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## A Living Metaphor of Differentiation: A Meta-Ethnography of Cognitively Guided Instruction in the Elementary Classroom

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**ABSTRACT** This meta-ethnography explores qualitative studies around the Cognitively Guided Instruction (CGI) framework of mathematics and illustrates how CGI epitomizes differentiation. The meta-ethnographic process is used to synthesize CGI as differentiation, specifically within the elementary mathematics classroom. Thomas P. Carpenter is credited as one of the foundational researchers of this instructional model, along with his team of Fennema, Franke, Levi, and Empson. Six qualitative pieces from this author group are synthesized to create a reciprocal translation, described by Noblit and Hare (1988) as a generation of a metaphor across similar studies. In this case, the pieces work together to form a metaphor of differentiation across the themes of student-centered pacing, alternative forms of assessment and teacher scaffolding.

**KEYWORDS** *Cognitively Guided Instruction (CGI), differentiation, meta-ethnography*

### Introduction

Differentiation is a much-contested buzzword in today's educational arena. Many educators, as well as researchers, disagree about what it entails and how it looks when enacted in the classroom. In a current debate surrounding the feasibility of differentiation, Delisle (2015) argued it doesn't work. He claimed that differentiation is not the magical elixir it is touted as and that it is unrealistic to "toss together into one classroom every possible learning strength and disability and expect a single teacher to be able to work academic miracles with every kid" (para. 13). Differentiation, in his eyes, is a farce that the educational community has bought into.

Tomlinson (2015), a prominent researcher and author around the topic of differentiation, responded to Delisle's commentary by stating she never claimed differentiation as a solve-all for classroom teachers. Rather, she touts differentiation as creating and modifying instructional

approaches that will benefit diverse learners. She acknowledged that this takes effort on a teacher's part, but that it does, in fact, work. Tomlinson then presented differentiation through the lens of Hattie's (2012) "plus-one learning" (para. 14), in which teachers must ensure that all learners move forward from their starting points—from the most needy student to the most advanced student.

This view of differentiation as plus-one learning is epitomized in work around Cognitively Guided Instruction (CGI) mathematics (Carpenter, Fennema, Franke, Levi, & Empson, 1999, 2014). We view CGI as progressing student learning in mathematics based on the students' needs, acknowledging that when the phrase progressing learning is used in this meta-ethnography it is meant to embody Hattie's view of plus-one learning. We began the meta-ethnographic process with the intention to synthesize across qualitative studies of the CGI movement, and it soon became clear that CGI could be interpreted as a working-model of differentiation.

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## Positionality and Context

As a research team, our personal instructional experiences with CGI made it an entry point for meta-ethnography. CGI is founded on how “teachers use research-based knowledge about children’s thinking and problem solving to make decisions as they plan and implement instruction, and how this instruction affects their students’ learning” (Carpenter, 1992, p. 458). Based on our previous experiences as generalist and special education teachers, we believe CGI allowed for the possibility of all learners to experience mathematical success, as it is a student-centric instructional model. Therefore, we felt that both the educational research and practitioner communities could benefit from an in-depth analysis of CGI classrooms.

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## Study Search and Inclusion

According to Noblit and Hare (1988), a meta-ethnography provides a means to “derive understanding from multiple cases, accounts, narratives, or studies” (p. 12). Although the original intent of meta-ethnographies was to synthesize understanding across ethnographic accounts, the method has now evolved to synthesizing other forms of qualitative work. For our purpose, a meta-ethnography allowed for “systematic comparison of case studies to draw cross-case conclusions through the lens of a new metaphoric interpretation” (Noblit & Hare, 1988, p. 13). This metaphoric interpretation “refer[s] to what others may call themes, perspectives, organizers, and/or concepts revealed by qualitative studies” (Noblit & Hare, 1988, p. 14), but allows for even deeper analysis as a topic can be seen in manners it previously was not. In other words, we could analyze and synthesize studies around a topic and let it live in a new way.

We deemed the origin of CGI research in the late 1980s as a significant point in reform-based mathematics instruction and therefore felt warranted to limit this analysis to works by the originators of CGI research. Thomas P. Carpenter is credited in this area as one of the foundational researchers, along with his colleagues Fenema, Franke, Levi, and Empson who wrote *Children’s Mathematics: Cognitively Guided Instruction* (1999, 2014). We focused on qualitative work of CGI in the years just before and after the 1999 edition of the book.

