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In-Class Purposes of Flipped Mathematics Educators

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ABSTRACT This paper provides empirical findings from a study that examined how three grade 7-12 flipped mathematics educators utilized class time when implementing a flipped learning model. Additionally, the researchers investigated the educators' purposes for various in-class tasks as well as their general use of class time. The data revealed flipped mathematics educators engaged students in multiple mathematical activities throughout each class period so as to provide the opportunity for students to develop their (1) introductory content knowledge and procedural fluency, (2) conceptual understanding of mathematical topics, and (3) mathematical problem-solving abilities. Implications from this study contribute to the literature on secondary mathematics classroom teachers' purposes and provide insights into the classroom practices of flipped mathematics educators.

KEYWORDS *flipped classroom, mathematics education, in-class purposes*

Flipped learning is a

pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter.

(Yarbro, Arfstrom, K. McKnight, & P. McKnight, 2014, p. 5)

Flipped learning advocates argue that a flipped classroom is more than a mere reversal of a traditional classroom (i.e., schoolwork is done at school and homework is done at home). A true flipped classroom should support a student-centered educational environment where students take greater responsibility for their learning; it is not an environment where teachers are merely talking at their students (Bergmann, 2013; Bormann, 2014; Hamdan, P. McKnight, K. McKnight, & Arfstrom, 2013; Reinhardt, 2014). Additionally, a flipped

classroom is a highly flexible environment, which may support both group work and independent study (Yarbro et al., 2014). As such, what occurs during the in-class portion of a flipped mathematics classroom is arguably more important than what happens outside of the classroom.

A commonly cited benefit of the flipped classroom is that, by moving direct instruction into the individual learning space, more class time becomes available to the flipped educator. As such, the flipped classroom theoretically affords educators more time for content depth and creative learning. While there is an abundance of preliminary evidence regarding the impact that flipped learning has on mathematics classrooms (Goodwin & Miller, 2013; Hamdan et al., 2013; Vaughan, 2014; Yarbro et al., 2014), the authors of the National Council of Teachers of Mathematics' (NCTM) *Principles to Actions* (2014) note that, "at the present time, no consistent scientific research evidence suggests that flipping, in the absence of an increased focus on conceptual learning

and student sense making, is an instructional practice that improves student learning” (NCTM, 2014, p. 80). Consequently, in order to better understand how flipped educators make use of additional class time—the time gained by moving direct instruction into the individual learning space—it is important that more research is done on the structure and use of the in-class portion of the flipped classroom model.

To better understand how flipped mathematics educators (FMEs) utilize class time in the flipped learning model, this research study aimed to investigate the types of tasks FMEs facilitate and their corresponding purposes (i.e., a FME’s intention, objective, or reasoning for a decision or action) for the in-class portion of the flipped classroom. In this paper, we report on the purposes grade 7-12 FMEs have for the in-class portion of the flipped classroom in order to classify why FMEs are facilitating this type of environment. This classification is a first step towards developing an awareness of what other FMEs are doing in their classrooms.

For the presented study, a classroom was considered a flipped classroom if (1) direct instruction occurred, in some capacity, in the individual learning space (i.e., on the students’ own time outside of class or independently during class time). This did not mean that the learning had to occur at home, but rather that direct instruction was not facilitated in a large group setting, and (2) technology was used to facilitate direct instruction in the individual learning space. It is important to note that “technology” was intentionally loosely defined, because participants had access to a variety of technological resources.

Literature Review

Many researchers, flipped classroom advocates, and flipped classroom critics have commented on the current lack of data-driven flipped classroom research (i.e., Bormann, 2014; Goodwin & Miller, 2013; Love, Hodge, Grandgenett, & Swift, 2014; Moore, Gillett, & Steele, 2014; Muir & Chick, 2014; Pierce & Fox, 2012; Reinhardt, 2014; Yarbrow et al., 2014). Consequently, even though the focus of this paper is on flipped secondary mathematics classrooms, flipped classroom literature in non-mathematics disciplines and higher education are also discussed so as to provide a comprehensive overview of the existing flipped classroom knowledge base. A review of existing flipped classroom literature revealed three notable research foci: Student perceptions of the flipped

classroom, the impact of the flipped classroom on student engagement, and the impact of the flipped classroom on student achievement.

