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# TABLE OF CONTENTS

## PREFACE

- v *Matthew DeGraaf, Teachers College Columbia University*  
*Beatriz S. Levin, Teachers College Columbia University*

## ARTICLES

- 1 **Resources Needed for Addressing Common Core Standards in Mathematics and Language Arts and Next Generation Science Standards**  
*Margaret B. Cozzens, Rutgers, The State University of New Jersey, Center for Discrete Mathematics and Theoretical Computer Science (DIMACS)*
- 9 **The Effects of Interdisciplinary Practice on Elementary Preservice Teachers' Mathematics Teaching Self-Efficacy and Pedagogy: An Intervention Study of Music-Mathematics Integrated Education**  
*Song An, University of Texas at El Paso*  
*Daniel Tillman, University of Texas at El Paso*  
*Carlos Paez, Navajo Technical University*
- 25 **Patterns in the Pythagorean Configuration and Some Extensions: The Power of Interactive Geometry Software**  
*José Contreras, Ball State University*
- 37 **Choosing High-Yield Tasks for the Mathematical Development of Practicing Secondary Teachers**  
*James A. Mendoza Epperson, University of Texas at Arlington*  
*Kathryn Rhoads, University of Texas at Arlington*
- 45 **Classroom Experience about Cartooning as Assessment in Pre-service Mathematics Content Course**  
*Hoyun Cho, Capital University, Columbus, Ohio*  
*Carolyn Osborne, Capital University, Columbus, Ohio*  
*Tobie Sanders, Capital University in Columbus, Ohio*

## Music-Themed Mathematics Education as a Strategy for Improving Elementary Preservice Teachers' Mathematics Pedagogy and Teaching Self-Efficacy

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**ABSTRACT** This study investigated the effects upon elementary preservice teachers' mathematics teaching self-efficacy and interdisciplinary lesson design strategies, which resulted from an educational intervention that emphasized integrated music-mathematics instruction. The participating elementary preservice teachers (n=152) were recruited for this study from the College of Education students enrolled at a medium-sized southwestern research university in the United States. Several sources of data were collected, including pre- and post-intervention surveys assessing self-efficacy for mathematics teaching and interdisciplinary-themed instructional design. The overall findings from both surveys indicated that the participating preservice teachers' mathematics teaching self-efficacy and their interdisciplinary teaching strategies were improved as a result of the intervention. This paper concludes with a discussion of the potential applications to pedagogical practice that accrue from the study's findings, as well as suggestions for further research extending this line of inquiry.

**KEYWORDS** *interdisciplinary curriculum, arts-based education, mathematics education, teacher education, teaching efficacy*

One of the foremost recognized barriers to student mathematics learning is low self-efficacy, with the negative relationship between low self-efficacy in mathematics and mathematics achievement firmly established within previous empirical literature (e.g., Sloan & Giesen, 2002; Satake & Amato, 1995). Factors contributing to low self-efficacy that have been recognized within traditional mathematics instructional methods and curriculum include: (a) teachers assigning the same problem to every student, (b) teachers utilizing the majority of class time to lecture about content from the textbook, (c) teachers insisting on there being only one "correct" way to solve a problem, and (d) teachers neglecting students' conceptual understanding in favor of improving performance on standardized tests (Furner & Berman, 2005). Contextualizing mathematics education within constructivist problem solving activities which use

simulations, challenges, discoveries, and games have been examined as effective methods for improving students' mathematics self-efficacy (Gresham, 2008). Teaching mathematics within a meaningful contextualization not only produces opportunities for students to improve their mathematics self-efficacy by obtaining a personalized comprehension of abstract concepts, but also provides teachers resources for designing and implementing problem-based mathematics lessons that are student-centered (Berlin & Lee, 2005; Hargreaves & Moore, 2000; Lampert, 2006).

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### Conceptual Framework

Over the past decade, mathematics educators in collaboration with mathematics education researchers have developed and field tested curriculum contextualizing

mathematics within numerous fields, including science (e.g., Berlin & Lee, 2005; Stinson, Harkness, Meyer, & Stallworth, 2009), language arts (e.g., Casey, 2004; Keen, 2003), history (e.g., Charalambous, Panaoura, & Philippou, 2009), athletics (e.g., Noubary, 2010), visual arts (e.g., Jarvis & Naested, 2012), and drama (e.g., Duatepe-Paksu, & Ubuz, 2009). Among the various mathematics oriented interdisciplinary pedagogies and curricula, music-themed mathematics lessons are one of the research areas that has been empirically investigated and evaluated, using both (a) psychological methodologies for measuring intellectual and behavioral tasks within laboratory or informal education settings (e.g., Costa-Giomi, 1999; Rauscher, Shaw, & Ky, 1993, 1995), as well as (b) pedagogical methodologies addressing attitudinal and academic tasks within formal class settings (e.g. An, Capraro, Tillman, 2013; An, Tillman, Boren, & Wang, 2014, An & Tillman, 2015; Cox & Stephens, 2006).

Within the present study, the authors categorized the numerous interrelationships between music and mathematics into three different levels: (a) the subject area level, (b) the cognitive level, and (c) the pedagogical level. Most published studies examining connections between music and mathematics have occurred at either the subject area level or at the cognitive level. Subject area level studies have investigated the mathematics in music and the application of mathematics to improving music composition and instrument design. Studies performed at the cognitive level have investigated the impacts of musical experiences upon mathematical cognition. In contrast, there is a need for more studies to be done performed at the pedagogical level, creating a lacuna deserving of further investigation into music as a context for classroom educational activities that help students conceptually understand mathematics.