We began our search for qualitative studies by combining research interests, which initially meant searching with the key terms of mathematics, elementary, middle school, special education, race studies, and CGI. We con-

ducted twenty-one article searches in Article + and Google Scholar from January to February 2015 with the key terms listed above. We discovered that a majority of the studies that surfaced with these terms used quantitative methodologies, were predominantly elementary-focused, and were predominantly focused in the general classroom and not the special education setting. While race studies and special education studies were included in some of the research, there was not enough qualitative work around CGI in these areas to provide a solid grounding for the meta-ethnography process.

After consulting with one another and other researchers involved with CGI, and reviewing the searches, we concluded that due to few hits with all of the search terms it would be beneficial to our meta-ethnographic process to hone searches to CGI in the elementary classroom. We also decided that to best maintain the integrity and original intentions of CGI, we would analyze research from the original author group. Thus, we chose to strategically focus on pieces specifically from Thomas P. Carpenter since he was lead author on much of the original research and both editions of the book. We then went back to the same databases using Thomas P. Carpenter in the author search field, along with two key word combinations: 1) elementary and mathematics and 2) elementary and math.

Using Thomas P. Carpenter in the author search field with the key search terms of elementary and mathematics resulted in 108 records in Google Scholar in February 2015. The search with Thomas P. Carpenter, elementary and math resulted in 45 records. Within the 108 records, we found duplication of the 45. We also found that the 45 records were listed consistently between Article + and Google Scholar, thus we began to analyze the 45 for relevance. In order for a study to be considered for our meta-ethnography, we set the parameters that it must be peer-reviewed, must have Carpenter in the authorship so as to maintain CGI framework integrity, and must utilize qualitative methodologies. A meta-ethnography requirement is that the data must be qualitative in nature, but our qualitative goal was even more specific in that we required rich CGI qualitative data. For us, this meant that studies included excerpts of elementary classrooms utilizing CGI and teacher and student dialogue.

Once we reviewed the 45 records with our meta-ethnographic parameters listed above, we were able to abandon 39 pieces that did not meet all of our qualifications. This meant we had six studies remaining that did meet our parameters. Rather than expand the search with other terms or into other databases, we chose to maintain a limit of these six pieces that exemplified

teachers' voices around CGI. This limit would allow for comprehensive synthesis that would lead to the generation and maintenance of a new metaphor across the pieces. It is important to note that among the six pieces that were selected, some of the qualitative work is found within multi-year mixed method studies and was extracted for synthesis to uphold the qualitative qualification.

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### Coding and Analysis Process

Settling into the six peer-reviewed studies using the aforementioned parameters, we read each for research purposes, participants, settings, and contexts. From there, we wrote summaries and then went back to the articles to collect key quotations that exemplified the CGI framework. These quotations became the first round of meta-ethnographic data and were examined across articles for commonalities. The commonalities led to three inductively generated themes: student-centered pacing, alternative forms of assessment, and teacher-scaffolding.

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### Results

#### Initial Synthesis: A Third Order Construct

After establishing the initial three themes, we re-read and coded each study for theme specificities. A synthesis chart allowed us to see how the authors discussed the themes across their research base. As expected, perhaps due to the potential groupthink nature of the CGI research program and the key author perspective, aspects that represented these themes emerged within each piece, although they differed depending on the context of the study. Thus we synthesized across the themes to find moments that were similar to one another regardless of context and most pivotally represented CGI in terms of these themes. The inductively generated codes and their pivotal excerpts are third order constructs. Atkins et al. (2008) explained a third order construct as "the synthesis of both first and second order constructs into a new model or theory about a phenomenon" (p. 6). The first and second order constructs reflect the participants' understandings and the authors' interpretations of participants' understandings.

