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Examining the Classroom Experience

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Designing and Testing a Mathematics Card Game for Teaching and Learning Elementary Group Theory

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ABSTRACT This paper explores the viability and development of the first edition of the researcher's mathematical card game, *Groups*, as a learning tool for elementary group theory, a topic in abstract algebra. *Groups* was play-tested by six undergraduate students in late 2016 who provided feedback on *Groups* from both utility-centric and design-centric perspectives. This paper addresses how well undergraduates with no prior group theory experience understand the fundamentals of group theory after playing several games of *Groups*, and how well undergraduates with prior knowledge of group theory related the *Groups* game play and mechanics to the fundamentals of group theory. Based on interview and questionnaire data, players found *Groups* to be an engaging and effective learning tool with both strengths and weaknesses in the tested iteration.

KEYWORDS *abstract algebra, group theory, educational game*

Design Inspiration and Introduction

In 2016, to find a new way to introduce students to abstract algebra, I designed and tested the first iteration of *Groups,* a card game (and *supplementary* tool) for the teaching and learning of elementary group theory at the secondary and undergraduate level. Salen and Zimmerman (2004) define a game as a "system in which players engage in an artificial conflict, defined by rules, that results in a quantifiable outcome" (p. 80). This definition is compatible with one way that modern scholars categorize mathematical exploration, as illustrated by many game-based mathematics-course interventions spanning the elementary to university levels (Ke, 2008; Kebritchi, Hirumu & Bai, 2010; McCue, 2011; Wijers, Jonker & Drijvers, 2010).

A staple of most undergraduate mathematics programs, group theory is a topic in abstract algebra involving the study of structures known as "groups," sets whose member elements interact with one another under a designated operation and achieve certain properties when considered in conjunction with the operation. The field first emerged in the 19th century and its origins are often linked to the work of prolific mathematicians Lagrange, Cauchy, and Galois. However, few deviations have been made in moving the teaching and learning of group theory away from the norm of lecture-based university-level mathematics courses. Leron and Dubinsky (1995) comment that "the teaching of abstract algebra [via lectures] is a disaster, and this remains true almost independently of the quality of the lectures…resulting inadequacies of the student [are]…an artifact of a too narrowly conceived view of instruction" (p. 227). Expanding on this point, Freedman (1983) elaborates on the weaknesses of a purely lecture-based instructional style when considering the teaching of abstract algebra, opting instead for approaches that force students to actively grapple with the content and evaluate it with peers. Offering alternatives from the lecture-norm, Pedersen (1972) and Lesh (1976) both emphasize an introduction to group theory that includes vivid, concrete examples with which students can physically interact (e.g. the transformations of paper polygons), ideally containing examples and counterexamples of the targeted mathematical ideas. Weber and Larsen (2008), in stark opposition to the traditional lecture method, recommend introducing students to group theory by building a strong informal knowledge base for student consideration before introducing students to that informal knowledge's formal mathematical-content counterpart; they describe this as a process of "mathematical reinvention" aimed at facilitating learners' content acquisition (p. 147). Drawing from several of these texts while designing *Groups*' game play and mechanics, I attempted to construct a learning experience emphasizing physical interaction and peer-to-peer discussion that promotes a gradual, discovery-based buildup from informal mathematics knowledge to formal mathematics knowledge.

This phenomenological study assesses the utility of an early iteration of *Groups* (specifically, the 1.1 edition) as a tool for teaching group theory at the secondary and undergraduate level, with a joint intent of finding ways to improve, for future iterations, the immersive abstract algebra experience *Groups* aims to provide. Simply put, the study carries the intent of fully determining the essence of *Groups*' game play experience, and how it may be refined.

Methodology

The 52-card *Groups* deck features four card types: group elements (represented as colors), group "blueprints" (e.g. Cayley tables for the groups of order 4), element-inverse signifiers, and effect cards (meant to deepen game play strategy). Each player's goal is to combine the element and inverse cards as stipulated by a blueprint card to form a group and verify the group's properties for both players.

Research Questions

In accordance with the learning goals as defined, the following two questions guided the study:

1. How well do undergraduates without prior exposure to group theory understand the fundamentals of group theory after playing several games of *Groups?*
   a. What is their opinion of *Groups* as a learning tool?
2. How do undergraduates with prior knowledge of group theory relate the *Groups* game play and mechanics to the fundamentals of group theory?
   a. What is their opinion of *Groups* as a review/learning tool?

