics education (Shih, Reys, & Engledowl, 2016; Shih, Reys, Reys, & Engledowl, 2019), a survey of faculty members actively involved in doctoral programs in mathematics education (Reys, et al., 2019) and some periodic reviews of job shortages in the field (Reys, 2002; Reys, Reys, & Estapa, 2013) and production of doctoral graduates (Reys & Reys, 2016). A brief summary (a baker's dozen) of findings from those studies includes:

1. The majority of doctoral programs in mathematics education are in the college/school of education, some institutions offer a doctorate in mathematics education in both the college/school of education and mathematics department, and a few institutions offer their doctorate in mathematics education exclusively in the mathematics department.

2. During the last 50 years, doctoral programs in mathematics education have been established or grown at some institutions and declined or eliminated at others. Overall, the number of different institutions graduating at least one doctorate in mathematics education has increased from about 37 during 1960-1962 to about 130 during 2010-2014.
3. The total number of doctorates in mathematics education averaged about 50 during the 1960s and about 130 during the period 2010-2014. Whereas the majority of doctoral graduates in mathematics education in the 1960s were male (about 80%), currently about two-thirds of the doctoral graduates are female.
4. Most doctoral programs in mathematics education are very small. Few institutions graduate a doctorate in mathematics education annually, and only two institutions (Teachers College and University of Georgia) have averaged at least 5 or more graduates annually since 2000.
5. There was an acute shortage of doctorates in mathematics education for more than two decades (1990-2010). There now seems to be an equilibrium between jobs available in mathematics education and new graduates. The exception is the continued shortage of doctorates in mathematics education in mathematics departments of private and regional institutions.
6. The number of faculty members actively involved in doctoral programs in mathematics education varies from 1 in some institutions to more than 10 in other institutions, with a mode of 4 faculty members. Over one-third of the institutions graduating doctorates in mathematics education have 3 or fewer faculty members in mathematics education.
7. The majority (nearly two-thirds) of faculty members working in doctoral programs are female, and this parallels the percent of new female doctorates in mathematics education that graduated during the last 15 to 20 years.
8. About one-fifth of the faculty members in doctoral programs in mathematics education have no K-12 teaching experience and most have three or fewer years of K-12 teaching experience.
9. The number of graduate level mathematics courses required for completion of a doctorate in mathematics education varies among institutions from none to at least a master level degree in mathematics.
10. The number of graduate courses specifically focused on mathematics education available at institutions offering a doctorate range from 0 to more than 10 with 4 to 6 courses being most typical.

11. Doctoral graduates in mathematics education were generally very positive about their doctoral program. The two areas most often cited in need of strengthening were opportunities to gain first-hand experience in preparing proposals for funding and sustained involvement in active research projects.
12. About one-third of faculty members working in doctoral programs reported regularly soliciting feedback about the program from their graduates, while about 15% reported they do not have a system for seeking feedback. At least one faculty member from over 90% of the institutions graduating the most doctorates in mathematics education was familiar with the *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* (Association of Mathematics Teacher Educators [AMTE], 2003).
13. About one-half of faculty members reported carefully reviewing their doctoral program in mathematics education within the last two years, and about one-quarter of the faculty members indicated they did not know when their doctoral program had been last reviewed.

Some reflections on these findings

More institutions and smaller programs. For the last 50 years there has been an increase in the number of different institutions offering a doctorate in mathematics education. Yet most doctoral programs in mathematics education are small and expensive to operate. That is, many smaller programs cannot afford to offer specific courses unique to mathematics education. Instead, they offer general education courses on curriculum, history, and the psychology of learning that serve graduates from multiple disciplines, and they may offer independent study courses focusing on mathematics education issues (Bay, 2001). These faculty members likely have major responsibilities in their undergraduate programs as well, so it raises questions about the amount of time they have to engage in research, mentor doctoral students through all phases of their program, in addition to other ongoing responsibilities (committee work, scholarship, proposal writing, etc.) that are typically expected of faculty members at doctoral granting institutions (Foley, 2014). This situation of small graduate programs reflecting few students and faculty members prompted

Levine (2007) to comment that we have “too many under resourced doctoral programs for the preparation of education scholars” (p. 60). Levine was particularly critical of the research mentoring and preparation provided to doctoral students in programs with faculty members that were not engaged in scholarly research. Levine went on to “recommend the establishment of high and clearly defined standards for education research and doctoral preparation in research; close doctoral programs that do not meet those standards” (p.75). We concur that limited resources pose severe challenges for establishing and maintaining a high-quality doctoral program in mathematics education.