**Student-centered pacing.** Student-centered pacing was coded as experiences when either the authors or the teacher participants of the studies stated that children entered classrooms with inherent mathematical abilities

and that these abilities should inform instructional decisions. All studies we examined demonstrated that teachers considered the students' abilities first before instructing them. Ms. Statz, a fourth grade teacher in her third year, exemplified this notion. The following excerpt showcases teacher decision-making around student thinking:

Ms. Statz's growing knowledge of the basic character and inadequacy of her students' strategies, combined with a strong belief against grouping children by ability or even by type of errors, presented a dilemma for her: how to accommodate a wide range of children's thinking without resorting to ability grouping or remediation. Maintaining her belief in the centrality of student-generated strategies to the development of understanding and confidence, she started to think about how she could assist these children to grow mathematically without directly telling them how to solve problems. (Steinberg et al., 2004, p. 249)

The studies also acknowledged that within CGI classrooms, it is the students that set the pace for the curriculum. This is presented by a Carpenter, Ansell, and Levi (2001) case study of two first grade teachers utilizing CGI. In these teachers' classrooms,

students often solved the same problem, but the different strategies they used represented very different points in the evolution of their understanding of multi-digit concepts and operations. The concepts that some students were developing in October other students were learning in December or February. (p. 29)

Allowing students to set the pacing for topics demonstrated the teachers' beliefs that children's knowledge was the priority of the classroom.

In Franke, Carpenter, Levi, and Fennema (2001), teachers were interviewed regarding the presence of student engagement when CGI was implemented in their classrooms. The participants framed their teaching philosophy similar to that of one teacher, Ms. Sullivan, whose "constructivist perspective both reflects and is reflected in her knowledge of her own students and in her use of that knowledge in planning and implementing instruction" (p. 674). This student-centered constructivist philosophy was found across the studies and provides the basis for teaching with the CGI framework: understand students' mathematical thinking, use knowledge

of children's thinking, encourage children's mathematical thinking (Carpenter et al., 1999). We saw this framework embodied across studies when teachers discovered a student's knowledge through word problems and questioning, made strategic curriculum choices like number range usage based on the student's knowledge, then built upon the student's starting point to progress understanding through purposeful questioning and exposure to other strategies. It became apparent that this framework is paramount to the CGI classroom environment.

**Alternative forms of assessment.** This theme was present in the studies when the CGI teachers formatively assessed their students' understandings through their interactions with the children, their observations, and through genuinely listening to their discussions or problem-solving explanations.

Carpenter and Fennema (1992) best represent this belief when they share their overall reflection on CGI instructors:

The critical element in the classes in which we observed the most impressive levels of problem solving was that the teachers were able to assess what their students were capable of so that they could continue to expand the students' knowledge by giving them increasingly challenging problems that were not beyond their capabilities. By listening to their students, these teachers learned that their students were capable of solving much more challenging problems than they previously had anticipated. (p. 462)

They continued their reflection by stating, "To assess their students, the teachers did not rely on written tests or formal assessment procedures. Instead assessment was an ongoing part of instruction" (p. 462). With each stage of a lesson, the teacher attended to a student's thinking around a concept and assessed understanding by listening to the student's explanations. From there, mathematical problems were strategically modified according to the student's thinking and what was needed to encourage the development of understanding.

**Teacher-scaffolding.** Our final coding theme of teacher-scaffolding involved the mechanisms and strategies teachers used to allow all students to access the curriculum while still maintaining the integrity of the mathematics. This surfaced when CGI educators varied the

number choices within the problems for different learners, encouraged manipulative use, and allowed space for collaboration within the lesson segments.

Modifying the numbers within the problem is a common CGI teacher-scaffolding method. CGI educators ensure students' access to the mathematics by attending to the number choice and the context students used to solve problems. The following teacher excerpt highlights the various teacher-scaffolding strategies of a CGI teacher, including that of number choice:

Interwoven with Ms. J.'s knowledge of problem types and solution strategies were pedagogical concerns about the use of counters, relevance of the problem context to children, the language used in problems, choice of number size, and selection of problems for which a variety of strategies could be used. (Fennema et al., 1993, p. 563)

Furthermore, both manipulative use and collaboration for the benefit of student sense-making is evident in this CGI classroom:

Ms. M.: "Got it? How many fewer did the African elephant eat, Ubank?"

Ubank: "Six."

Ms. M.: "Does everyone agree with that?...How did you figure it out, Ubank?"

