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A Century of Leadership in Mathematics and Its Teaching

Fostering Positive Cognitive and Affective Growth

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Middle School Students' Mindsets Before and After Open-Ended Problems

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ABSTRACT Growth mindset is an important belief for students to be successful in mathematics and in their current and future lives. Solving open-ended problems also has many positive benefits for students. Little research has been conducted though on growth mindset and mathematics at the middle school level and particularly growth mindset with open-ended problems. This study explored middle school students' mindsets before and after a four-week Saturday program that incorporated open-ended problems. We also looked at the quality of solutions developed by the students. It was found that the students generally had growth mindsets with some fixed ideas at the beginning of the study that improved to strong growth mindsets at the conclusion of the four weeks. The students also improved on their quality of solutions from the first to the last open-ended activity. Implications for the implementation and research of open-ended problems are discussed.

KEYWORDS *growth mindset; open-ended problem solving; middle school*

Overview

At some point in every person's mathematical studies, mathematics becomes difficult. When these difficulties arise, students can decide that mathematics is not for them and they cannot do mathematics or they can persevere, put in effort, and continue to develop their understanding. One of the main factors that affects whether students do well in mathematics is their mindset. Mindsets are a collection of beliefs related to continual learning and malleability of intelligence. Beliefs are vital because they are the best indicators of the decisions that individuals make throughout their lives (Bandura, 1986; Pajares, 1992). Beliefs can be a lens through which people perceive the world and can be thought of as dispositions toward action (Ambrose, Clement, Philipp, & Chauvot, 2004).

If success in mathematics is a worthy goal for all students, then it is worthwhile thinking about how to encourage growth mindsets. The concept of a growth mindset has received more attention since Dweck's (2006) book on the subject. A growth mindset is the belief that intellectual skills can be cultivated through effort; on the opposite end of the spectrum, a fixed mindset is believing that your qualities are carved in stone or fixed (Dweck, 2006). Fixed mindsets are particularly troubling because "fixed mindset beliefs contribute to inequalities in education as they particularly harm minority students and girls; they also contribute to overall low achievement and participation" (Boaler, 2013, p. 150).

In addition to instilling in students a growth mindset, incorporating open-ended problems has been recommended as a way to reduce inequities in mathematics education and to give students the opportunity to demonstrate mathematical understanding that may not be captured on typical assessments (Lesh & Doerr, 2003). Using open-ended problems enables teachers to employ best practices for mathematics teaching including cooperative learning, assessment integrated in instruction, building on prior knowledge of students, and a focus on students' capabilities.

Since growth mindsets can be an important factor to continued success in mathematics, it is important to determine how to develop this belief in students. Researchers have developed growth mindsets in students by teaching them directly about growth mindset through computer programs, readings, and brain research (e.g. Esparza, Shumow, & Schmidt, 2014; Paunesku et al., 2015; Yeager et al., 2016). Little research though has focused on open-ended problems being used as an intervention to help students hold growth mindset beliefs. Participating in open-ended problems has the potential to develop growth mindsets. In one study, researchers found that when freshmen engineering students worked on an open-ended project, students were more likely to develop a growth mindset (Reid & Ferguson, 2014). Through open-ended problems students can persevere in problem solving, use multiple representations in their solutions, see there is more than one right answer to a problem, that there is not one type of person that can be successful in mathematics, and learn from others (Stohlmann, 2017a).

This study investigated middle school students' mindsets, using Dweck's (2006) growth mindset Likert questionnaire, before and after a four-week Saturday Science, Technology, Engineering, and Mathematics (STEM) program offered at a large research university in the Southwestern part of the United States. The program focused on having students participate in open-ended problems. For this study we define open-ended problems as real world or game-based tasks that have multiple possible answers. In these task students often use multiple representations. The research questions that guided this study were the following: *What are middle school students' mindsets before and after participating in a four-week Saturday program focused on open-ended problems? Using the Quality Assurance Guide (Lesh & Clarke, 2000), what is the quality of solutions developed by the middle school students?*

Growth Mindset

Defined as core assumptions of the malleability of personal qualities, implicit theories refer to a person's commonsense explanations for everyday events (Dweck et al, 1995; Dweck & Leggett, 1998; Molden & Dweck, 2006, Yeager & Dweck, 2012), and vary from an entity theory to incremental theory (Yeager & Dweck, 2012). Students who hold entity theory of intelligence or fixed mindset tend to see intelligence as an inherent and unchangeable trait, while students who hold incremental theory of intelligence or growth mindset see intelligence as an ability that can be developed over time (Dweck, 2006). Theories of mindset, or the implicit theory of intelligence, allow us to understand how mindset fosters goals, attributions, and reactions to setbacks (Yeager & Dweck, 2012; Dweck, 2017). In theories of mindset, students who hold growth mindsets set self-improvement as achievement goals, attribute failures to something that is under their control, and work harder when faced with setbacks. These students actively try new learning strategies and seek all available resources. However, students who hold fixed mindsets aim for performance-oriented goals, see failures as something that is beyond their control, and give up when they experience setbacks.