Intellectual communities. To strengthen doctoral programs, some have called for the establishment of intellectual communities (Golde, 2008; Hiebert, Lambdin, & Williams, 2008). An intellectual community is formed around domains of knowledge that involve active faculty participation and leadership that provide models, mentoring, and apprenticeships for doctoral students. Examples include intellectual communities focused on teaching, curriculum, or equity/diversity as were fostered through the NSF Centers for Learning and Teaching initiative in the first decade of this century¹. Some institutions with a large number of faculty members may have several different intellectual communities operating simultaneously. While the minimum number of people needed to form intellectual communities may vary, it would certainly be a challenge for such communities to exist with three or fewer faculty members.

K-12 teaching experience. Arguments have been made that doctoral candidates in mathematics education should have PreK–12 teaching experience prior to entering a doctoral program (Reys, 2018a; AMTE, 2003). For example, PreK–12 classroom teaching experience provides essential grounding and ensures first-hand experience with PreK–12 students working in school environments. Such PreK–12 teaching experience provides valuable credibility for mathematics educators working with future and in-service teachers.

While the majority of people entering doctoral programs in mathematics education have some K-12 teaching experience, about 20% of current mathematics education faculty in higher education have no K-12 teaching experience. This lack of K-12 teaching experience may put the doctoral student at a disadvantage in

¹ Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM), Center for Mathematics Education of Latinos/as (CEMELA), Center for Mathematics in America’s Cities (MetroMath), Center for Proficiency in Teaching Mathematics (CPTM), Center for the Study of Mathematics Curriculum (CSMC), Diversity In Mathematics Education (DIME), and Mid-Atlantic Center for Mathematics Teaching and Learning (MAC-MTL).

some courses that call upon relating content to teaching and learning mathematics. It may also prove troublesome in establishing credibility with college students in teaching methods courses and perhaps even supervising student teachers in schools. It may also eliminate some job opportunities, as some states, such as Alabama and Minnesota, require faculty members supervising field experience and/or teaching pre-service teachers to have had K-12 teaching experience.

A focus on mathematics content knowledge. What is an appropriate level of mathematics content knowledge for graduates of doctoral programs in mathematics education? This was a major area of attention at the first national conference on doctoral programs in mathematics education (Reys & Kilpatrick, 2001). There was general agreement that doctoral graduates in mathematics education should have foundational knowledge of mathematics, although there was not consensus on the extent or nature of that knowledge. Mathematics content was one of the common core recommendations reported in the *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* (AMTE, 2003). Some details have been spelled out about mathematics content background with the amount of mathematical knowledge being a function of whether the doctoral student is focusing on elementary, secondary or collegiate levels (Dossey and Lappan, 2001). Nearly twenty years after this conference the amount of foundational mathematics content required for a doctorate in mathematics education continues to vary across, and oftentimes within, institutions. Most institutions require some graduate level mathematics for completion of a doctorate, but this requirement may depend on the major advisor and doctoral committee. While the mathematics content required in the colleges/schools of education may vary, doctorates in mathematics education awarded by mathematics departments typically require at least the equivalent of a master's degree in mathematics.

Improving the system. Doctoral programs in mathematics education have been called a 'complex system' (Hiebert, Kilpatrick, & Lindquist, 2001). Improving complex systems cannot be done easily or quickly, and one framework for improving complex systems consists of four steps: assess current conditions; clarify goals; develop strategies for moving from current conditions to goals; and document and share information about the effects of improvement strategies (Hiebert, et al., 2008). While specific goals of institutional programs may vary, the *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* (AMTE, 2003) provides a blueprint. Faculty members at most institutions are

familiar with this document, so it might serve as a helpful guide. Obtaining feedback from doctoral graduates to help shape and strengthen a program reflects the first step toward improvement. Yet less than one-third of the institutions reported gathering feedback from graduates annually, and over 15% of the faculty members said they have never gathered such feedback. It has been argued that such regular feedback would provide valuable information to integrate into programmatic reviews and make progress toward future steps in improving doctoral programs (Reys & Reys, 2017). Our hope is that this commentary will encourage all faculty members involved in doctoral programs in mathematics education to become involved in shaping requests for feedback from doctoral graduates, agree upon the frequency of this effort, and periodically contribute to a careful review of their doctoral program.