Ubank: "Well, I had 43 here" (*pushing out 4 stacks of ten cubes and 3 additional cubes joined together*), "and I had 37 here" (*pushing out 3 stacks of ten cubes and a stack of 7*). "I put 30 on top of these 30. I took 3, and I put them here. There were 4 left, so I took 4 off, and there were 6 left." As he described what he did, he took 3 of the ten stacks from the collection of 43 and put them on top of the 3 ten stacks in the collection of 37. Then he took the 3 single cubes from the original set of 43 and put them on top of the 7 cubes in the set of 37. Then he took the remaining stack of ten cubes from the original 43 and broke off 4 cubes. He put these 4 cubes on the 4 cubes in the set of 37 that were not covered. He was left with 6 cubes from the set of 43 that did not match up with cubes in the set of 37.

Ms. M.: "Did he do it a good way?...Did anyone do it a different way?"

Marci: "I took 37, and I needed 43. So I counted up 3 more. That was 40. Then I took 3 more to 43"

Ms. M.: “Good. Does her way work well?...It sure does. Did anybody do it differently?”

Linda: “Well first I got 37. Then I got 43” (*pushes out collections of 37 and 43 cubes joined together in stacks of ten, with the extra cubes also connected together*). “See, I know it couldn’t be 10, because if you had 10 it would be 47 instead of 43. So I realized that it had to be less than 10. So what I did was I imagined 3 more cubes here” (*points to the top of the stack of 7 cubes in the set of 37*), “and I imagined 3 more right here” (*pointing to a space next to the collection of 37 that corresponds to where the 3 cubes are in the collection of 43*). (Carpenter, & Fennema, 1992, p. 463)

By allowing children to use manipulatives if necessary, and by highlighting a variety of strategies, the teacher honored varied pathways to a mathematical solution. The quotation above illustrates teacher-scaffolding by demonstrating how modification of class work is done through acknowledging students’ differing abilities and strengths.

### Analysis Through the Lens of Delisle: Challenging a Stance by Establishing a Reciprocal Translation

It is through our inductively generated themes that we initially recognized and then re-analyzed CGI as a metaphor of differentiation. After coding the studies for our inductively generated themes that seemed to establish tenets of differentiation, we felt it necessary to refer to the context surrounding differentiation. In doing so, we discovered the current debate between Delisle and Tomlinson. In Delisle’s (2015) Education Week commentary, he did not specifically define differentiation, but acknowledged that differentiation takes into account the following three factors:

- It seeks to determine what students already know and what they still need to learn.
- It allows students to demonstrate what they know through multiple methods.
- It encourages students and teachers to add depth and complexity to the learning/teaching process. (para. 5)

However, in his commentary, these three aspects are also viewed as the downfall of differentiation, in that he believes it is nearly impossible to ask teachers in heterogeneous settings to comply with these standards at a

level of competency. Therefore, Delisle viewed “differentiation [as] a failure, a farce, and the ultimate educational joke played on countless educators and students” (para. 4).

We suspected that Delisle’s stance could be challenged, as the research around CGI classrooms suggested that these three factors were indeed possible to implement. We chose to explore to what extent our inductively generated themes and data supported these factors of differentiation by recoding within themes through the process of reciprocal translation. Noblit and Hare (1988) explained a reciprocal translation synthesis as the formation of a new metaphor across similar studies, in our case, we would generate a metaphor of CGI as differentiation. Reciprocal translation synthesis is a process that is “facilitated by the emerging conclusion about how the studies in question relate to each other. Once we know that the studies are similar and the metaphor the authors employ, we proceed to construct the ‘reciprocal’ translation” (p. 39). Within our original synthesis chart, we now recoded excerpts specifically for the three points of differentiation presented in Delisle’s article and created a reciprocal translation chart with the data. Table 1 highlights the process our synthesis followed.

Table 1  
*Definitions and Process of Meta-Ethnography*

TERMS	DEFINITIONS
1st order construct	Constructs that reflect participants’ understandings, as reported in the included studies (usually found in the results section of an article).
2nd order construct	Interpretations of participants’ understandings made by authors of these studies (and usually found in the discussion and conclusion section of an article).
3rd order construct	The synthesis of both first and second order constructs into a new model or theory about a phenomenon.
Reciprocal translation	The comparison of themes across papers and an attempt to “match” themes from one paper with themes from another, ensuring that a key theme captures similar themes from different papers.
<b>Note.</b> Excerpted from Atkins <i>et al.</i> (2008).	