Of the studies that focused on analyzing student perceptions of the flipped classroom, multiple researchers found that a majority of students held positive views of the flipped classroom instructional model. Notably, students perceived the flipped classroom as a more collaborative environment than the traditional classroom (Butt, 2014; Foertsch, Moses, Strikwerda, & Litzkow, 2002; Love et al., 2014; Schultz, Duffield, Rasmussen, & Wageman, 2014); students perceived themselves as having more control over their learning (Foertsch et al., 2002; Muir & Chick, 2014; Love et al., 2014; Schultz et al., 2014); and students perceived themselves as being more engaged in a flipped classroom environment compared to a traditional classroom (Butt, 2014; Love et al., 2014; Wong & Chu, 2014).

A smaller subset of the flipped classroom literature investigated the impact of flipping on student engagement (McGivney-Burelle & Xue, 2013; Moore et al., 2014; Schultz et al., 2014; Vaughan, 2014). Specifically, several researchers found that instructional videos that were clear, concise, and easy to follow helped to keep students engaged when learning new material in the independent learning space (McGivney-Burelle & Xue, 2013; Schultz et al., 2014). Additionally, a number of researchers found that students were more engaged in a flipped classroom than in a traditional classroom (Moore et al., 2014; Vaughan 2014). For example, Vaughan (2014) discovered that, overall, undergraduate students participated in more discussion, collaboration, and raised higher-level questions in the flipped classroom than students in previous semesters. Similarly, Moore et al. (2014) found that a majority of secondary mathematics students were more engaged in a flipped classroom than in a traditional classroom. Specifically, secondary mathematics students reported spending less time taking notes in class and more time engaged in problem-solving activities in the flipped classroom than in the traditional classroom.

Given that the flipped classroom model is relatively new, there is little longitudinal data regarding the impact that flipped classrooms have on student achievement. However, several studies at the higher education and secondary levels have shown that students in flipped classrooms perform at least as well as, if not better than, students in traditional classrooms (Love et al., 2014; McGivney-Burelle & Xue, 2013; Pierce & Fox,

2012; Schultz et al., 2014; Wong & Chu, 2014). For instance, Love et al. (2014) found that undergraduate students in a flipped Linear Algebra course saw a greater average change in score between the first and second course exams ($p < 0.034$) as well as between the second and third exams ($p < 0.012$) than students in the traditional section of Linear Algebra. Similarly, McGivney-Burelle and Xue (2013) found “some evidence that students learning via a flipping pedagogy outperform students who are in a traditional lecture” (p. 484). The authors also reported that students in the flipped classroom outperformed students in the traditional classroom on exam two.

It is appropriate that a large majority of the existing flipped classroom literature has been student-focused given that an essential characteristic of flipped learning is that the classroom must transition from a teacher-centered to a student-centered learning environment (Hamdan et al., 2013). However, there is a significant lack of research on the flipped educator. Notably, there is limited information regarding FMEs’ in-class purposes. As such, it is difficult to determine whether or not flipped educators *intend* to foster educational environments that promote the development of students’ conceptual understandings and mastery of mathematical curricula.

Methodology

The present study was designed as a holistic multiple-case study where each case was a flipped mathematics classroom. According to Yin (2003), “case studies are the preferred strategy when ‘how’ or ‘why’ questions are being posed, when the investigator has little control over events, and when the focus is on a contemporary phenomenon within some real-life context” (p. 1). Therefore, by collecting data from and analyzing specific cases of the flipped mathematics classroom, detailed information about in-class purposes were collected and compared across cases.

Table 1

Summary of Participant Identifying Information

Pseudonym	Years of Teaching Experience	Highest Level of Education	Type of School	Grade/Content Area	Flipped Classroom Experience	Type of Flipped Classroom
Adam	4	Master’s Degree	Charter school	High school/Integrated	< 6 months	Fully flipped
Jason	17	Master’s Degree	Public school	7th grade/Integrated	5 years	Fully flipped
Rachel	3	4-year College Degree	Public school	7th grade/Algebra I	< 1 year	Periodically flipped

FME Participants

Three teachers from the Mid-Atlantic region in the United States were invited to participate, as they met the study’s inclusion criteria (i.e., their classroom was considered a flipped classroom as defined above). Of the three participants, two taught seventh grade mathematics in public schools and one taught general high school mathematics at a charter school. Two of the teachers had fully flipped classrooms, and one periodically flipped her classroom. With respect to technology, two of the FMEs, Adam and Jason, taught in one-to-one classrooms. In other words, each student had access to a technological device meaning that the technology to student ratio was one-to-one. Specifically, Adam had access to a class set of desktop computers and Jason’s school district provided each student with a laptop; alternatively, Rachel’s students did not have regular access to school-provided technology. All three educators had varying degrees of teaching and flipped classroom experience. See Table 1 for a summary of participant background information.