Accreditation. Is it time for the field to consider establishing an accreditation system for doctoral programs in mathematics education? Program accreditation is widespread in many areas of higher education. It was a topic discussed at the second national conference that stimulated thoughtful discussion on both sides (Lappan, Newton, & Teuscher, 2008). It was agreed that an accreditation process would require guidelines and standards that could be used to develop and assess the quality of doctoral programs in mathematics education, and to better define what is meant by a doctorate in mathematics education. Furthermore, it was agreed that external reviews from an accreditation would encourage more regular self-examination and thoughtful discussions by faculty members leading the doctoral program. Music education provides an existence proof that accreditation of doctoral programs in an education discipline can be successfully carried out (Reys, 2018b).

The ultimate goal of reviewing and accrediting doctoral programs in mathematics education would be to strengthen doctoral preparation. The accreditation process should be constructive. It should also provide a pathway to help new doctoral programs become established as well as strengthen and help keep established doctoral programs dynamic. An accreditation report would summarize program strengths and weaknesses and this feedback could be used by faculty members to develop an action plan going forward. Such information could be used by faculty members to leverage support from administrators to strengthen their doctoral program. It might also be used to denote accreditation status for their doctoral program in mathematics education, thereby attracting more doctoral students.

Some possible directions for future research

Research related to doctoral preparation in mathematics education has been limited and rarely reported in peer-reviewed journals (Kilpatrick & Spangler, 2016; Reys, 2017). Yet, if the mathematics education community is to grow and become stronger, research is needed on many fronts related to doctoral preparation in the field. Some possible directions for future research are offered here:

- Identify institutions that offer the option of a Ph.D. or an Ed.D. in mathematics education. What program requirements are the same? How are they different? Do career paths for recipients of these degrees differ? If so, in what ways?
- Examine the number of applications, acceptance rate, attrition rate, and graduation rate of students entering doctoral programs in mathematics education by institution. Are the attrition-graduation rates different across institutions? If so, why?
- Identify syllabus/content descriptions of graduate level courses for doctoral students in mathematics education. How similar/different are courses focusing on similar topics, such as mathematics curriculum or learning mathematics? Is there a required minimum type or number of these courses that all doctoral graduates in mathematics education must complete? Is there a rationale for this requirement?
- Collect and analyze required readings, courses, internships, and other experiences of current doctoral programs. Use this to identify a common core of knowledge, if it exists.
- Examine the core knowledge summarized in the *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* and see how the core knowledge aligns with the syllabus/content descriptions of the graduate level courses for doctoral students in mathematics education.
- How do programs that focus on preparing researchers in mathematics education differ from institutions that focus on preparing collegiate teachers of mathematics? Are there differences in course requirements? Internships? Clinical experiences? Job opportunities?
- Determine how the pathway to a doctorate in mathematics education is different at the same institution when earned in a mathematics department or in a college/school of education. How are the pathways similar/different across several institutions? What are the career aspirations of the respective programs' graduates?
- Identify institutions with doctoral programs in mathematics education that have established intellectual communities and investigate their nature and effect on their program and doctoral students.

- Select a mathematics education doctoral program that has shown significant growth in number of graduates during the last decade and carefully examine the factors that facilitated the growth. Select a program that has declined in number of graduates during the last decade and document the factors that contributed to its decline.
- Survey doctoral graduates in mathematics education to learn about how their doctoral preparation aligned with their post-graduation job expectations.
- Explore accreditation systems in other similar education and non-education areas. Identify arguments for and against an accreditation system for doctoral programs in mathematics education.
- Identify institutions that have invited external examiners to review their doctoral program and document the nature of the review as well as how the review process has impacted their doctoral program in mathematics education.