Method

This study was conducted from a transcendental phenomenological perspective. Exploring group theory through card game play allowed participants to reflect internally on the relevant content while also having a shared experience with other players/learners.

Field Setting. All phases of the study involving participants took place on an urban university campus in the northeastern United States, with specific work spaces in common areas, dormitories, and academic buildings. Game play sessions and interviews were held outside of classes.

Participants. Six undergraduate students were pseudorandomly chosen to participate in this study based on responses to an interest survey at the institution. Three of these participants ("experts") had prior experience with formal group theory coursework, while the other three ("novices") did not (though they did have university-level mathematics experience including, minimally, some calculus).

Data Collection. Video footage of study participants playing *Groups* in three pairs—a pair of novices, a pair of experts, and a mixed pair—was collected and used to facilitate discussion with participants during individual interviews. The individual semi-structured interviews (see Appendix A) were also video recorded. Interviews aimed to collect participants' experiential responses on the game play experience. Further, participants responded to an open-ended questionnaire (see Appendix B) following game play and interviews. Questionnaires were designed to help participants further reflect on their experiences during the study, particularly with respect to the formal mathematics content associated with game play.

Data Analysis. I triangulated data by introducing my observations of game play into participants' interviews, and comparing interview transcripts alongside questionnaire data. In Table 1, questionnaire responses were analyzed through an open coding procedure that identified and grouped ideas, as per Merriam and Tisdell's (2016) tree-forest method (p. 208).

Limitations. Due to time constraints, a sample size of six participants was chosen for manageability. Further, the six participants were all in their third or fourth year of undergraduate studies at the same institution, a university in an urban setting. Additionally, because the learning tool in question was of my own design, I had to table my own pre-existing biases and opinions on *Groups*.

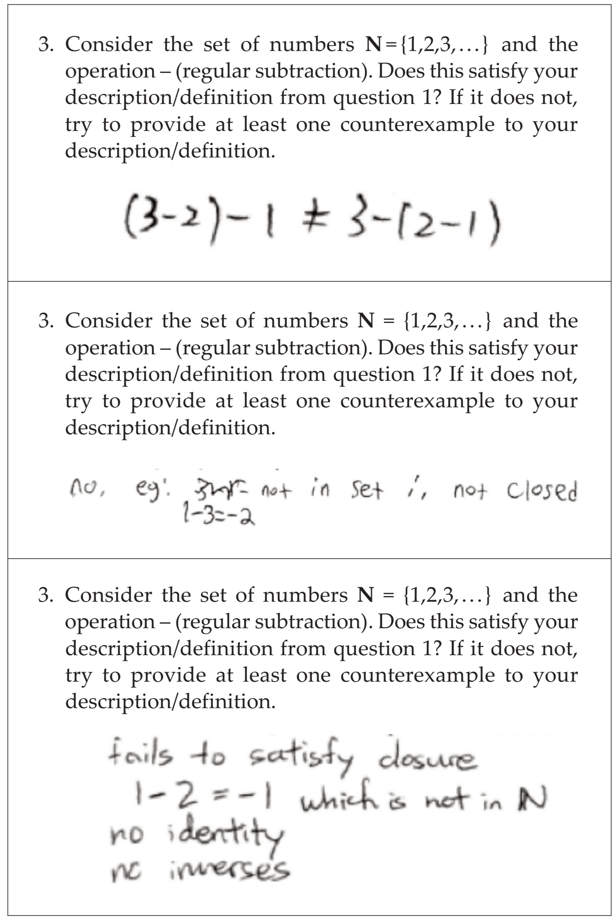
Findings

The Novice Experience

Associativity Confusion. All the novices felt that associativity was the most confusing property among the four designated as defining a group, although Lyoko felt that closure was equally as confusing for players. Of interest was Jay's interview, wherein he expressed relative clarity with all the properties except for associativity.