Research has shown that fostering growth mindsets improves students' academic performance, increases students' motivation, and reduces social, gender, and social class gaps. For example, a mindset intervention significantly helped at-risk students raise their semester grade point average in core academic courses (Paunesku et al. 2015). Yeager et al. (2016) also found that students who were in the transition to high school in the United States showed academic progress after receiving a growth mindset intervention. In addition, growth mindset helped African American students resist stereotype threat (Steele & Aronson, 1995), increased their enjoyment of academic success, and improved their academic engagement and GPA (Arson, Fried & Good, 2002). In a sample across all of the socioeconomic levels in Chile, Claro, Paunesku, and Dweck (2016) found that growth mindset was a relatively strong predictor of math and language performance. It is suggested that students' growth mindset might play a role in mediating the effects of economic disadvantage (Claro et al., 2016).

Growth Mindset and Mathematics

Few studies have examined how growth mindset impacts students' mathematics performance particularly at the middle school level, but there have been promising results. Good and her colleagues found that a growth mindset intervention increased both 7th grade boys' and girls' mathematics performance, and that such increase was higher for girls (Good et al., 2003). Blackwell, Trzesniewski, and Dweck (2007) examined the role of growth mindset in 373 seventh grade students' mathematics achievement. The study did not find a significant correlation between students' mathematics test scores in 6th grade and their belief in a growth mindset at the beginning of the year. However, a growth mindset was a significant predictor of students' mathematics achievement for the students as they were followed into 8th grade though. Bostwick et al. (2017) adopted an integrative approach to analyze the impact of growth construct (growth mindset, self-based growth goals, and task-based growth goals) on mathematics outcomes from a dataset of 4,411 Australian students in 7th grade to 9th grade. Results found that even when students' background factors were included, students' growth orientations were positively associated with both their academic engagement and achievement. This previous research demonstrates the importance of helping students to develop a growth mindset.

Open-Ended Problems

For open-ended problems to be used, classroom activities need to be structured to help students mathematize situations appropriately, find mathematical rules or relations, solve problems, and check results. In open-ended problems the process, ways to develop solutions, and end products are all open. Students get the opportunity to see other students' discoveries or methods, compare and examine different ideas, and modify and further develop their own ideas (Sawada, 1997).

There are advantages of this approach that are worth the time investment for teachers to try implementing open-ended problems. Students participate more actively in the lesson and express their ideas more frequently. They also have more opportunities to make comprehensive use of their mathematical knowledge and skills (Sawada, 1997). Students can develop their higher-order thinking skills through the use of authentic mathematics (Varygiannes, 2013).

Research on open-ended problems has found promising results on students' ability to participate in these tasks and develop solutions. A book by the National Council of Teachers of Mathematics (NCTM) summarizes multiple classroom tasks done in Japanese schools in which students successfully develop multiple solutions to open-ended tasks (Becker & Shimada, 1997). Little information is provided on the mathematical ability of these students or their past experience with open-ended problems. A quasi-experimental study conducted by Fatah, Suryadi, & Sabandar (2016) found that mathematical creative thinking ability and self-esteem of the treatment group was statistically significantly higher after participating in open-ended problems.

More research is needed specifically on growth mindset with open-ended problems. Two main questions need to be looked at in regards to this that this study helps to answer. What is the potential of open-ended problems to support students in their development of growth mindset? How does growth mindset affect the quality of solutions that students can develop during open-ended problems? (Stohlmann et al., 2016).

Method

This study was conducted with 19 middle school students (age 11-13) that voluntarily enrolled in a Saturday STEM program at a large research university in the Southwestern part of the United States. The students were ethnically diverse and from a large urban school district. They were recruited with an email announcement of the program sent to all parents in the school district. The purpose of the Saturday STEM program was to provide a series of inquiry experiences designed to provide interesting and exciting opportunities in STEM education. Fourteen out of the nineteen students reported typically receiving an A or A- in mathematics, with the other five students typically receiving a B+ or B. The first and third authors were the instructors for this program and had been instructors for this program for several years.