### **Subject Area Level: Mathematics in Music Creation**

The first integration of mathematics with music that historians are aware of occurred more than 2500 years ago when the Greek mathematician Pythagoras identified the ratio of percussion instrument sizes and weight among harmonic music intervals (an interval is the difference between two pitches), specifically that the corresponding ratio between the intervals of an octave is 1:2, and between the intervals of a fifth is 2:3 (Ferreira, 2002). In the time since Pythagoras's discovery, musicians and mathematicians have often collaborated together towards common goals such as improving acoustics, standardizing auditory measurement tools,

and creating complex musical instruments such as piano (Fauvel, Flood, & Wilson, 2006).

Fundamental mathematical concepts such as arithmetic computations, geometrical transformations, and algebraic patterns are utilized in contemporary music composition and instrument design, as are more advanced mathematical concepts taught in college-level mathematics courses such as calculus, linear algebra, and analytical geometry (Harkleroad, 2006). As an illustration, both classical and present-day composers have incorporated Fibonacci numbers, the golden section, transpositions, and inversions into their musical arrangements during the music composition process (Beer, 1998; Loy, 2006). Likewise, logarithms and differential equations have been utilized to create equal temperament (a system of tuning that is extensively used among western musical instruments, in which all pairs of notes have an identical frequency ratio that is determined artificially) within music by distributing inharmonic errors in an octave across all individual intervals (Cho, 2003).

### **Cognitive Level: Music Impacts Mathematical Cognition**

Experimental research in lab settings has found measureable correlations between music-themed activities and proficiency at mathematical cognition, including music listening improving spatial-temporal reasoning (Cheek & Smith, 1999; Hetland, 2000; Ivanov, & Geake, 2003; Rauscher et al., 1993) and learning to play a musical instrument improving mathematics achievements (Bilhartz, Bruhn, & Olson, 2000; Rauscher & Zupan, 2000). One influential series of studies among these investigations has been the so-called "Mozart effect" studies (Rauscher et al., 1993). Rauscher and his colleagues discovered that participants who were assigned to listen to Mozart's music significantly outperformed their peers who listened to generic relaxing music, as well as another group listening to silence, on spatial-reasoning assessments. Follow-up studies found similar results from the Mozart effect for several additional mathematical ability areas including capability of mental visualization of three dimensional figures, mathematics problem solving skills, and memory abilities (Hui, 2006; Ivanov & Geake, 2003; Rauscher, Shaw, & Ky, 1995; Rideout & Laubach, 1998; Nantanis & Schellenberg, 1999).

Additionally, researchers have found significant statistical correlations between learning to play a musical instrument and achievement gains in mathematics. In experimental studies comparing a treatment group

with a control group, K-12 students who participated in learning to play certain musical instruments demonstrated significantly higher mathematics achievement scores than their peers who did not receive music lessons (Costa-Giomi, 1999; Cox & Stephens, 2006; Haley, 2001; Kafer & Kennell, 1998; Rauscher & Zupan, 2000; Whitehead, 2001). A key component in explaining why some musical experiences improved mathematical cognition is that music can stimulate areas of the brain also responsible for mathematical reasoning (Rauscher et al., 1995). Specifically, Spelke (2008) demonstrated that the cognitive mechanisms responsible for processing melody, harmony, and rhythm also activate brain functions improving capacity at mathematical cognition including numerical calculation and estimation.

### **Pedagogical Level: Using Music to Teach Mathematics Content in Context**

The natural connections between music and mathematics provide opportunities for mathematics educators to create pedagogical tactics based upon meaningful inquiry activities wherein learners apply, represent, and understand mathematics through contextualized approaches (Lesser, 2001; An, Ma, Capraro, 2011; An, et al., 2013, 2014). Mathematics lessons integrated with musical themes such as music composition and musical instrument design have been used to address a wide variety of mathematics topics including operations, geometry, and statistics—resulting in statistically significant improvements in participating K-12 students' mathematics achievement and dispositions (An & Tillman, 2015; Carrier et al., 2011; Colwell, 2008; Costa-Giomi, 2004; Courey et al., 2012; Johnson & Edelson, 2003).

Mathematics education using music-themed activities offers teachers the opportunity to present challenging mathematical concepts to their students via an alternative approach that encourages students to analyze and understand mathematics within a meaningful context (An & Tillman, 2014). Numerous benefits have been reported for both students and teachers as results from music-mathematics integrated instruction, including: (a) motivating students to participate in mathematics tasks (Glastra, Hake, & Schedler, 2004); (b) engaging students in investigation of mathematical relationships (Parson, 2005); (c) providing students' with a collaborative learning environment (Robertson & Lesser, 2013); and (d) providing a context that serves to minimize language barriers for English-language learner (ELL) students (An et al., 2013).

Previous empirical literature on the benefits of integrated music-mathematics education has focused primarily upon the impacts on students, but more studies are needed investigating the effects of such interdisciplinary experiences on teachers, both inservice and preservice, and any impacts upon their mathematics teaching self-efficacy and instructional design strategies. As part of a larger NSF-funded research project created to improve preservice and inservice K-8 primary school teachers' pedagogical content knowledge by teaching STEM content through interdisciplinary approaches, the current mixed-methods study investigated impacts upon participating preservice teachers' mathematics teaching self-efficacy and their development of music-themed mathematics pedagogy design strategies resulting from an intervention involving professional development in how to use interdisciplinary music-mathematics teaching strategies.

The two guiding research questions for this study were:

1. What were the impacts from interdisciplinary music-mathematics professional development upon elementary preservice teachers' mathematics teaching self-efficacy?
2. How did elementary preservice teachers' mathematics teaching strategies change as a result from learning to teach mathematics integrated with music activities?