The six qualitative studies about CGI taken together formed a metaphor for differentiation epitomizing the three factors and because of this, they work together to challenge Delisle's stance on the infeasibility of differentiation. We found that Delisle's factors as the new themes for translation across the studies actually seemed to strengthen the metaphor of CGI as differentiation.

**Delisle factor 1: It seeks to determine what students already know and what they still need to learn.** Delisle's first factor of differentiation was perhaps the most overtly present across the six studies. CGI teachers believe that students enter a classroom with mathematical knowledge and abilities and that it is their job to elicit knowledge through questioning and observation and then center instruction around it. The responsibility of the teacher, in this case, would not be to directly teach mathematical skills and methods, but rather to bring the children's strategies into the classroom and allow them to guide the learning progression. In our initial coding, we categorized examples of this teacher belief in the student-centered learning theme. We looked within this theme to develop our argument around the first factor in Delisle's commentary. We found that the reciprocal argument expressed throughout the studies is best described by the teachers in the following quotation: "...it was their understanding of their students' thinking that allowed them to interpret students' responses and modify questioning or instruction accordingly" (Carpenter & Fennema, 1992, p. 462).

In one of the CGI classrooms, the teacher, Ms. J, uses the strategy of Big Sheets, big sheets of newsprint paper used by students to solve mathematical problems, to determine what students know and decide how to progress student learning. Big Sheets as described by Fennema et al. (1993) can be interpreted as a way to "determine what students already know and what they still need to learn" (Delisle, 2015, para. 5). An excerpt about Ms. J utilizing Big Sheets confirms that CGI teachers consider ways to expose what students know and where to move next:

Another activity that we observed regularly involved Big Sheets. Starting early and continuing throughout the year, these were used as a way for Ms. J. to listen to, assess, and record individual children's thinking; and to plan future instruction for each child. Big Sheets were exactly what their name implies—big sheets of newsprint on which were written story problems or number sentences for a child to solve. Ms. J. constructed several

sheets per week for each child to work on whenever his or her previous one was completed. During Big Sheet time, Ms. J. worked with individuals. She seated herself at a table, and children presented themselves with their Big Sheets. Rarely were there children waiting to talk to Ms. J. The students seemed to have learned to go on to their other work until she was available. Ms. J. would ask how the problems had been solved and if the problems appeared to be too easy or too hard. She would write notes on the big sheet itself about the child's solutions. She used these notes when creating new big sheets for the child and as a record of the child's growth in understanding. (Fennema et al., 1993, p. 566)

By using Big Sheets, Ms. J. continually assessed what students knew and built on students' knowledge to expand the curriculum. Students were able to express their ideas and thought processes on the Big Sheets and this became a safe way to expose mathematical knowledge about the problems at hand. As a result, Ms. J was able to determine how they achieved their results, using their thoughts to ultimately drive the lesson planning and the time spent during the mathematics class period.

**Delisle factor 2: It allows students to demonstrate what they know through multiple methods.** Delisle's second factor of differentiation was coded by our team as students' being able to demonstrate their knowledge through various forms of assessment, often non-traditional methods and non-standardized. Many times, students in the studies' CGI classrooms were administered problem-solving interviews, as well as being formatively assessed through their class interactions with peers and with the teacher.

In a case study following two first grade teachers, one-on-one interviews were used to get a clear picture of students' understandings. "During the individual interviews conducted at the beginning of the year, eight students consistently solved a variety of addition, subtraction, multiplication, and division problems using modeling or counting strategies" (Carpenter, Ansell, & Levi, 2001, p. 33). This knowledge provided a point for teachers to begin their instruction, rather than to directly instruct with no account for students' abilities. Utilizing interviews as an assessment strategy allows a teacher to better understand their children compared to that of a traditional paper-pencil assessment form. Interviews allow for probing and further questioning in order to give students the opportunity to demonstrate all they understand.

In Ms. Statz's class (Steinberg, Empson, & Carpenter, 2004) the method of student notebooks is also highlighted. When asked to reflect on her student's work within a certain lesson, Ms. Statz says, "the way she solved this is kind of strange... Can I go see what she's got in her journal?" (p. 248). Ms. Statz used notebooks as a way for her students to record their thinking, and this excerpt showed how she relied on the notebooks to better understand her students rather than judging them on one account. Not only does this express the differentiation factor that students demonstrate their knowledge through various methods, it also shows that these various methods can be actually honored and used by an educator.