The program lasted four Saturdays (Table 1) and involved concept development activities, open-ended activities, and videos of general social skills that students need to work effectively in groups. Each day had an overall topic: day 1 focused on equations and expressions, day 2 on ratios and proportions, day 3 on linear equations, and day 4 on systems of equations.

Table 1

Saturday STEM program activities by day.

Day	Activities
1	• Growth mindset Likert questionnaire (Dweck, 2006) • Communicating and listening video (FlowMathematics, 2012) • Dirt Dash (Calculation Nation, 2017a) • Ker-splash (Calculation Nation, 2017b)—open-ended activity
2	• Decision making video (FlowMathematics, 2011) • Waffle choices activity (Ehlert, 2014b)—open-ended activity • Marcellus the giant (Desmos, 2017a) • ST Math 7th grade proportional relationships—monster ratios and build a monster (ST Math, 2017)
3	• Polygraph lines (Desmos, 2017b)—open-ended activity • Polygraph lines part 2 (Desmos, 2017c) • Marbleslides line (Desmos, 2017d) • Lego prices (Desmos, 2017e)
4	• Stairs or elevator problem (Ehlert, 2014a)—open-ended activity • Polygraph linear systems (Desmos, 2017f)—open-ended activity • Systems of two linear equations (Desmos, 2017g) • Growth mindset Likert questionnaire (Dweck, 2006)

Data Collection

The data collection involved student work, a pre and post growth mindset Likert questionnaire (Dweck, 2006), and researcher field notes. Of the 19 students, 15 fully completed the pre- and post-growth mindset Likert questionnaire.

Data Analysis

Each answer choice on the growth mindset questionnaire was assigned a point value of 0 to 5, with a higher score being more closely aligned to a growth mindset. For example, two questions are listed below with the point values included.

No matter who you are, you can significantly change your intelligence level.

Strongly agree (5) Agree (4) Mostly agree (3)
Mostly disagree (2) Disagree (1) Strongly Disagree (0)

You have a certain amount of intelligence, and you can't really do much to change it.

Strongly agree (0) Agree (1) Mostly agree (2)
Mostly disagree (3) Disagree (4) Strongly Disagree (5)

The students pre and post questionnaire was summarized using descriptive statistics and a paired t-test was conducted to see if there was a significant difference between the pre and post scores. Table 2 summarizes general categories for individual total scores on the growth mindset questionnaire.

Table 2
Growth mindset questionnaire categorizations.

Categorization	Points value
Strong growth mindset	61-80 points
Growth mindset with some fixed ideas	41-60 points
Fixed mindset with some growth ideas	21-40 points
Strong fixed mindset	0-20 points

The student work and researcher field notes were analyzed using the Quality Assurance Guide (QAG) to give students' solutions on the open-ended activities a quality ranking (Lesh & Clarke, 2000). The QAG was designed to evaluate products that are developed from mathematical modeling activities, a type of open-ended problem (Table 3). Two of the researchers coded the students' solutions. The Cohen's K coefficient of inter-rater agreement was .80, and thus within an acceptable range (Fleiss, 1981; Landis & Koch, 1977). Once coding differences were identified, the raters came to an agreement on the discrepancies so that full agreement was reached.

Table 3
Saturday STEM program activities by day.

Performance Level	Description
(0) Requires redirection	The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher.
(1) Requires major extensions or refinements	The product is a good start, but a lot more work is needed to respond to all of the issues.
(2) Requires only minor editing	The product is nearly ready to be used. It still needs a few small modifications, additions, or refinements.
(3) Useful for the specific situation given	No changes will be needed for the current situation.
(4) Sharable or reusable	The solution not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.

Results

A paired t-test indicated that there was a statistically significant difference between the pre and post growth mindset questionnaire: pre-test ($M=59.47$, $SD=15.9$) and the post-test ($M=66.07$, $SD=14.55$), $t(14)=1.576$, $p=0.069 < .10$. The pre-questionnaire mean could be categorized as growth mindset with some fixed ideas while the post-questionnaire mean could be categorized as evidence of strong growth mindsets. The Cohen's d effect size was .43, which is a medium to small effect size. Table 4 details the descriptive statistics.

There were five groups for each of the open-ended problems the students completed. Each group was given a score based on the Quality Assurance Guide (Table 5). A majority of scores of 2 or greater on the activity means indicates that the students on average developed useable solutions or only needed small modifications or refinements. The students also improved in their scores from the first to the last modeling activity. A Wilcoxen signed-rank test indicated that students did better on the last activity (mean=3.2) than the first activity (mean=1.8), $Z=-1.84$, $p=.066<.10$.