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### **Methods**

A convergent mixed-methods design is used to collect both quantitative and qualitative data, each distinctly analyzed; however, the two kinds of data were merged during interpretation as evidences to support each other. Quantitative data from participants included a 30-item survey, used as empirical evidence to address the first research question. Qualitative data from participants included written answers for open-response tasks, and these answers were analyzed to address the second research question. The two research questions were designed so that the quantitative evidences from the first research question determined the development of the key themes that were coded and analyzed as qualitative evidences to address the second research question.

## Participants and Setting

As part of a larger line of research focused upon improving preservice elementary teachers' knowledge of mathematics pedagogy as well as self-efficacy towards mathematics teaching, the preservice teacher participants in this project focused on learning and exploring musi-contextualized mathematics teaching approaches. The general framework was that the participants were: (a) introduced to sample music-themed mathematics lessons as an intervention, (b) assigned the task of designing arts-themed interdisciplinary mathematics curriculum, and (c) evaluated the arts-themed mathematics instructional designs created by their peers and their own group. We report here the first part of this three part process, investigating the effects of the intervention portion of the framework upon the preservice teachers' self-efficacy towards mathematics teaching and knowledge of how to contextualize mathematics education within music activities. Data collected quantitatively and qualitatively from the participating teachers for this study included: (a) a pre-survey and post-survey measuring their mathematics teaching self-efficacy, and (b) a pre-survey and post-survey measuring their knowledge of utilizing interdisciplinary pedagogy as a strategy for contextualizing mathematics education.

The current study took place at a research university located within a southwestern metropolitan area of the United States. The university has an enrollment of approximately 23,000 students, with over 75% of all student body predominantly bilingual in Spanish and English. The participants in the current study were 152 undergraduate preservice teachers enrolled in one of four sections of elementary education methods courses within an elementary teacher preparation program. Among the 152 participants, there were 136 female preservice teachers and 16 male preservice teachers. Demographic data collected from the participants indicated 125 preservice teachers self-reported as Hispanics, 24 self-reported as Caucasian, and 3 self-reported as African American. Consent for participation in the study was obtained by graduate teaching assistants, and the consent forms were collected anonymously so that the instructors would not know which of their preservice teacher students were participating in the study.

## Intervention

The sample music-themed mathematics lessons that were demonstrated during the intervention were co-developed by four college professors having specializations in different areas relevant to the study—including mathematics education, music education, mathematical science, and educational technology respectively. The intervention was delivered to the participants during their regular class meetings and lasted eight-hours in total for each participant. Although the participants from the four sessions had different time schedules for participating in the intervention activities, the researchers took due diligence to try and ensure that all sections of courses that participated received an equivalent repertory of activities during the intervention. In general during the intervention the participants had opportunities to tryout, practice, and evaluate various mathematical concepts and skills embedded within a series of music-themed activities. The two primary interdisciplinary activity formats that the intervention offered were contextualizing mathematics education through: (a) music composition and playing processes, and (b) musical instrument design and construction processes. Throughout various music activities within these two themes, mathematical problems based upon the participating preservice teachers' own original musical creations were proposed for them to solve, and mathematics teaching methods were discussed based on these interdisciplinary activities.

Throughout the intervention, a series of progressively more advanced music-mathematics interdisciplinary activities were presented to preservice teachers (see sample activities in Appendix). Towards the beginning of the intervention activities, the preservice teachers interacted with professional musicians, mathematicians, music educators, and mathematics educators with a goal of identifying naturally occurring music-mathematics connections that supported interdisciplinary pedagogical opportunities. Preservice teachers were also introduced to mathematics activities that included music composition and performance, including:

- Examining and evaluating musical masterpieces by investigating wave phenomena, mathematical relationships of complex music structures, rhythm patterns, waveform shapes, frequency components, timbre, and wave modulation such as sustain and tremolo;

- Composing original music based upon algebraic patterns and geometrical transformations using graphical notation systems such as colors to represent musical notes that are then played on color-matching handbells.

Participating preservice teachers were also introduced to mathematics activities that integrated musical instrument design and construction, including:

- Constructing physical materials such as paper, wood, and recycled plastic into the parts for musical instruments including stringed, wind-based, and percussion instruments;
- Manipulating variables such as length, size, volume, shape, material, and tension to determine the impact on the sound properties of pitch, tone, timbre, loudness, and resonance time; and
- Designing blueprints for musical instruments based upon combinations of geometric shapes within coordinate systems.

### Instrument and Data Collection

A pretest-posttest data collection design was utilized to assess the effects of the intervention on the participants' mathematics teaching self-efficacy as well as their music contextualized mathematics teaching strategies. The instrument had two sections: (a) a survey measuring mathematics teaching self-efficacy utilizing 30 items and a Likert scale for response options, and (b) a survey assessing mathematics teaching strategies utilizing five questions asking for open-ended responses. Prior to the beginning of the intervention all of the preservice teacher participants completed both surveys as an assessment of their self-efficacy beliefs about interdisciplinary approaches to mathematics pedagogy and music contextualized mathematics teaching strategies. At the end of the intervention the preservice teachers again completed the same two survey instruments.

The survey with 30 items using a Likert scale of responses was developed based upon items from the *Attitudes Toward Mathematics Inventory* (Tapia, Marsh & George, 2004) and the *Mathematics Teaching Efficacy Beliefs Instrument* (Enochs, Smith, & Huinker, 2000) by choosing appropriate items, and then revising them to be more specifically orientated towards assessing interdisciplinary mathematics pedagogy. The final survey instrument consisted of 30 items, with five levels of responses available along a Likert scale ranging from

“strongly disagree” to “strongly agree.” The overall alpha reliability coefficient was 0.835 for the survey. Survey items focused on four major themes: (a) seven items were on self-efficacy for teaching mathematics via interdisciplinary pedagogy, (b) eight items were on self-efficacy for motivating students to participate in mathematics tasks, (c) seven items were on self-efficacy for teaching mathematics within music contextualized pedagogy, and (d) eight items were on providing a positive mathematics classroom environment.