Data Collection

To gain insight into the in-class purposes of FMEs, data for this study included an online survey; two semi-structured interviews, which were transcribed; and field notes. The online survey, designed by the first author, consisted of ten questions related to the participants’ educational and professional backgrounds and experiences with the flipped classroom. Upon completion of the online survey, each participant chose one flipped lesson for the first author to observe. During an audio-recorded pre-observation interview, participants were asked questions related to the purposes for the tasks in their respective flipped lessons. After each lesson observation, participants completed a one-hour stimulated-recall interview for the researchers to gain an understanding of how the participant perceived their flipped lesson and to determine whether the lesson objectives were met as initially intended. Each participant was first

asked to describe the lesson that was observed, including what tasks were completed, and what they believed the students took away from the lesson. The third data source, field notes, was collected during the in-person flipped classroom observations. For each observed lesson, detailed notes were taken on the types of student-teacher interactions and how the teacher addressed or did not address their intended purposes, as stated in the pre-observation interviews.

Data Analysis

The data were analyzed by first coding each line of the individual interviews into the code of *in-class purposes*, when appropriate. For example, for the *in-class purposes* code, the words “why,” “I think,” “the reason,” “purpose,” etc. were used to guide the identification of specific in-class purposes each teacher communicated in their interviews. In addition, incident to incident coding (Charmaz, 2006) was used to code the purposes identified in the field notes. Throughout the categorization process, subthemes were identified and descriptions of those themes were written. Data were first coded independently by the first author who then met with the

second author to refine descriptions of corresponding purposes. Consistency of coding was verified with three other researchers.

Results and Discussion

A variety of results relating to the FME in-class purposes emerged from the data analysis. See Table 2 for a complete detailing of all purpose-related findings. For the remainder of the paper, the focus will be on the FMEs in-class mathematical purposes (i.e., purposes that influence the way that a FME teaches mathematical content). Although all of the FME in-class purposes were of import, the FME mathematical purposes were of particular interest, because of their relationship with the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and NCTM’s “vision of high school classrooms in which the majority of the activity involves students working on rich mathematical problems and engaging in mathematical discourse” (Moore et al., 2014, p. 421). Three overarching mathematical purposes emerged from the

Table 2

Overview of FME In-Class Purposes

RELATED TO THE CLASSROOM ENVIRONMENT
<p><i>Fostering a collaborative classroom environment</i></p> <ul style="list-style-type: none"> • FME wants students to answer each other’s questions • FME wants to provide students with collaborative choice • FME wants students to collaborate to creatively solve problems <p><i>Fostering a differentiated classroom environment</i></p> <ul style="list-style-type: none"> • FME wants to differentiate the classroom to individualize student pace • FME works one-on-one with struggling students <p><i>Fostering a student-centered classroom environment</i></p> <ul style="list-style-type: none"> • FME wants students to take responsibility for their learning • FME wants to create a caring educational environment
RELATED TO ASSESSMENT
<ul style="list-style-type: none"> • FME wants to provide the opportunity for students to assess their understanding • FME wants to informally assess student understanding • FME wants to assess mastery learning
RELATED TO MATHEMATICS
<p><i>Provide the opportunity for students to develop introductory content knowledge and procedural fluency</i></p> <ul style="list-style-type: none"> • FME wants students to receive initial introduction to mathematical content • FME wants to provide opportunities for students to practice mathematical skills <p><i>Provide the opportunity for students to develop conceptual understanding of mathematical topics</i></p> <p><i>Provide the opportunity for students to develop mathematical problem-solving skills</i></p>

case study data: FMEs sought to provide the opportunity for students to develop (1) introductory content knowledge and procedural fluency, (2) conceptual understanding of mathematical topics, and (3) mathematical problem-solving skills.