> [4:47-5:06] *Jay:* Uh, so, the inverse was pretty straightforward, uh, they give the identity, which was pretty straightforward. Definitely it was the associativity concept because at first—
>
> [5:06-5:07] *Researcher:* For most confusing, right?
>
> [5:07-5:47] *Jay:* Yeah, yeah, for most confusing. So, at first I thought the win condition was actually that you had to have one of each property. I didn't realize that you just had to represent every combination on the blueprint, so I was actually trying to figure out how the associativity works. Like, if I have three cards and like if I, if these two go together first, and then it goes with the other one, does it give them the same result as if it was another way around—if it was grouped another way. So, I was actually trying to figure that out, and that took me a lot of time during my turns, so, yeah.

After giving a brief informal explanation on how he understood the associativity property, Jay explained how he spent long periods of time thinking during game play to check that associativity kept holding as his cards changed. However, Jay's grappling with associativity was a productive struggle; in the questionnaire, Jay was the only participant (among both the novices and the experts) who answered the third question using an associativity counterexample, correctly arguing that $(3-2)-1 \neq 3-(2-1)$ (see Figure 1).


3. Consider the set of numbers **N** = {1,2,3,…} and the operation – (regular subtraction). Does this satisfy your description/definition from question 1? If it does not, try to provide at least one counterexample to your description/definition.

(3-2)-1 ≠ 3-(2-1)

3. Consider the set of numbers **N** = {1,2,3,…} and the operation – (regular subtraction). Does this satisfy your description/definition from question 1? If it does not, try to provide at least one counterexample to your description/definition.

no, eg: ~~3-4=~~ not in set ∴, not closed
1-3=-2

3. Consider the set of numbers **N** = {1,2,3,…} and the operation – (regular subtraction). Does this satisfy your description/definition from question 1? If it does not, try to provide at least one counterexample to your description/definition.

fails to satisfy closure
1-2=-1 which is not in N
no identity
no inverses

Figure 1. From top to bottom: novice Jay's response, novice Marcus's response, expert Kevin's response.

Color as an Abstract. None of the novices expressed total support for, or opposition to, the use of colors as group elements. However, both Jay and Marcus verbalized their individual experiences of productive struggle while attempting to make sense of the game's use of color.

> [3:09-3:12] *Researcher:* So how did you feel being able to talk about these interactions with colors?
>
> [3:12-3:13] *Marcus:* At first it was awkward.
>
> [3:13-3:14] Researcher: Yeah.
>
> [3:15-3:24] *Marcus:* Like, I wasn't sure how to phrase—like, the first game I won, the first match I won, I could, like, see the relationships, but...
>
> [3:25-3:27] *Researcher:* Hard to articulate it, maybe?
> [3:27-3:28] Marcus: Yeah.

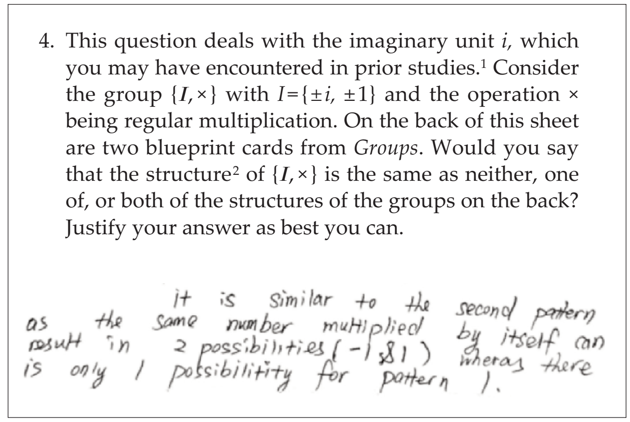
Marcus was more inclined to think of group elements as concrete objects, as hinted at earlier in his discussion of mathematical parallels; colors were objects that did not initially fit his mental picture of an element.

> [2:22-2:53] *Marcus:*…The [representations of properties] I've encountered have all been math-related, but uh, never like in as abstract a format as colors.
>
> [2:54-2:57] *Researcher:* Ah, as colors specifically?
>
> [2:58-3:08] *Marcus:* Well like, you know, as an abstract concept, as something that's not, like, 3. Like a set of languages—sets, I guess.

Jay took issue with the idea of colors "combining" in counterintuitive ways; in *Groups*, color combinations do not behave as they do in reality.