The survey with the open-ended questions was developed based upon National Council of Teachers of Mathematics (2000) content standards. This survey included five mathematics content areas—namely number and operations, algebra, geometry, measurement, and data analysis and probability—each as an instructional task for which the preservice teachers developed pedagogical strategies to teach that used music-themed activities as a context for presenting the mathematical concepts to students. Throughout this open-ended questions survey, the participating preservice teachers were requested to specify their use of music activities as a pedagogical resource for helping students make sense of challenging mathematical concepts, and their strategies for connecting music and mathematics for their classroom.

### Data Analysis

A paired-samples t-test was used to determine any statistically significant differences in mean scores and standard deviations between the close-ended survey administered before and after the intervention. Specifically, descriptive information including means and standard deviations were compared, and significance tests were analyzed to determine whether there existed any statistical differences between pretest and posttest results. Effect sizes utilizing Cohen's *d* were also calculated as an indicator of practical significance indicating the scope of any change as a result of the intervention.

Grounded theory was used to analyze qualitative data that was collected with the open-ended survey. This qualitative data was then coded into thematic clusters and/or categories using an inductive constant comparative approach (Patton, 2002). Specifically, first, data from the pretest and posttest were compared case by case while generating flexible categories via searches for different teaching strategies proposed by the preservice teachers. Next, the clustering and categorization was then further refined based upon responses that combined



similar items into one category. Finally, the clusters and categories were compared between the pre-survey and post-survey to check for any critical differences in each of the mathematics content areas.

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## Results

The preservice teachers demonstrated statistically significant improvement from pretest to posttest within the following themes: (a) the  $p$  value of self-efficacy for teaching math via interdisciplinary pedagogy was 0.002 for the seven items within this factor, and a medium effect size of  $d=0.42$  was found between pretest and posttest; (b) the  $p$  value of self-efficacy for motivating students to participate in math tasks was less than 0.001 for the eight items within this theme, and a medium-large effect size of  $d=0.75$  was found between pretest and posttest; (c) the  $p$  value of self-efficacy for teaching math in music contextualized pedagogy was less than 0.001 for the seven items within this theme, and a large effect size of  $d=1.13$  was found between pretest and posttest; and (d) the  $p$  value of self-efficacy for teaching math in music contextualized pedagogy was less than 0.001 for the seven items within this theme, and medium-large effect size of  $d=0.90$  was found between pretest and posttest. Overall the  $t$ -test results and calculated effect sizes presented in Table 1 indicated that after the intervention the preservice teachers displayed statistically significant higher scores on their self-efficacy for mathematics teaching.

The results from the open-ended survey presented in Tables 2 and 3 are consistent with the quantitative findings, which demonstrated that the preservice teachers' strategies for using music-themed activities as pedagogical contexts for mathematics concepts had noticeable changes in all five mathematics content areas. More teaching strategies were proposed by the preservice teachers, and they also presented further advanced music-themed strategies in the posttest than the pretest. In the pre-survey most of the strategies the preservice teachers provided were based on superficial relationships between music and mathematics, such as counting beats or singing a song with mathematics-themed lyrics, with number and operations as the principal content area that the preservice teachers focused upon. In the post-survey, the percentage of strategies based on

superficial relationships decreased. Instead, pedagogical strategies that the preservice teachers developed were based on more advanced music processes, such as music composition and instrument designing, with lessons that covered topics across all five mathematics content areas.

In terms of the content area of number and operations, in the pretest 116 (76.32%) preservice teachers proposed seven different music themed strategies to teach one of the concepts in content areas; in the posttest, nine more new music themed strategies were identified and all 152 (100%) preservice teachers proposed at least one way to teach one of the concepts in number and operations. In terms of the content area of algebra, in the pretest 43 (28.29%) preservice teachers proposed five different music themed strategies to teach one of the concepts in content areas; in the posttest, eight more new music themed strategies were identified and 106 (69.73%) preservice teachers proposed at least one way to teach one of the concepts in algebra. In terms of the content area of geometry, in the pretest 70 (46.52%) preservice teachers proposed four different music themed strategies to teach one of the concepts in content areas; in the posttest six more new music themed strategies were identified and 126 (82.89%) preservice teachers proposed at least one way to teach one of the concepts in geometry. In terms of the content area of measurement, in the pretest 62 (40.79%) preservice teachers proposed four different music themed strategies to teach one of the concepts in content areas; in the posttest six more new music themed strategies were identified and 119 (78.29%) preservice teachers proposed at least one way to teach one of the concepts in measurement. In terms of the content area of data analysis and probability, in the pretest 43 (28.29%) preservice teachers proposed two different music themed strategies to teach one of the concepts in content areas; in the posttest seven more new music themed strategies were identified and 117 (76.97%) preservice teachers proposed at least one way to teach one of the concepts in data analysis and probability.

As illustrative examples, the instructional design of music-mathematics integrated lessons by two preservice teachers, Cyndi and David (names have been changed to pseudonyms), will be exhibited and the differences between the lessons that they proposed in the pre-survey and the post-survey will be contrasted.