**Delisle factor 3: It encourages students and teachers to add depth and complexity to the learning/teaching process.** Finally, Delisle's third factor aligned with our teacher-scaffolding theme. We viewed scaffolding as the CGI educators as encouraging manipulative use, varying number size or context of problems for the students, and utilizing student collaboration, as these teaching moves encouraged the development of the sophistication of the students' mathematical strategies and progressed the development of student thinking. Regarding Delisle's point, teacher-scaffolding is best represented as the norms that CGI teachers established in their classrooms in order to create an environment conducive to all learners. These norms made certain that CGI classrooms provided both the supports and the challenges that various learners needed. This is evident in the following excerpt about Ms. J.:

Children were expected to be engaged in mathematics, usually solving word problems written by Ms. J., their peers, or themselves. They were also expected to persist in their work, to be able to report how they had solved the problems, and to reflect on their own thinking by comparing it with someone else's solution, or on the difficulty of the problem for them. In addition, they were expected to listen to others' solutions, to understand the solution when possible, and to respect the other children's solutions. (Fennema et. al., 1993, p. 565)

It is also seen in Ms. Gehn's class:

Ms. Gehn's class had well-established norms for mathematics instruction. Almost all instruction involved students in solving word problems, which generally were written around a theme for the day.

Ms. Gehn almost never showed students how to solve a problem or modeled a particular strategy. Typically, the class would solve and discuss three or four problems during the mathematics lessons, which would last about 1 hour. (Carpenter, Ansell, & Levi, 2001, p. 31)

CGI teachers also used thoughtful student partnerships to encourage rich dialogue and exploration of mathematical ideas and growth in understanding. These student partnerships are another factor of scaffolding that exemplify Delisle's factor of adding depth and complexity to the classroom. In Ms. Statz class, "each pair got a sheet with the problems and a space for two strategies. Each child could use his or her own strategy or the pair could generate two strategies together" (Steinberg, Empson, & Carpenter, 2004, p. 251). Then, "to help children move forward to using more sophisticated strategies, Ms. Statz told them she would ask both children from a pair to explain his or her partner's strategy at discussion time" (Steinberg, Empson, & Carpenter, 2004, p. 251).

The following is a more open exploration of numbers through student partnerships, but it also shows how collaboration adds depth to the mathematics time for the students.

The children were seen talking to each other, to Ms. J., or working alone. Ms. J. was almost always actively engaged either with one child or a group of children. The room was often noisy and the children were active, but as one walked around the room, mathematics was heard. [This example was included in footnote by the authors: One casual conversation (Year 3) between two children involved a discussion of whether 20 times 20 was 40. An older child had told Edgar that it was, and he had obviously been thinking about it. Edgar said to another child: "20 times 20 can't be 40 because 2 times 20 is 40." The two children talked about it for a while, decided that 20 times 20 had to be 400, and returned to their other work.] (Fennema, et al., 1993, p. 565)

Through these scaffolding mechanisms of norms and partnerships, the CGI teachers ensured that all students received an accessible, yet appropriately challenging, learning experience.

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## Considerations and Limitations

The six studies showed that student-centric differentiation is possible. However, there are several possible limitations to the transferability of the CGI model to other classrooms.

### Teacher Resources

The teachers in the studies had access to professional development and access to participant-researchers that assisted with student interviews and with instructional decisions. Most of the studies also noted the flexibility with timing and pacing that the teachers were given within their schools in order to implement CGI to its fullest intention. If resources taper, then it potentially affects how teachers use CGI in the classroom.

However, the Levels of Engagement scale used in both the Franke, Carpenter, Levi, and Fennema (2001) and the Steinberg, Empson, and Carpenter (2004) studies demonstrated that once teachers progress in their understanding of, and engagement with, student thinking, their beliefs around teaching mathematics shift and teachers remain committed to instruction guided by student thinking. Therefore, it can be argued that permanent instructional changes are in place even after the extra support and resources are no longer available. This speaks to Delisle's concern that differentiation is impossible for teachers to attend to in heterogeneously grouped classrooms, as the meta-ethnography showed that for a teacher who believes in and enacts the CGI model of instruction, differentiation has the potential to naturally occur.