Provide the opportunity for students to develop introductory content knowledge and procedural fluency

A common mathematical purpose held by all three FMEs was to provide the opportunity for students to develop introductory content knowledge and procedural fluency. In order to address this purpose, the FMEs provided students with ample time to engage with mathematical content during class-time.

FME wants students to receive initial introduction to mathematical content. A primary way that Jason and Adam introduced their students to mathematical content during class was through instructional videos. Jason's students were instructed to watch a specific video lesson(s) at home, although some students chose to watch the videos during class. Alternatively, Adam's students all independently watched brief instructional videos during the in-class portion of his flipped classroom due to the technological resource constraints faced by his students. It is important to note that Rachel's students also watched instructional videos, but only outside of regularly scheduled class time. One of Jason and Adam's in-class purposes was to utilize the brief videos as tools for introducing students to new mathematical content in an independent learning space. During his pre-observation interview, Adam explained,

The purpose of the video would be to give them the raw skills of knowing what regular polygons are...I want them to have those skills so that when it comes time to looking at the pentagon corral problem somebody might say, "Hey, that looks familiar" and start trying to make that connection themselves.

Jason explained that he often reiterated important concepts at the beginning of class, in order to ensure that his students took the time to watch the videos and obtain the necessary skills to work on their assignments. Although Jason reviewed mathematical concepts with his students, he explicitly emphasized the importance of watching the videos. Jason explained to his students, "If you're not watching the videos then you're missing out...You're missing out if you are not getting the information first."

Both Jason and Adam provided the opportunity for their students' to develop introductory content knowledge by giving their students time to engage with new mathematical content, through a brief video lesson, during class.

FME wants to provide opportunities for students to practice mathematical skills. In all three flipped classrooms, students were observed working on a variety of activities, which served as practice material. Specifically, Jason and Rachel emphasized the importance of repeated exposure to mathematical ideas by having students practice their mathematical skills. In Jason's class, seventh grade students practiced naming triangles through three different mediums (i.e., written, verbal, and digital). Similarly, Rachel emphasized the importance of providing her seventh grade Algebra students with the opportunity to practice solving systems of linear equations using whiteboards. Rachel described her perspective of the whiteboard activity in the following way:

I didn't really have to do much explaining about what the actual content was and I could focus more on just practice, practice, practice. They didn't seem to have many questions, just getting that extra practice really helped solidify everything.

Furthermore, Rachel provided her students with an additional opportunity to practice solving systems of linear equations by allowing her students to spend fifteen minutes collaboratively working on their homework assignment at the end of class so that she could informally assess student progress and provide immediate feedback. Adam also created opportunities for his high school students to enhance their procedural fluency through questions that were built into his self-created instructional videos. Rachel and Adam both explained that one of the reasons they wanted their students to practice their mathematical skills was so that they would be able to eventually apply those procedural skills to solve more complex or real world problems.

Provide the opportunity for students to develop conceptual understanding of mathematical topics

Both middle school FMEs, Jason and Rachel, provided opportunities for their students to develop conceptual understanding of mathematics by exposing them to multiple representations of mathematical topics. Specifically, Jason and Rachel spent class time discussing how there are often multiple ways of solving a problem. Jason

accomplished this goal by providing his students with a variety of mediums through which they could solve problems (i.e., written, verbal, and digital). He explained his decision to use a combination of written and digital work during the pre-observation interview. He noted,

There are some teachers who have transferred mainly to digital where kids are just clicking answers online or typing them in. So, part of my assessment assignment is by hand. [Students] have to look at a triangle and write down what type of triangle it is. By hand, they will have to take the information given, write an equation, and solve for the missing variable... And then digitally using the IXL.com they will have to go and answer the same questions.

Jason explained that, although digital activities are useful for their time saving and unlimited questioning capabilities, he believed that paper pencil assignments were just as important for his students to complete. He stated, "I do believe in the value of paper pencil still... when they get to higher levels of math (i.e., Algebra I, Keystone Algebra, Algebra II) they will be required to write out their steps of how to solve problems." By writing out their steps, Jason's students had to demonstrate an understanding of the concepts they were learning. He similarly discussed the importance of students being able to explain their mathematical work and articulated,

We talk about the importance of being able to write it out, especially when you get into algebra. It's like writing a sentence. You can't say "dog let the out back door." You can't put a sentence together like that. You have to know how to put a sentence together in the right way.