Table 1

*Comparison of Pretest and Posttest Interdisciplinary Math Teaching Self-Efficacy of Participating Preservice Teachers*

<i>n</i> =152 Themes	No. of Items	Pretest Mean (SD)	Pretest Mean (SD)	<i>p</i> ( <i>t</i> ) Value	Cohen's <i>d</i>	Sample Items
Efficacy for teaching math through interdisciplinary pedagogy	7	3.71 (0.50)	3.90 (0.41)	0.002 (3.189)	0.42	I need to learn more about how to integrate math with other subjects I will teach.
Efficacy for motivating students to participate in math tasks	8	3.32 (0.45)	3.64 (0.40)	<0.001 (6.249)	0.75	I know of teaching methods that will develop student's interest in mathematics.
Efficacy for math teaching via music contextualized pedagogy	7	3.22 (0.34)	3.60 (0.33)	<0.001 (8.674)	1.13	I can think of many ways to teach math using music.
Efficacy for providing a positive math classroom environment	8	3.73 (0.46)	4.16 (0.49)	<0.001 (6.594)	0.90	I can think of ways to teach math that are fun for students.

Table 2

*Comparison of Music-Mathematics Interdisciplinary Teaching Strategies that Emerged Between Pre-Survey and Post-Survey (Part I)*

Math Content Area		Major Music-Themed Strategies
Number & Operations	Strategies emerged in pre-survey	<ul style="list-style-type: none"> <li>Recognize number by linking numbers with specific tones/notes</li> <li>Sing songs with operation facts/process as lyrics</li> <li>Count beats/notes in songs</li> <li>Play music video with number as contexts</li> <li>Compose and play music by using number as notations</li> <li>Represent number and operation through music/rhythm playing</li> <li>Practice additions and subtraction based on value of notes</li> </ul>
	Strategies emerged in post-survey	<ul style="list-style-type: none"> <li>Count/group number of instruments in contexts of concert</li> <li>Conduct operations based on number of keys/strings/frets on instruments</li> <li>Demonstrate number sequences on piano keyboard</li> <li>Introduce base-eight number system through music scales</li> <li>Explore fraction/percentage through analysis of music works</li> <li>Represent negative numbers on staff/musical instruments</li> <li>Compare differences of notes within and among musical works</li> <li>Explain operation rules through demonstration of chords variations</li> <li>Explore fractions through musical notation/instrument development</li> </ul>
Algebra	Strategies emerged in pre-survey	<ul style="list-style-type: none"> <li>Represent unknowns/variables through musical notation or performances</li> <li>Apply formulas in composing music</li> <li>Investigate patterns in existing music works/musical instruments</li> <li>Create patterns through music composition</li> <li>Play music video with algebra as context</li> </ul>
	Strategies emerged in post-survey	<ul style="list-style-type: none"> <li>Introduce ratio and proportion through analysis of music works</li> <li>Demonstrate ratio and proportion on musical instruments</li> <li>Find unknowns/missing values in music works</li> <li>Introduce functions through music composition</li> <li>Explore sequence through music composition</li> <li>Explore factors through music composition</li> <li>Construct equations through investigation of musical notations/forms</li> <li>Use coordinate system to design instruments and compose music</li> </ul>

Table 3

*Comparison of Music-Mathematics Interdisciplinary Teaching Strategies that Emerged Between Pre-Survey and Post-Survey (Part II)*

Math Content Area		Major Music-Themed Strategies
Geometry	Strategies emerged in pre-survey	<ul style="list-style-type: none"> <li>• Play music video with geometry as context</li> <li>• Sing songs with names of geometrical shape as lyrics</li> <li>• Explore areas and volumes in musical instruments</li> <li>• Introduce shapes through musical instruments</li> </ul>
	Strategies emerged in post-survey	<ul style="list-style-type: none"> <li>• Explore geometrical transformation in music works</li> <li>• Design musical instruments based on 2-dimensional shapes</li> <li>• Design musical instruments based on 3-dimensional shapes</li> <li>• Construct musical instruments based on solid materials</li> <li>• Compose music through geometrical transformations</li> <li>• Introduce prosperities of shapes through musical instrument design</li> </ul>
Measurement	Strategies emerged in pre-survey	<ul style="list-style-type: none"> <li>• Identify frequency of vibration through music instrument playing</li> <li>• Explore concept of time through music listening and playing</li> <li>• Play music video with measurement as contexts</li> <li>• Sing songs with characteristics of units as lyrics</li> </ul>
	Strategies emerged in post-survey	<ul style="list-style-type: none"> <li>• Measure angle, length, areas, volumes, and weight of musical instruments</li> <li>• Measure speed and dynamic through music composition and playing</li> <li>• Converting measurement units through music instrument development</li> <li>• Apply non-standard units to measure musical instruments</li> </ul>
Data Analysis & Probability	Strategies emerged in pre-survey	<ul style="list-style-type: none"> <li>• Play music video with data analysis and probability as context</li> <li>• Sing songs about concepts in data analysis and probability as lyrics</li> </ul>
	Strategies emerged in post-survey	<ul style="list-style-type: none"> <li>• Collect and analyze data based on music works</li> <li>• Develop statistical graphs based on music works</li> <li>• Explain statistical concepts through analysis of music works</li> <li>• Conduct analysis of events within musical compositions</li> <li>• Explore combination and permutation through musical instrument playing</li> <li>• Develop probability problems based on music contexts</li> <li>• Explore combinations of notes/chords/forms within music pieces</li> </ul>

### Cyndi's Lessons

Cyndi, a former flute player in high school orchestra, is a preservice teacher with a medium level music background. In the pre-survey, she proposed a strategy of teaching fractions connected with musical notations of note values. The pedagogy was developed based on the natural fraction-themed relationships between music and mathematics but without further exploration, and the role of auditory representations of music in the lesson was limited to only having the students clapping notes with different lengths of time. The lesson is more like a music lesson that is based on fraction concepts instead of being an actual mathematics lesson with music activities. Cyndi described her own lesson as:

The goal of my lesson is to help students understand how musical notes relate to fractions and clap a measure of music by assigning appropriate values to

notes. In the lesson I will have following steps: (1) present note names and their values and introduce lesson vocabulary such as whole note [4 beats], half note [2 beats], quarter note [1 beat], eighth note [1/2 beat], sixteenth note [1/4 beat]; (2) present a music note chart to demonstrate the values each note holds and students will be clapping along to gain understanding of the values of the notes in a measure; (3) discuss the value of different notes to help students "hear" the value of those notes, clap a 4-beat measure, a 2-beat measure, and a 1-beat measure and have students join in; (4) ask questions regarding the counting values of music notes (How many beats are in 1 whole note? How many beats are in 2 half notes?) and check and correct the student answers; and (5) on scratch sheet of paper, in groups, students will solve some math problems such as "two eighth notes equal to \_\_\_\_\_", and "four quarter notes equal to \_\_\_\_\_".

After being inspired by the methods that the instructors presented during the intervention (i.e., of using graphical notation wherein colors represented music notes), Cyndi adopted a revised graphical notation system into her own lesson designs. Her lesson reported in the post-survey taught probability based on music composition activities, and she also developed activities wherein students conduct probability experiments as a way to understand prediction methods and possible outcomes. Handbell playing activities were also offered for students to experience the music that they had composed during the probability experiments. Cyndi described her revised lesson as:

In this lesson I will prepare several bags of skittles [sic], brown paper bags, and index cards for my students to compose music through skittle drawing activity. First, I will explain to the students that they are now going to conduct a music color skittles drawing experiment and record the data on the index card. Then, I will model the process of the experiment: (1) Put different color of skittles in a big bag (e.g. 5 red, 4 purple, 3 green); (2) Find out the result—which color skittles is drawn (red or purple or green?); (3) Record the result pattern on the index card; (4) As the skittles are being drawn and record the result on the sample record chart (After drawing a skittles, the skittle should be back in the bag); (5) After finishing the experiment, count how many red or purple or green skittles were drawn and write down the total numbers. Based on the probability experiment results of what color of skittles they got, students will make music by playing bells. After finishing the experiment, the class will make an inquiry based conclusion using the data they recorded from the experiment, and during this time I will introduce the vocabulary “experimental outcome” and “theoretical outcome.”

### David's Lessons

In the pre-survey, David proposed a mathematics lesson with a music listening activity as the introduction of the lesson. In this proposed lesson, decimal computation was contextualized into birthday gift purchasing activities, and the *Birthday Song* was used to introduce the target mathematics topic. However, this music listening activity was only used at the beginning of the class as a means to introduce a cover-story for mathematics, and there was no follow up afterwards within the lesson integrating the music activity into the mathematics learning process. David provided the following description of the instructional process in his lesson plan:

The objective of this lesson is to help students learn how to budget through addition and subtraction of decimals. First, I will introduce the lesson by playing the *Birthday Song* on a piano and letting my students know this lesson is about shopping for their favorite toys or on their birthday. After my piano playing, I will have a little review lesson to explain how adding and subtracting thousands is very similar to subtracting hundreds. Then, I will initiate a discussion about birthday shopping, and I will clarify there is a solution to get all you want without running out of money, which is called budgeting. By knowing how much the budget each children have for their birthday they can figure out what they will buy before going to the store. Finally, my students will be given an amount they can consume for their birthday, and they will then decide how many items they want and compute the total value of their favorite items.

Different from the superficial use of music in David's original design for a lesson, his revised lesson offered a more substantial degree of integration between the music theme and mathematics content by utilizing music to illustrate mathematics concepts at a more sophisticated level. Rather than passively listening to a piece of music, David's revised lesson connected the algebraic concepts of number pairs and coordinate systems with the musical concepts of harmonic intervals. Within his lesson's pedagogy, number pairs were represented via audio by playing musical note pairs forming different harmonic intervals. Within this lesson, David asked the students to compose music within a geometric coordinate system by placing a sequence of numbers on an x-axis and a y-axis. Thereby the students get to hear their own original algebraic compositions as a series of auditory harmonic intervals. David provided the following description of the lesson he designed:

The objective of this lesson is for students to reinforce their basic math and to make algebra concepts easier to comprehend. First, I will start by playing a single note and harmonic intervals on a piano keyboard and I will let my students to know the differences of sound effects between single note playing and harmonic intervals. After that I will start playing more harmonic intervals with variations. For example I will play a major third (Do & Mi), a perfect fourth (Re & So) and a perfect fifth (Do & So) for my students to understand the sound effect differences. Then, I will start introducing the coordinate system and to explain how to represent number pairs in the algebraic format such

as  $(x, y)$ . I will show a variety of number pairs in the coordinate system, and then find the corresponding musical notes based on the numeration notation and play the music. After a few practices and a little fun, I will ask my students to compose music by locating number pairs on the y-axis and the x-axis in a coordinate system and they will play music based on harmonic intervals that they created.

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## Discussion

During the intervention described in the current study, the preservice teacher participants were offered opportunities to systematically experience interdisciplinary mathematics teaching strategies, with a focus on developing their understanding of how to contextualize elementary level mathematics concepts into engaging age-appropriate music activities for young students. The current study was undertaken using a pretest-posttest research design so as to investigate the effects of music-themed interdisciplinary mathematics education experiences on preservice teachers' mathematics teaching self-efficacy and their interdisciplinary mathematics pedagogy strategies. The overall findings from both surveys indicated that the participating preservice teachers' mathematics teaching self-efficacy and their interdisciplinary teaching strategies were improved as a result of the intervention. The current study added empirical evidence on how interdisciplinary pedagogy, which in this case was a combination of music and mathematics, can positively impact preservice teachers' self-efficacy and capacity for meaningfully contextualized instruction that generates student engagement with difficult academic content.