### Student Needs and Demographics

Only the Steinburg et al. (2004) study provided detailed demographics of the classroom in which the case study took place. The other studies in this meta-ethnography did not provide demographic information beyond surface level, leaving questions about whether or not Culturally and Linguistically diverse (CLD) students, students with special needs, or low socioeconomic students respond well to the CGI pedagogy. Although the 2014 edition of the Carpenter et al. book begins to address CLD and special needs students, further studies are needed to examine the impacts of CGI specifically on these demographics in order to fully understand all students' thought processes. In the meantime, since all classrooms are comprised of students with differing understandings, one could infer that even without demographics overtly listed across this set of studies, CGI could offer an optimal pedagogical approach for students of different demographics because the framework uses each student's unique background and knowledge to inform teaching.

### High-Stakes Assessments

In our current high-stakes assessment climate in which students need to have mastered content to a certain level, we acknowledge that CGI may not seem feasible to some teachers and school systems. The pressures of standardized assessments and standards-based instruction lead educators and administrators to believe that they do not have flexibility of pacing and planning. However, the meta-ethnography findings may serve to ease some of these standardized concerns in that the studies showed that students did learn the mathematics needed even when they were allowed to do so in their own time. Although their pacing may have been different, students arrived at similar understandings when their own thinking was guided, rather than all following a lockstep pace. Carpenter, Ansell, and Levi (2001) believed that this sort of self-paced learning lends itself to students then understanding mathematics conceptually and warned that when students do not learn with understanding "they perceive each topic as an isolated skill and they cannot apply their skills to solve problems not explicitly covered by instruction, nor extend their learning to new topics" (p. 27). While this does not take away the pressures of high-stakes testing, it does expose the value in allowing students to learn the content through their thinking in their own time, rather than be rushed only for test-sake.

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### Looking to the Future

Perhaps if teachers were to receive professional development support, more curriculum flexibility, and time, they would naturally implement the living metaphor, CGI as differentiation, within their classrooms. While some teachers may be able to implement CGI without outside support, we acknowledge that more teachers would be able to implement it with support. In addition, there is a need for more qualitative studies that specifically address CLD students and their response to CGI as this would reinforce the metaphor of differentiation. As for our synthesis, we found that Delisle's stance against differentiation could be challenged with the research as it stands, because the meta-ethnographic metaphor illustrated that his commentary's three factors are evident and seem to thrive within CGI classrooms.

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## References

- Atkins, S., Lewin, S., Smith, H., Engel, M., Fretheim, A. & Volmink, J. (2008). Conducting a meta-ethnography of qualitative literature: lessons learnt. *BMC Medical Research Methodology*, 8(1), 21.
- Carpenter, T. P., & Fennema, E. (1992). Cognitively guided instruction: Building on the knowledge of students and teachers. *International Journal of Educational Research*, 17(5), 457-470.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L. & Empson, S. B. (1999/2014). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.
- Carpenter, T. P., Fennema, E. & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 3-20.
- Carpenter, T. P., Ansell, E. & Levi, L. (2001). An alternative conception of teaching for understanding: Case studies of two first-grade mathematics classes. In *Beyond Classical Pedagogy in Elementary Mathematics: The Nature of Facilitative Teaching* (pp. 27-46). Mahwah, NJ: Lawrence Erlbaum Associations, Inc..
- Delisle, J.R. (2015). Differentiation doesn't work. *Education Week*. Retrieved from <http://www.edweek.org/ew/articles/2015/01/07/differentiation-doesnt-work.html>.
- Fennema, E., Franke, M. L., Carpenter, T. P. & Carey, D. A. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal*, 30(3), 555-583.
- Franke, M. L., Carpenter, T. P., Levi, L. & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal*, 38(3), 653-689.
- Hattie, J. (2012). *Visible learning for teachers: Maximizing impact on learning*. New York, New York; London, UK: Routledge.
- Noblit, G.W. & Hare, R.D. (1988). *Meta-ethnography: Synthesizing qualitative studies* (Vol. 11). Newbury Park, CA: Sage.
- Steinberg, R. M., Empson, S. B. & Carpenter, T. P. (2004). Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education*, 7(3), 237-267.
- Tomlinson, C.A. (2015) Differentiation does, in fact, work. *Education Week*. Retrieved from <http://www.edweek.org/ew/articles/2015/01/28/differentiation-does-in-fact-work.html>.