Conclusion

Our paper has reported some research findings related to doctoral programs and doctoral preparation in mathematics education. We noted an increasing number of institutions producing doctorates in mathematics education. Very little information is known about the nature and quality of the over 125 programs that have graduated doctorates in mathematics education during the last five years. While some programs graduate doctorates in mathematics annually, the overwhelming majority of institutions graduate one student every few years. The small number of annual graduates raises questions about the resources available to provide focused course work related to mathematics education and valuable research experiences. Do institutions graduating someone ever few years have a viable doctoral program in mathematics education? Research is needed to examine and learn more about doctoral preparation programs and the extent to what core-knowledge exists among all doctoral graduates in mathematics education. Accreditation has been suggested as a means of gathering more detailed information from institutions about the nature and scope of their doctoral program in mathematics education.

In an earlier JRME Research Commentary, Schoenfeld focused on the need for and value of replications of research in mathematics education (Schoenfeld, 2018). We agree that replication of research is valuable. However, we argue that so little research related to doctoral preparation in mathematics education has been reported in scholarly journals, that it is a bit early for replication.

Simply put, much more research focusing on multiple aspects of doctoral programs in mathematics education is needed.

Our paper has made clear that limited research on a few facets of doctoral programs in mathematics education has been reported. We offered some possible directions for future research. Our hope is that this paper will stimulate discussion in the mathematics education community that will lead to more research focusing on various components of doctoral preparation in mathematics education. This could include faculty members actively involved in doctoral mathematics education programs doing case studies, i.e., self-examination, of their own or other programs and then sharing their process and what has been learned both internally and externally. This may encourage faculty members in different institutions to collaborate and spearhead efforts that might move toward accreditation of institutions purporting to have doctoral programs in mathematics education. We hope in the future there will be many quality research studies focusing on different aspects of doctoral programs in mathematics education. Such research could provide much needed foundational knowledge to guide the preparation of future generations of stewards of our discipline of mathematics education.

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References

Association of Mathematics Teacher Educators (AMTE). (2003). *Principles to guide the design and implementation of doctoral programs in mathematics education*. San Diego, CA: AMTE.

Bay, J. (2001). Appropriate preparation of doctoral students: Dilemmas from a small program perspective. In R. Reys & J. Kilpatrick (Eds.) *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education*, (pp. 129–132), Washington, DC: American Mathematical Society/Mathematical Association of America.

Donoghue, E. (2001). Mathematics education in the United States: Origins of the field and the development of early graduate programs. In R. Reys & J. Kilpatrick (Eds.) *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education*, (pp. 3–18), Washington, DC: American Mathematical Society/Mathematical Association of America.

Dossey, J. & Lappan, G. (2001). The mathematical education of mathematics educators in doctoral programs in mathematics education. In R. Reys & J. Kilpatrick (Eds.) *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education*, (pp. 67–73), Washington, DC: American Mathematical Society/Mathematical Association of America.

Foley, G. D. (2014). Mentoring doctoral students. In A. Howley & M. B. Trube (Eds.), *Mentoring in the professions: Orienting toward the future* (pp. 205–221). Charlotte, NC: Information Age.

Glasgow, R. (2000). *An investigation of recent graduates of doctoral programs in mathematics education* (Unpublished doctoral dissertation). University of Missouri: Columbia.

Golde, C. (2008). Creating a broader vision of doctoral education: Lesson from the Carnegie initiative on the doctorate. In R. Reys & J. Dossey (Eds.) *U.S. Doctorates in Mathematics Education: Developing Stewards of the Discipline*, (pp. 53–63), Washington, DC: American Mathematical Society/Mathematical Association of America.

Golde, C. & Walker G. (Eds.). (2006). *Envisioning the future of doctoral education: Preparing stewards of the discipline*. San Francisco, CA: Jossey-Bass.

Hiebert, J., Kilpatrick, J., & Lindquist, M. (2001). Improving doctoral programs in mathematics education. In R. Reys & J. Kilpatrick (Eds.) *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education*, (pp. 153–162), Washington, DC: American Mathematical Society/Mathematical Association of America.

Hiebert, J., Lambdin, D., & Williams, S. (2008). Reflecting on the conference and Looking toward the future. In R. Reys & J. Dossey (Eds.) *U.S. Doctorates in mathematics Education: Developing Stewards of the Discipline*, (pp. 241–254), Washington, DC: American Mathematical Society/Mathematical Association of America.