> [7:37-7:43] *Researcher:* Were you comfortable with dealing with colors and talking about the way that colors interacted, and things like that?
>
> [7:43-7:44] *Jay:* Yeah, it's a little counterintuitive.
>
> [7:44-7:45] *Researcher:* Counterintuitive? Why?
>
> [7:45-8:16] *Jay:* For two colors to produce, like, you have to keep looking at the blueprint otherwise there's no way to figure out what it could possibly be. I don't know if it's possible to make it so that it's a little bit intuitive. Maybe, so, I don't know, Red and Blue gives purple, like you're mixing the colors together or something like that. I don't know if it's possible with two blueprints though, but yeah, the colors combining were a little counterintuitive.

Strong Formal Mapping. The novices performed well on the questionnaire, which checked to see if players of *Groups* could answer formally-posed group theory questions using only the knowledge derived from game play. Marcus and Lyoko could correctly describe a group, identify a group, and explain why a structure was a non-group. However, they were unable to identify two isomorphic groups. Jay had not given a complete definition of an algebraic group, omitting a mention of closure and identity properties, although he was able to identify a group and non-group. However, he could correctly identify two isomorphic groups, as he described the structural similarity between $\{I, \times\}$and $\mathbb{Z}_4$ (See Figure 2). In his questionnaire response, Jay recognizes that the products $i \times i$, $(-i) \times (-i)$, 1×1, "and" $(-1) \times (-1)$ have two possible outcomes: either 1 or –1. He used this as his justification for likening $\{I, \times\}$ to $\mathbb{Z}_4$, noting that in the Klein four-group, all such products must produce the same result.



4. This question deals with the imaginary unit *i*, which you may have encountered in prior studies.[1] Consider the group $\{I, \times\}$ with $I=\{\pm i, \pm 1\}$ and the operation × being regular multiplication. On the back of this sheet are two blueprint cards from *Groups*. Would you say that the structure[2] of $\{I, \times\}$ is the same as neither, one of, or both of the structures of the groups on the back? Justify your answer as best you can.

it is similar to the second pattern as the same number multiplied by itself can result in 2 possibilities (-1 &1) wheras there is only 1 possibilitity for pattern 1.

Figure 2. Jay's recognition of an isomorphic property.

The Expert Experience

Fundamentally Faithful. All three expert participants—Kevin, Neil, and Oscar—found the *Groups* game play experience to adequately represent some of the essential ideas of group theory. Kevin's interview comments were particularly interesting because of the way he related game mechanics to group properties' "relative necessity."

> [3:11-3:34] *Researcher:*…Do you feel that the game's representation of group theory concepts was faithful?
>
> [3:35-4:09] *Kevin:* Yeah, definitely. Again, I mentioned before the issue with the size, the issue with having certain [prerequisites], like having the identity. You need to have inverses, and I think that's a really important point that people don't realize, that a lot of sets aren't groups because they don't have inverses...
>
> [4:09-4:15] *Researcher:*…So, you liked the emphasis that was put on…inverses.
>
> [4:16-4:32] *Kevin:* Yeah, and also on the identity because there's only four *Blacks* in the game, and it makes it kind of hard to get the *Black* when you need it, but at the same time, you don't want two identities, so I think that the number of cards is a good indicator of, kind of, the necessity and the rarity.

Kevin was also the only expert who commented on closure and associativity requiring further work.

> [4:56-5:16] *Kevin:*…I think the only property that we probably don't see in this game is associativity…but I feel like that's because…non-associative attributes are quite rare…
>
> [21:01-21:22] *Kevin:*…I think that maybe the [property] that wasn't really stressed is…closure.

Commutativity Omission. When questioned about *Groups'* algebraic faithfulness, Oscar immediately stated that every group property seemed well defined besides commutativity, which is not a mandatory group property, although it is one which both the groups featured in game play hold. During the design process, commutativity stuck out as a property that could quickly complicate game play interactions; because it is not an essential property of all groups, a treatment of it was omitted. All expert participants agreed that the property did not require a full explanation for game play, though their reasons varied. For Oscar, the property was self-evident.

> [5:40-5:48] *Oscar:* I think that since there's the blueprint, right, I think that a [moderately knowledgeable] player would pick [it] up pretty quickly.

Neil's initial concern was with game play manageability, as an explicit discussion of commutativity might force game play mechanics to include cards describing elements' relationships in each of two directions; in the studied iteration, relationships are described jointly in two directions. Later, Neil also expressed his view that commutativity could be confusing to new players/learners, and is not worth implementing, given *Groups'* current goals.