Table 6 displays the pre and post assessment averages for each group on the growth mindset Likert questionnaire. Results of a Spearman correlation indicated that there was not a significant correlation between the groups' post questionnaire average and the groups' activity average, ($r_s(3)=-.564$, $p>.10$)

To provide further detail on the Quality Assurance Scores and the groups' solutions we present details for the waffle choices activity. In this activity students are shown a short video of two different waffle brands, one a mix and the other frozen waffles. Students are asked what they notice from the video and which waffle package they would buy. The students mentioned many things to consider by answering this question including the ingredients, the price, the number of waffles, the dimensions of the waffles, and the health information. The following question was then posed to the students: Which waffle box is the better value and what choice would you make based on this information? Students were then provided information on the price of each waffle box ($4.99 for the mix and $2.79 for the frozen waffles), the diameter of a waffle from each box (6.75 inches for the mix and 3.75 inches for the frozen waffles), the serving size (1/2 a cup for the mix with 1 cup equaling 2 waffles and 2 waffles for the frozen waffles), and number of servings in each box (11 servings for the mix and 3 for the frozen waffles). We present each group's solution to this problem starting with the lowest QAG ranking.

Table 4

Pre and post-questionnaire descriptive statistics.

Pre-Questionnaire					Post-Questionnaire				
n	M	Min	Max	SD	n	M	Min	Max	SD
15	59.47	20	80	15.9	15	66.07	38	80	14.55

Table 5

Quality assurance guide scores per group for the open-ended problems.

	Ker-splash	Waffle choices	Polygraph lines	Stairs or elevator	Polygraph linear systems	Group mean
Group 1	1	2	1	1	2	1.4
Group 2	4	2	4	4	4	3.6
Group 3	1	2	1	1	2	1.4
Group 4	2	4	4	3	4	3.4
Group 5	1	0	2	2	4	1.8
Activity mean	1.8	2	2.4	2.4	3.2	

Table 6

Pre- and post-group averages on the growth mindset Likert questionnaire.

	Group 1	Group 2	Group 3	Group 4	Group 5
Pre score	48.3	59.6	67	56	65.6
Post score	70.3	65.3	74	65	63.6

Group 5

This group was given a QAG ranking of 0 because their solution path needed redirection to be successful. The students in this group did an Internet search on the brands of the waffles and looked at the nutritional information. They discussed the context of the problem but had difficulty in figuring out how to use mathematics to help them develop a solution. Their attempt at a solution involved multiplying the diameter of each waffle by pi and then subtracting the price of each box. Students struggled explaining their solution method when asked. After hearing other groups' present their solutions, this group stated that they would use the price per waffle to make their decision and that they would buy the mix box.

Group 1

This group came up with a method that would work for the given situation, but made a minor mistake in the number of waffles in the mix box, resulting in a QAG score of 2. This group took the price of each box and divided each by 6. They did not realize that there were 11 waffles that could be made from the mix box. It was determined that price per waffle was 83 cents for the mix and 46 cents for the frozen waffles. Informally, the students then compared the diameters of the waffles noting that the diameter of the mix was almost two times the diameter of the frozen waffles, while the mix waffles were only 37 cents more per waffle. The group also noted that they thought the mix waffles would taste better so their decision was to buy the mix box. If the students had correctly determined the mix box could make 11 waffles it would have strengthened their case for buying the mix box.

Group 2

This group was able to develop a useable solution for the given situation, but needed some clarification of their work to make it clearer, resulting in a QAG score of 2. This group also spent time looking at the nutritional information, and initially thought there were 8 waffles in the frozen box because on the outside of the box was a big number 8. They eventually realized this referred to 8 whole grains. This group noted that the mix box had a greater amount of protein per waffle. They noted the difference in price ($2.20), the difference in number of waffles (5), and the difference in diameter (3 inches). Next, they determined that the price per waffle was roughly equivalent. They decided to use a point system by giving each box 3 points on price since they were about the same. Next, they gave the mix 6 points for the diameter and the frozen box 3 points for the diameter. It was not clear where the point differential came from, but might have been from the 3-inch difference in diameter. Using this information, they determined that the mix box was the better value. They stopped at this point because they did not even need to consider the specific number of waffles since the mix could make more waffles.

Group 3

This group had a solution method that would work for the specific situation and other closely related situations but needed to make one correction on the number of waffles in the frozen waffles box, resulting in a QAG score of 2. This group also spent some time looking at the nutritional information between the two brands of waffles. They noted that the mix had less saturated fat, less total fat, and less sodium. The group determined the area of one waffle for each of the waffle brands by using the formula for the area of a circle. Next, they took the respective areas and multiplied by 11 for the mix and 3 for the frozen waffles. When this group presented their solution, the instructor noted that there were six waffles in the box and 3 servings. The group then explained that they would take the price and divide it by the total area of the waffles in each box. From this they determined that the mix box would be their choice because the price per square inch of waffle was lower.