Jason wanted to expose his students to mathematics through a variety of mediums. He wanted his students to be comfortable viewing mathematics in different ways with the ultimate goal of helping to build his students' conceptual understanding.

Rachel took a different approach to demonstrating multiple ways of solving a single mathematical problem. At the time of this study, Rachel's Algebra I students were learning how to solve systems of linear equations by graphing and algebraic manipulation (i.e., substitution and elimination). Throughout the observation, Rachel let her students choose which method they wanted to use. She explained that she wanted to demonstrate multiple representations of mathematical

concepts so as to encourage her students to think and recognize the creative aspect of mathematics:

I'm trying to teach them all of these different options. Choose which one you understand more or choose which one you think would fit better, but all of them will work. I want to give them that flexibility and that creativity so they don't get stuck saying here's step 1, step 2, step 3, step 4... And then they have no idea how to solve the harder problems that actually take some thought.

During the observed lesson, one student asked Rachel if she had to solve the systems by elimination and Rachel reiterated that the student was allowed to use whichever method she preferred. In a second instance near the end of class, a pair of students asked Rachel if they should write their solution to the system of linear equations as an ordered pair. The students justified their question by relating the answer they obtained through algebraic manipulation to the solution they received when solving the systems of equations by graphing. During her post-observation interview, Rachel explained,

We did practice graphing. Here are two lines, here's the solution. This is the same thing when you substitute and it'll be the same thing when you eliminate... [I do this so] they don't get stuck and say "Oh, here are my two numbers, but I have no idea what they actually mean or why those two numbers are my answer."

Rachel's students clearly understood that there were multiple ways to solve a system of linear equations, and no matter which method was used, the answer should have the same meaning. Rachel further commented,

I think [the students] understood the different methods. Now, they're kind of thinking about it and you even heard some kids asking during the whiteboards if they had to use that method or could they use something else. So I think they are starting to understand that all three methods do the same thing... and how to be a little bit more creative.

Although Rachel and Jason provided their students with opportunities to develop conceptual understanding in different ways, both FMEs wanted to expose their students to multiple representations of mathematical topics.

Jason further provided the opportunity for his students to engage with multiple representations of mathematical concepts by facilitating hands-on and digital

activities that stretched students' spatial visual thinking (i.e., a gears and LEGO® station, which applied the concepts of ratio and proportion, and the Spatial-Temporal [ST] Math program).

During his pre-observation interview, Jason stated that he wanted to regularly provide his students with Science, Technology, Engineering, and Mathematics (STEM) related projects they might not typically encounter in a textbook or traditional mathematics lesson. During the observed lesson, Jason exposed students to seeing mathematics visually when they were working on a project related to gears. Small groups of students worked with tangible LEGO® gears, tangible bicycle gears, and with an online website where they needed to use gears to build a functioning clock. In this STEM project group, the students were engaging with ratios not only algebraically, but also modeling ratios with manipulatives and then building them on a website.

A second way that Jason exposed his students to visual mathematics was through the ST Math program, which is an online game-based mathematical program, designed to improve mathematical proficiency through visual learning. Jason stated, “[ST Math] is a program that does math visually. It doesn’t have any written directions. It’s almost more problem-solving and puzzle based.” See Figure 1 for an example of a fraction activity from ST Math, which was designed to develop students’ visual understanding of fractions. In the example, students observe there are four portions of the fraction circle on the post where the penguin stands. They determine that the value of the combined parts is one and then place the balloon stand on the number one on the ground. The program animates the penguin moving from zero to one by dropping one quarter circle at the appropriate intervals until the penguin lands in the balloon stand. Jason had his students engage with the ST Math program during class time in order to provide them with an opportunity to visually analyze fractional pieces to determine the combined value in a non-written form.

Provide the opportunity for students to develop mathematical problem-solving skills

Given that Adam was the only high school FME, it was expected that some of his purposes would differ from the two seventh grade teachers. Perhaps the most dramatic difference between the FMEs was that Adam intentionally fostered an environment for students to develop their mathematical problem-solving skills, and the other two teachers did not emphasize this problem-solving approach. During Adam’s lesson, his students