> [4:14-4:52] *Neil:*…It just adds, it basically not quite doubles, but close to doubles the number of things you have to specify for the group.

> [5:29-6:04] *Neil:* I think if you were really to talk about abstract algebra in a way that was sort of meant to convey the difficulties of it, I think commutativity would be an important thing to make note of…not all groups are commutative, and that's why a lot of this is difficult, but if the game is to show like these are what algebraic structures are, these are what groups are, I don't think it's necessary to basically say…they are not commutative in general because they sometimes are.

After some consideration, Kevin's overall viewpoint closely resembled Neil's opinions.

> [6:13-6:23] *Kevin:*…The thing is, because [*Groups* aims to teach at a] very basic level—maybe for high-school students or middle-school students to get exposure to groups—it's not really necessary to introduce [commutativity].

Color as an Abstract. Like the novices, the experts were also very opinionated about the use of color for representing group elements. Neil praised the representations of both group elements and group operators as they appeared in game play.

> [7:47-8:41] *Neil:*…I think the different choices of operations are both sort of abstract enough that—and the choice of colors and *o* and * are abstract enough away from math that—people who don't really know it's math won't say like "oh … this is the cyclic group, this is like Z_4…" It shows that these are ideas that are independent of what the actual members of the sets are and what the actual operations are. It has to do more with the properties of how they work…

Oscar's defense of colors as elements was short, but resonated with *Groups'* goal of extending abstract mathematics to a wider audience.

> [6:37-6:42] *Oscar:*…The use of color is more user-friendly and approachable.

> [6:58-7:01] *Oscar:*…If you put numbers here, you might fend off some people, perhaps.

Kevin was less optimistic about the use of colors, as he felt that they were too rigidly defined, starkly contrasting the opinions of his fellow experts. While considering the potential effects this representation of elements could have on first-time learners' game play experiences, Kevin articulated a preference for a new means of assigning properties.

> [8:31-9:07] *Kevin:* Well I think the [use of] colors is ok in the sense that they're just trying to represent neutral ideas, like in the blueprint layout. I think colors are fine, but perhaps, maybe, in a general sense of what the colors are, maybe don't use red, blue, yellow; maybe just have a different way to assign properties without having it be so fixed because in this way… [players] might never think "you can use any colors to be a group…"

Table 1

Cross-Subject Characteristics Found via Questionnaires

Characteristics	Participants					
	Jay [Novice]	Kevin [Expert]	Lyoko [Novice]	Marcus [Novice]	Neil [Expert]	Oscar [Expert]
Produced a clear and comprehensive definition of an algebraic group		x	x	x	x	x
Identified a structure external to *Groups* as a group	x	x	x	x	x	x
Identified a structure external to *Groups* as a non- group via a closure counterexample		x	x	x	x	x
Identified a structure external to *Groups* as a non- group via an associativity counterexample	x					
Identified two isomorphic groups	x	x			x	x

Discussion

One of the most important findings is that most players were comfortable dealing with the properties of group closure, group inverses, and group identities. However, four of the six players—including all the novice participants—found associativity to be poorly represented relative to the representations of the other three essential group properties. Jay's strong understanding of associativity does seem to indicate that the game has adequate support for acquiring this knowledge; this is particularly important to note, as Jay was the only player who claimed to have not engaged with any parallels to the group properties in formal coursework. It is likely, then, that the other participants entered the game with some pre-existing notion of associativity, and that the way associativity is implemented in the 1.1 edition of *Groups* is inelegant to the extent that it forces players to rely on their internalized ideas of associativity without really reinventing the concept or approaching it from a new perspective. This indicates that future iterations of *Groups* should revisit the implementation of associativity, and to a lesser extent, the implementation of closure, which attracted some concern from Lyoko and Kevin. Notably, five of the six participants directly utilized and discussed the property of closure on their questionnaires.