### Improvement of Preservice Teachers' Mathematics Teaching Self-Efficacy

The statistically significant higher scores on the posttest than the pretest for the mathematics teaching self-efficacy survey demonstrated that the preservice teachers improved their mathematics teaching self-efficacy in all four major themes measured. These findings were consistent with findings from previous studies (e.g., An, Ma, & Capraro, 2011; Gresham, 2007; Furner & Berman, 2005; Gresham, 2008) that showed offering preservice teachers' opportunities for contextualizing mathematics education within inquiry-based activities reduced preservice teachers' anxiety towards mathematics and improved their mathematical dispositions and self-efficacy for mathematics teaching. In this current study, the natural connections between mathematics and music

were developed into educational materials, and a highly engaging learning environment was created based upon these lessons.

Throughout the intervention, the preservice teachers experienced different levels of integration between the two school subjects of mathematics and music. The music-related creative processes enabled the preservice teachers to further understand the relationships between music and mathematics—after the preservice teachers finished their creation of musical products they evaluated their works, and from these procedures the preservice teachers learned how to represent mathematical concepts in multiple mediums. From these enjoyable sense making experiences the preservice teachers learned that teaching mathematics integrated with music is not a difficult approach to implement, resulting in their original negative perception of mathematics pedagogy becoming reshaped, and their self-efficacy for mathematics teaching improved.

Our results showed that the participating preservice teachers not only improved their self-efficacy for teaching mathematics via music contextualized pedagogy, but also improved their self-efficacy for more general interdisciplinary pedagogy as well. As illustrations, the preservice teachers' self-efficacy for teaching mathematics through interdisciplinary pedagogy, as well as their self-efficacy for providing a positive mathematics classroom environment to students, were both improved. This finding suggests that the pedagogical strategies that the preservice teachers learned from the intervention empowered them to: (a) create an enjoyable classroom environment by using the arts as a medium to aesthetically engage students, (b) propose meaningful inquiry-based activities based on interdisciplinary themes that can serve to motivate students to participate in mathematics related tasks, and (c) develop multiple types of mathematics lessons that can facilitate students understanding of mathematics concepts via the various connections between mathematics and other school subjects.

### Development of Preservice Teachers' Mathematics Teaching Strategies

The increased diversity and amount of strategies presented in the open-ended survey from pretest to posttest indicated that the preservice teacher participants in this study improved their knowledge of utilizing music activities as educational resources for developing contextualized mathematics lessons. In the pre-survey, the preservice teachers proposed music strategies that linked with all five of the mathematics

content areas, but almost all of the strategies presented were fairly limited in scope regarding mathematical depth, with music often only treated as a superficial cover-story for mathematics word problems. The two most prevalent music-mathematics integration strategies that the preservice teachers reported in the pretest were using music lyrics to teach basic arithmetic topics, or playing background music during math class to make the experience more pleasant but with no connection to content or pedagogy. For example, some of the teachers on the pretest stated that they planned to play a music video with song lyrics that introduced mathematics vocabulary to the class. On the pretest almost all of the preservice teachers failed to go beyond music-mathematics connections deeper than background noise or song lyrics about math, with only less than 10% of preservice teachers developing music components that could be considered inquiry-based activities that would facilitate students in developing conceptual understand of mathematics rather than simply support them in rote memorization.

In the post-survey, however, the findings demonstrated that the preservice teachers not only demonstrated more types of music-themed activities integrated into mathematics lessons, but also covered broader mathematics content areas than their responses in the pre-survey. For musical processes, a variety of activities were all used as inquiry-based interdisciplinary themes for contextualizing mathematical concepts, including: music elements exploration such as planning rhythm, investigating intervals, and transferring chords; music composition such as creating melody, organizing musical form, and arranging instrumentation; and musical instrument making such as designing and crafting instruments. As the cases of Cyndi and David's lesson plans illustrated, rather than merely using music to create a more pleasant learning environment for students to learn number and operations (as indicated on the pre-survey), instead the preservice teachers presented (on the post-survey) a range of pedagogy within a variety of mathematics content areas, including: (a) exploring algebraic patterns and geometrical transformations within existing musical works; (b) applying statistical knowledge such as measurement and data analysis as mathematical tools for supporting music analysis and creation processes; and (c) representing mathematical ideas through multiple representations including singing, playing, composing, decomposing, and recomposing of music works.

The differences between the strategies that the preservice teachers anticipated employing during the

pre-survey versus those they described during the post-survey can be reasonably attributed to the intervention, and the impacts it had upon the preservice teachers as they learned a variety of music-mathematics integration strategies. Apparently the innovative mathematics teaching strategies presented during the intervention—such as students employing graphical notation using music composition cards, as well as utilizing mathematics manipulatives and contextualizing mathematics concepts into musical instrument construction processes—helped the preservice teachers to understand alternative methods for developing mathematics lessons. From the results it appears that, to some extent, these intervention activities: (a) enlarged the preservice teachers' understanding of interdisciplinary pedagogical content knowledge, and particularly curriculum knowledge about teaching meaningfully contextualized mathematics that is designed to maintain student engagement; and (b) guided the preservice teachers to design innovative interdisciplinary lessons that utilized the relationships between music and mathematics. Furthermore, the research findings also suggested that the preservice teachers' improved their knowledge for preparing mathematics lessons that incorporate meaningful connections to subjects outside of the strict domain of abstract mathematics concepts, and therefore are more engaging to students. Specifically, the preservice teachers learned how to find and develop potential educational resources like music activities into a part of their mathematics instruction.