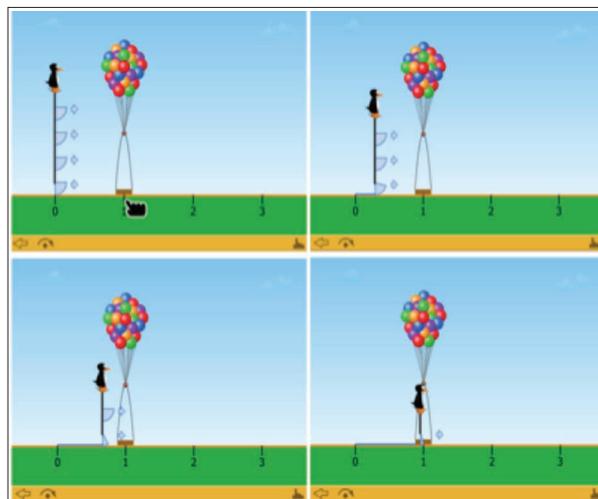


Figure 1. Screenshot of a fraction activity from a free demo available from the ST Math program found at <http://www.mindresearch.org/play>.

worked on a pentagon investigation activity where they were asked to construct a pentagon of maximum area with 300 feet of fencing. During his pre-observation interview, Adam explained,

The [students] will wrestle with [a] problem where they actually have to find the area of a corral in the shape of the regular pentagon with 300 feet of fencing to use. I’m not going to tell them how to do that. I want to see if anybody can make the connection between triangles and the pentagon and see if they can figure that out.

It was evident through Adam’s pre- and post-observation interviews that the main purpose of the pentagon investigation activity was to engage students in the problem-solving process. Additionally, Adam allowed his students to develop their problem-solving skills by not providing them with immediate answers to their questions. He explained, “letting them wrestle [with the problem] was going to help them build their reasoning and critical thinking skills.” Instead of giving his students immediate answers, Adam prompted them with questions, and encouraged them to be creative and test out their theories. A significant portion of Adam’s in-class time was spent having his students investigate the pentagon problem and apply mathematical concepts so as to develop mathematical problem-solving skills.

Over the past few years, secondary mathematics teachers have been encouraged to engage students in mathematical discussions and exercises which develop higher-order problem-solving skills (e.g., Moore et al.,

2014). Although the researchers did not study student-learning outcomes, data from the present study suggests that the three participants intentionally tried to foster engaging mathematical classroom environments focused on providing the opportunity for students to conceptually understand the mathematical topics under discussion. Adam further utilized class time to engage his students in activities designed to develop their problem-solving skills. These findings relate to a concern raised in *Principles to Actions* (NCTM, 2014) that without an increased focus on conceptual understanding, the flipped classroom has not been proven to be an instructional strategy that benefits student learning.

The authors of *Principles to Actions* (NCTM, 2014) state that research “supports the characterization of mathematics learning as an active process” (p. 9) and more specifically that “learners should have experiences that enable them to...acquire conceptual knowledge as well as procedural knowledge, so that they can meaningfully organize their knowledge, acquire new knowledge, and transfer and apply knowledge to new situations” (p. 9). Notably, one of the in-class purposes of two of the FMEs was to provide the opportunity for students to develop conceptual understanding, which directly relates to providing students with experiences that enable them to *acquire conceptual knowledge as well as procedural knowledge*. Rachel and Jason addressed this purpose by emphasizing multiple representations of mathematical concepts through discussions related to solving problems in multiple ways and activities that provided opportunities for students to develop spatial visual mathematics.

Conclusion

The intent of this work is to provide the mathematics education community with a classification of in-class purposes that three FMEs articulated. The highlighted purposes are representative of their reasons for engaging students in various mathematical learning experiences in a flipped classroom environment. This classification may help the mathematics education community to develop an awareness of FMEs’ thought processes related to the purposes for a flipped classroom—perhaps aiding mathematics educators in structuring their own classroom practices. The list of purposes may not be exhaustive and may vary across different settings based on the experiences of the FMEs; however, these empirical data provide a foundation on which other FMEs may build their practice.

Teachers are faced with a myriad of decisions to facilitate their classroom instruction. Above, three FMEs’ purposes reveal that through a flipped learning environment they engaged students in multiple mathematical activities throughout each class period so as to provide the opportunity for students to develop (1) introductory content knowledge and procedural fluency, (2) conceptual understandings of mathematical topics, and (3) mathematical problem-solving skills. Although the results of this study may not be generalized to every flipped mathematics classroom, the data serves as an important first step toward documenting and understanding how FMEs structure the in-class portion of the flipped mathematics classroom.

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