- Inglis, M. & Foster, C. (2018). Five decades of mathematics education research. *Journal for Research in Mathematics Education*, 49(4), 462–500.
- Kilpatrick, J. & Spangler, D. (2016). Educating future mathematics education professors. In L. D. English & D. Kirsher (Eds.), *Handbook of International Research in Mathematics Education*, (3rd ed). (pp. 297–309). New York: Routledge.
- Lappan, G., Newton, J., & Teuscher, D. (2008). Accreditation of doctoral programs in mathematics education. In R. Reys & J. Dossey (Eds.) *U.S. Doctorates in mathematics Education: Developing Stewards of the Discipline*, (pp. 215–222), Washington, DC: American Mathematical Society/Mathematical Association of America.
- Levine, A. (2007). *Educating researchers*. Princeton NJ: The Education Schools Project.
- McIntosh, J. & Crosswhite, J. (1973). *A survey of doctoral programs in mathematics education*, ERIC: ED091250. Retrieved from <https://eric.ed.gov/?q=ED091250&id=ED091250>
- Reys, R. E. (2002). Mathematics Education Positions in Higher Education and Their Applicants: A Many to One Correspondence, *Notices of the American Mathematical Society*, 49(2), 202–207.
- Reys, R. (2017). Doctorates in mathematics education-How they have evolved, what constitutes a high-quality program, and what might lie ahead? In J. Cai (Ed.). *Compendium for Research in Mathematics Education*. (pp. 934–948). Reston, VA: National Council of Teachers of Mathematics.
- Reys, R. (2018a). The Preparation of a Mathematics Educator: The Case of Carey. *Canadian Journal of Science, Mathematics and Technology Education*. 18(1), 56–67.
- Reys, R. (2018b). Doctoral programs in mathematics education: Should programs be accredited? *Journal of Mathematics Education at Teachers College*. 9(1), 15–21.
- Reys, R. E. & Dossey, J. A. (Eds.). (2008). *U.S. Doctorates in Mathematics Education: Developing Stewards of the Discipline*, Washington, DC: American Mathematical Society/Mathematical Association of America.
- Reys, R., Glasgow, R., Ragan, G. & Simms, K. (2001). Doctoral programs in mathematics education in the United States: A status report. In R. Reys & K. Kilpatrick (Eds.) *One Field, Many Paths: U.S. Doctoral Programs in Mathematics Education*, (pp. 19–40). Washington, DC: American Mathematical Society/Mathematical Association of America.
- Reys, R. E., Glasgow, R., Teuscher, D., & Nevels, N. (2008). Doctoral programs in mathematics education in the United States: 2007 Status Report. *Notices of the American Mathematical Society*, 54(10), 1283–1293.
- Reys, R. E. & Kilpatrick, J. (Eds.). (2001). *One Field, Many Paths: U. S. Doctoral Programs in Mathematics Education*, Washington, DC: American Mathematical Society/Mathematical Association of America.
- Reys, B. & Reys, R. (2017). Strengthening Doctoral Programs in Mathematics Education: A Continuous Process. *Notices of the American Mathematical Society*, 64(4), 386–389.
- Reys, R. & Reys, B. (2016). A recent history (5 decades) of the production of doctorates in mathematics education, *Notices of the American Mathematical Society*, 63(8), 936–939.
- Reys, R., Reys, B., & Estapa, A. (2013). An update on jobs for doctorates in mathematics education at institutions of higher education in the United States, *Notices of the American Mathematical Society*, 60(4), 470–473.
- Reys, R., Reys, B., Shih, J. & Safi, F. (2019). Doctoral programs in mathematics education: A status report of size, origin of program leadership, and recommended institutions. *Notices of the American Mathematical Society*, 66(2), 212–217.
- Shih, J. Reys, R., & Engledowl, C. (2016) Profile of research preparation of doctorates in mathematics education in the United States. *Far East Journal of Mathematical Education*. 16(2), 135–148.
- Shih, J., Reys, R., Reys, B. & Engledowl, C. (2019). Examining career paths of doctorates in mathematics education working in institutions of higher education. *Investigations in Mathematics Learning*, 11(1), 16–28.
- Schoenfeld, A. (2018). On replications. *Journal for Research in Mathematics Education*, 49(1), 91–97.
- Soonabend, T. (1981). *A survey of doctoral programs in mathematics education*, ERIC: ED201518. Retrieved from <https://files.eric.ed.gov/fulltext/ED201518.pdf>