The representation of group elements as colors was a key discussion point for nearly every participant, and both groups had members experience productive struggle. While two of the experts praised this implementation for its abstracted nature, the novices spoke about it as a sort of acquired-understanding. This is likely because formal abstract algebra study trained the experts in dealing with multiple representations of elements of a group, whereas the blank-slated novices had to focus on the colors as their first representations of group elements. The predominant concern was that the colors used in the tested iteration of *Groups*—red, blue, and yellow—have realistic interactions among themselves (if one considers a real-world operation like "mixing"), which differ from their interactions in the game. Participants' experiences indicate that my choice of representing colors as group elements carried over some of the properties most closely associated with colors as concrete ideas, which is overall undesirable—but perhaps unavoidable; as Oscar had commented, the choice to utilize colors was made so that the game elements would seem neutral and inviting. Although an initially counterintuitive sensation was reported by some participants, every participant seemed to overcome this, indicating that although this aspect of game play is non-obvious, it can be fully understood in a short amount of time following practice, exposure, and consideration.

Participants developed an understanding of how group elements worked in conjunction with (most) group properties based on questionnaire items 2 and 3 being answered satisfactorily by all participants; these questions required element manipulation and group property usage to prove or disprove that a structure was a group. This result strongly indicates that novice participants could grasp the fundamentals of group theory after only a few rounds of *Groups*. Going further, the fourth questionnaire item dealt with group isomorphisms, and was only answered correctly among the novices by

Jay (although Marcus' response was partially correct). While defining and describing a group's structure is integral to standard game play, spotting structural similarities between groups (the task of identifying group isomorphisms) is not. Jay's correct answer and Marcus' partially correct answer indicate that *Groups* can allow new learners to push even a little bit farther than the bare fundamentals of a group's definition and properties, given accompanying resources; in this case, the questionnaire item contained a textual description of the group $\{I, \times\}$ and a pictorial description of the groups featured in game play. It also highlights the depth of *Groups'* learning experience, as a relatively small but reasonably complicated amount of content was assimilated by individuals with little to no prior knowledge of the information.

Further, the expert participants indicated that *Groups* game play appropriately reflected the fundamentals of group theory, as can be seen by their interview and questionnaire comments. When questioned regarding whether commutativity merits treatment in *Groups*, all of the expert participants agreed that, for the purposes of covering just the fundamental aspects of what constitutes a group, a discussion of commutativity could be omitted, as was the case in the studied game iteration. This reflects *Groups'* design policy of depth over breadth.

All participants agreed that *Groups* would be a valuable addition to a secondary level course. Many participants commented on the game offering insight to mathematics beyond simple numeracy in accessible and enjoyable format. Others were pleased with the design, but still felt that the time commitment required to explain game play, play through several hands, and then discuss game play was just too much for any session of a university course and might be viewed as wasteful. Overall, the novice players seemed to enjoy *Groups* as a learning tool, and the expert players seemed to enjoy it as a learning and review tool, even given its extant shortcomings.

Groups is still a long way from being ideal, and this study has highlighted several avenues for potential further research. First, it would be useful to replicate this study with a larger and more diverse population (up to gender, ethnicity, prior mathematics achievement, etc.), this time including secondary school students. Although this study found that participants were comfortable with most of the representations of group properties, further fine-tuning of associativity and closure is required.

Therefore, the study should also be repeated with future iterations of *Groups* that specifically address these representational issues. Because the choice of colors to represent group elements caused some confusion, the study could possibly be repeated with a new means of informally representing group elements in a straightforward, intuitive fashion. In all future research, I recommend maintaining the research questions that were used to guide this study.

Conclusion

The essence of *Groups'* game play is derived in part from the perspectives and experiences of both group theory novices and experts. Participants describe the game as an engaging experience that imparts fundamentally faithful information on elementary group theory through a novel and enjoyable format. The 1.1 edition's implementations of group properties associativity and closure are somewhat unclear and require revision. Not being an essential group property, commutativity is never discussed, but is visible, within game play. The representation of group elements as colors is somewhat counterintuitive because of the way that element relationships are described, but this can lead to productive struggle; once prior knowledge of colors as they interact in reality is suspended and colors themselves are abstracted, colors serve as an inviting means of representing mathematically challenging ideas. *Groups* can take a fair amount of time to learn and often requires at least one individual to be acquainted with formal group theory on some level. Because of this, initial game play rounds can be lengthy as players begin processing game flow, but this process speeds up dramatically in future rounds. Because of the learning curve and somewhat high initial maintenance level, it's unclear if game play would be attractive for university level courses, but it seems ideal at the secondary level because of its exploratory nature. Players of *Groups* find that they have a strong understanding of what a group is, what a group's properties are, and why structures might not be considered groups. The non-traditional treatment of elementary group theory provided by *Groups* encourages critical thinking and mathematical thought, and provides a learning experience that both appeals to new learners and offers a fresh perspective to group theory experts.