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## Conclusion

Improving teacher quality has been identified as one of the key factors in closing the mathematics achievement gap between minority and non-minority students (The Education Alliance, 2006). Teachers who are highly qualified in mathematics are those who have strong pedagogical and mathematics content knowledge, as well as a rich understanding of how student best learn mathematics, and such teachers produce significantly higher teaching outcomes than their less competent peers (Singham, 2003). However, existing studies provide evidence that a majority of elementary preservice teachers are not sufficiently prepared in their content knowledge, pedagogical content knowledge, or mathematics teaching self-efficacy before they start teaching in classrooms (Ball, 1990; Darling-Hammond & Baratz-Snowden, 2007; Fuller, 1997; Swars, Daane, & Giesen, 2006). For the most part, teacher education programs

have failed to provide the appropriate experiences to preservice teachers needed for them to develop high quality mathematics pedagogy, engage students in meaningful mathematics activities, and design and implement innovative mathematics teaching strategies (Gresham, 2007; Knoblauch & Hoy, 2008). Additionally, many of the existing teacher education resources that claim to provide entertainment-contextualized mathematics pedagogy are in actuality primarily entertainment-oriented (e.g., Moomaw, 2011, 2013). Such curricular materials, which offer only superficial connections between the mathematics being instructed and the contextualization of the mathematics within games, erroneously de-emphasize the importance of conceptual understanding. Teacher educators should strive to assist preservice teachers in developing familiarity with authentically contextualized mathematics instructions that does not focus on trivial adornments, which are neither helpful nor relevant to the pedagogical process. Instead, during the mathematics learning process the activities chosen should provide structured opportunities for applying high-level thinking processes such as analysis, synthesis, and evaluation.

The findings from this study contribute empirical evidence that could serve to extend teacher educators' understanding of potential strategies for developing preservice teachers, particularly their knowledge of contextualized mathematics pedagogy, and interdisciplinary instructional design abilities. The development of future mathematics teachers' knowledge of content, curriculum, and pedagogy, as well as self-efficacy for teaching mathematics, should not be unnecessarily limited by artificial subject area boundaries. Meaningful cross-discipline approaches to mathematics pedagogy deserve further attention from the educational research community, and can be further examined as an effective intervention towards developing preservice teacher's understanding of teaching, curriculum and students. The current study had several limitations, including: (a) the small sample size, (b) the ethnically homogenous participants, and (c) the survey instrument is not fully validated through the confirmatory factor analysis, and (d) the brevity of the intervention, all of which limited the generalization of our findings to other teacher education programs serving a different academic or ethnic demographic. The findings from this study invite further research tracking the impacts of similar minded interventions upon preservice teachers' future teaching practice in their classrooms, as well as impacts upon

their elementary students' mathematics dispositions and achievement. More so, the results encourage further research on strategies for the pedagogical implementation of instruction contextualizing mathematics education within creative contexts supported by the arts.

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## Appendix

To facilitate participants in composing their own music and using handbells to play music, graphic musical representation were adopted—we used the color of red, orange, yellow, green, turquoise, blue and purple to represent the music notes of C(Do), D(Re), E (Mi), F(Fa), G(Sol), A(La), and B(Si). Based on this graphic notation system, participants will compose music by placing a group of color cards on the table, and play music by shaking the handbells with corresponding colors (see An & Capraro, 2011).

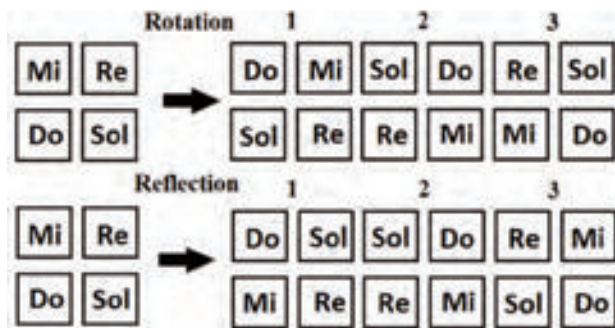


Note: Original colors of graphical notation have been replaced with musical note name for the sake of black-and white printing.

### Sample Activity 1: Geometrical Transformation

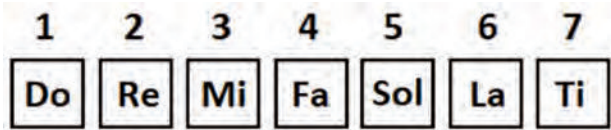
In this activity, geometric transformations such as rotation and reflection are involved as the music composition process for participants to compose their own music. For example, participants will compose music by putting four cards with different colors together as a large square and then making geometrical transformations, including translation, rotation and reflection to compose music.

One of possible geometrical transformations and music compositions:



### Sample Activity 2: Whole Number Computation

In this activity, whole number addition and multiplication is involved as one of the music composition requirements for participants to create their own music. Each music note (color) will be assigned a value, and participants will compose a piece of music based on these values.



For example, participants will be asked to compose a piece of music by using 24 color cards. In this music composition activity each color should be used at least once and the sum of your music value should be equal to 100 by adding the value of each cards together. After the participants finished their composition they can play their music by using handbells.

One of possible music compositions and computation solutions:



Total value of the music:

$$(1 \times 4) + (2 \times 1) + (3 \times 3) + (4 \times 4) + (5 \times 7) + (6 \times 1) + (7 \times 4) = 100$$

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## References for Additional Activities Used in the Intervention

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