Group 4

This group came up with a solution that worked for the situation given and could be used in other closely related situations resulting in a QAG score of 4. They noted that each box had whole grains included. To determine their choice they calculated the price per waffle and noted that they were very similar. Since the mix had more waffles and larger waffles they would buy the mix box. If needed they would have gone on to use the area of one of each type of waffle divided by the price per waffle.

Activity follow-up

To have students see the importance of assumptions and approximations, the instructor posed the following question to the students after the group presentations: Would you rather have 7.2 hours of more free time over the course of a year or $150.02? All of the students stated that they would rather have the money. The instructor then went on to explain, using assumptions based on how often the waffles are eaten, how buying the mix waffles could save a person $150.02 over a year. Next the instructor showed, by making assumptions on the prep, cooking, and clean up time, how a person could save 7.2 hours over the course of a year by buying the frozen waffles. All of the groups stayed with their decision to buy the mix waffles.

Discussion

This study was conducted to determine the mindsets of middle school students before and after a 4-week Saturday program that incorporated open-ended problems, as well as the quality of solutions the students developed. In regards to the first research question, the class average significantly increased from the pre to the post assessment on the growth mindset questionnaire. The Saturday program helped improve the students' mindsets. This is an important finding as having students participate in open-ended problems can be another way to help students develop growth mindsets.

Students participating in open-ended problems need to persevere in problem solving, try new approaches, use all of their resources, and continue to develop their ideas when encountering setbacks or failures. These are all characteristics that are connected with a growth mindset. The students in this study were on-task while working on the open-ended problems and used the Internet when needed, their group members, and other groups to persevere in problem solving.

Regarding the second research question, there was not a significant correlation between the groups' growth mindset average and the quality of solutions. It is worth noting that each group held strong growth mindsets at the conclusion of the study but varied in the quality of their solutions. Overall, the groups improved on the quality of their solutions from the first to the last activity. Since this study had a small sample size, future research is needed to investigate how growth mindset is related to the quality of solutions developed during open-ended problem solving.

This study supports previous research that found that mathematics achievement was connected to a growth mindset (Blackwell et al., 2007; Bostwick et al., 2017; Good et al., 2003). The majority of the students in our study generally received A's in their mathematics classes. Students who hold growth mindsets set self-improvement as achievement goals and work harder when faced with setbacks (Dweck, 2006).

Past research has shown that middle school students can have difficulties when first experiencing open-ended problems (Ang, 2013; Gould and Wasserman, 2014; MaaB & Mischo, 2011). It may be useful in future research to provide students with information on growth mindset prior to their first experience with open-ended problems. Stohlmann (2017a) has also developed messages and questions for students to metacognitively monitor themselves while working through open-ended problems. These messages and questions were used with middle school students in their first experience with open-ended problems and led to success (Stohlmann, 2017b).

Understanding context is an important part of open-ended problems. Gould & Wasserman (2014) found that high achieving mathematics students struggled with identifying the most important variables and making assumptions when solving a problem on deciding the best gas station to buy gas from. The students in this study could have received more support on understanding the context. In our study we found that students had difficulties in using serving sizes to correctly determine the number of waffles in the box. Most middle school students do not do a lot of grocery shopping and were probably unfamiliar with this. If this problem is used in future research, it would help to discuss this aspect with students before the group work time. The students in Gould and Wasserman's (2014) study were also inexperienced with open-ended problems, which contributed to their difficulties. Research has shown that students have improved solutions the more open-ended problems they participate in over time (Biccard & Wessels, 2011; Grunewald, 2013; Ikeda & Stephens, 2010). The results of our study support this, as the students improved on their Quality Assurance Guide scores from the first to the last problem.

Open-ended problems have many benefits to students including developing students' communication, teamwork, and presentation skills (English & Watters, 2005); as well as helping students see mathematics as real life and applicable (Yanagimoto & Yoshimura, 2013) and developing mathematical understandings (Arleback, Doerr, & O'Neil, 2013; Lesh & Harel, 2003). Students will need these competencies to be successful in their current and future lives. This study showed that open-ended problems improved students' growth mindsets, which is another possible benefit of open-ended problems. Future research can focus on open-ended problems with students who have strong fixed mindsets and/or lower mathematics ability, as well as explore further the relation between growth mindset and success in open-ended problems with a larger sample size.

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