References

Freedman, H. (1983). A way of teaching abstract algebra. *The American Mathematical Monthly, 90*(9), 641–644.

Ke, F. (2008). Computer games application within alternative classroom goal structures: cognitive, metacognitive, and affective evaluation. *Educational Technology Research and Development, 56*(5–6), 539–556.

Kebritchi, M., Hirumi, A., & Bai, H. (2010). The effects of modern mathematics computer games on mathematics achievement and class motivation. *Computers & education, 55*(2), 427–443.

Leron, U., & Dubinsky, E. (1995). An abstract algebra story. *The American Mathematical Monthly, 102*(3), 227–242.

Lesh, R. A. (1976). The influence of an advanced organizer on two types of instructional units about finite geometries. *Journal for Research in Mathematics Education, 7*(2), 82–86.

McCue, C. M. (2011). Learning middle school mathematics through student designed and constructed video games (Doctoral Dissertation). Retrieved from Digital Dissertations. 3460893.

Merriam, S. B., & Tisdell, E. J. (2016). *Qualitative research: A guide to design and implementation.* San Francisco, CA: John Wiley & Sons.

Pedersen, J. J. (1972). Sneaking up on a group. *The Two-Year College Mathematics Journal, 3*(2), 9–12.

Salen, K., & Zimmerman, E. (2004). *Rules of play: Game design fundamentals.* Cambridge, MA: MIT press.

Weber, K., & Larsen, S. (2008). Teaching and learning group theory. *Making the connection: Research and teaching in undergraduate mathematics education, 73,* 139.

Wijers, M., Jonker, V., & Drijvers, P. (2010). MobileMath: exploring mathematics outside the classroom. *ZDM, 42*(7), 789–799.

Appendix A

Interview Protocols:

1) Have you played any games similar to this in the past? How about educational games in general?

2) Did you feel that the ideas represented meshed well with a card-game format? Could you see this as a digital game?

Let's talk about the content of the game.

---

3) [Novice] As a new learner of this content, do you feel that you have a basic understanding of some of the concepts discussed? Did anything in the game remind you of things you'd seen elsewhere in your studies?

4) [Novice] Which concept or concepts do you think were clearest, and which do you think were the most confusing?

5) [Novice] Did any of the representations or vocabulary in the game confuse you? How about the mechanics?

---

3) [Expert] As an individual with experience in university-level mathematics, including algebra, do you feel that the game's representation of group theory concepts was faithful?

4) [Expert] Would you say that the game is compatible with the way you think about groups and group properties? Particularly, what is your opinion on the game's exclusion of commutativity discussions?

5) [Expert] What was your opinion on the representations and vocabulary used within the game?

---

6) Could you see a refined version of this game being used in a classroom setting, and what are your views—if any—on incorporating game play into mathematics curricula at the elementary, secondary, and university levels?

7) What would you do to improve the game for learning, if anything?

Appendix B (Page 1)

Groups Study Questionnaire

1) Construct a description or definition of an algebraic group.

2) Consider the set of numbers $\mathbb{Z} = \{0, \pm1, \pm2, \pm3, \ldots\}$ and the operation + (regular addition). Does this satisfy your description/definition from question 1? If it does not, try to provide at least one counterexample to your description/definition.

3) Consider the set of numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$ and the operation – (regular subtraction). Does this satisfy your description/definition from question 1? If it does not, try to provide at least one counterexample to your description/definition.

4) This question deals with the imaginary unit i, which you may have encountered in prior studies.[1] Consider the group $(I, \times)$ with $I = \{\pm i, \pm 1\}$ and the operation × being regular multiplication. On the back of this sheet are the two blueprint cards from *Groups*. Would you say that the structure[2] of $(I, \times)$ is the same as neither, one of, or both of the structures of the groups on the back? Justify your answer as best you can.

5) What is your final overall opinion of the educational gaming experience presented by *Groups*?

[1] Recall that i is traditionally used to represent $\sqrt{-1}$.

[2] The way(s) the elements of I interact among themselves under operation ×

Appendix B (Page 2)