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Mathematical Modelling in European Education

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Teaching and learning of mathematical modelling has become a key competence within school curricula and educational standards in many countries of the world. The term mathematical modelling, its meaning, and how it can be implemented in mathematics lessons have been intensively discussed during several Conferences of the European Society for Research in Mathematics (CERME), particularly in the last few years. The aim of the article is to show the similarities and the differences in the teaching and learning of mathematical modelling within education in Europe.

Keywords: mathematical modelling, European perspective, didactic-traditions

Introduction

The relevance of promoting mathematical modelling in schools and within teacher education is currently under consensus in most parts of the world. Still, one can find different meanings of the notion "mathematical modelling" and what is meant by teaching and learning mathematical modelling. This paper discusses mathematical modelling in European education, shows a broad range of different understandings of modelling, and describes various, what are known in Europe as, "didactical traditions." In section one, justifications for including modelling into everyday teaching are described, based on early discussions of modeling. In section two, the current theoretical perspectives on modelling are presented, a majority of which are European. The underlying didactical traditions in several countries in Europe have influenced the teaching and learning of mathematical modelling. This is discussed and illustrated with an example in section three. Based on this tradition, fictive lessons in selected European countries are described in section four. A short overview of the state of empirical studies in mathematics education in Europe is presented in section five. Last, implications for teacher education are presented in section six.

Justifications for Inclusion in Everyday Teaching

Looking at debates on mathematical modelling and applications over the past twenty or thirty years, it becomes clear that at this time various perspectives could be distinguished nationally and internationally (see, e.g., Burkhardt, 2006). All early perspectives depend on the aims due applications and modelling. Kaiser (1995) and Blum (1996) formulated dimensions of goals for teaching modelling. These goals do not focus on a special age or grade, but can be used from primary through high school in order to improve learners' modelling competencies. Blum (2011) emphasizes the word "justifications" as a reason for the inclusion of mathematical modelling and applications in every day teaching—rather than using the word "goals" as he did twenty years ago. The following four justifications (see Blum, 2011) show the importance of modelling for learning or deepening mathematical content, as well as making a contribution to general education: (1) The "pragmatic" justification comprises ordering, understanding, and mastering real world situations, which means treating applications and modelling examples explicitly and continuously; (2) the "formative" justification means to advance competencies by engaging in modelling activities; whereas (3) the "cultural" justification focuses on the contributions that the real world can make in providing an adequate picture of mathematics; additionally, (4) real-world examples should make a contribution towards raising students' interest in mathematics, to motivate or to better understand mathematical content, which Blum named the "psychological" justification. The next section will provide an elaboration of the justifications formulated by Blum by presenting a descriptive classification of perspectives on mathematical modelling based on discussions during several CERME (see Kaiser & Sriraman, 2006).

Theoretical Perspectives

What does mathematical modelling mean? What does teaching and learning of mathematical modelling mean? Different conceptions of mathematical modelling become apparent when looking at the current educational discussion of the last ten years (Blum, Galbraith, Henn, & Niss, 2007). Although the same terms are used within the international debate, various individuals and groups consider them differently. Consequently, mathematical modelling is not only linked with diverse goals, but also receives different justifications within the following theoretical perspectives. Similarly, various modelling activities are understood differently.

The following classifications were developed during several European Conferences within the group "Mathematical Modelling and Applications." Yet, one should not speak strictly from the European perspective because participants came from all over the world. It follows then, that many of the views and the approaches of mathematical modelling are influenced by researchers of non-European countries. Theoretical perspectives on mathematical modelling (Kaiser & Sriraman, 2006; Borromeo, Kaiser, & Blum, 2011) include:

- *Realistic* or *applied* modelling
- *Educational* modelling
- *Epistemological* or *theoretical* modelling
- *Socio-critical* modelling
- *Model-Eliciting Activity* approach (MEA)
- *Cognitive and affective* modelling (as a meta-perspective)

The *realistic* or *applied* modelling perspective has a strong focus on using realistic and authentic real world examples (see section three). The *educational* modelling perspective focuses on two modeling sub-perspectives: didactical and conceptual modelling. The didactical perspective means structuring learning processes. The conceptual modelling perspective focuses on the introduction of a mathematical concept through modelling and meta-level development such as introducing and clarifying the terms "real model," "mathematical model," or "validation" within the modelling cycle. The educational modelling perspective has strong pedagogical and subject-related goals. *Epistemological* modelling (or *theoretical* modelling) on the other hand, has more theory-oriented goals. The strength of this perspective within the Roman speaking countries will be discussed later. The *socio-critical* modelling perspective has pedagogical goals, but in particular is concerned with seeing and understanding the surrounding world critically (see Barbosa, 2006 or Rosa, in press). The *Model-Eliciting Activity* (MEA) approach can be seen as an important outcome from the debate on modelling in the United States and from the extensive work of Lesh and his "MEA-group" (see, e.g., Lesh & Doerr, 2003). According to Lesh, MEAs should start in kindergarten so learners can adopt positions typical of various professions, like engineering or economics, for purposes of understanding how mathematics is needed in real life. Finally the meta-perspectives—*cognitive and affective* modelling—deserve a mention. They are labeled as meta-perspectives because in most of the research studies these approaches are an integral part of the investigation. Hence, the research aims for the cognitive perspective are the analyses of cognitive processes of learners and teachers while undertaking modelling activities (see Borromeo Ferri, 2010).

Modelling Education in Europe: Didactical Concepts

Although these perspectives are considered as research perspectives, they evolved from the interplay of theory, practical experiences, and empirical research, concerning the teaching and learning of mathematical modelling. Stronger efforts within the research field of modelling in Europe were motivated by the results of the Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) testing. Even though the International Community of Teachers of Mathematical Modeling and Applications (ICTMA)[1] has existed for thirty years, outcomes from the international large-scale assessment studies have provided new inputs for European mathematics education. It has become clear that most learners should be supported to solve problems in a way that empowers them as citizens and future workers, so most countries in Europe have revisited their "old" curriculum in mathematics for all school levels.

Since 2004, German-wide education standards for mathematics (see Blum, Drüke-Noe, Hartung, & Köller, 2006) and new curricula have been developed in some parts of Germany. Mathematical modelling is one of the six main competencies that are taught within the main areas of content. In the United Kingdom, mathematical modeling has had a different status at numerous times within all levels of the curriculum, but presently one can find it as "using and applying" in a strand of the national curriculum. In Spain, mathematical modelling is implicitly included in the curriculum. Even though the latest curriculum from 2006 describes eight fundamental competencies, modelling is not one of them—there is only a phrase stating that pupils should gain the ability to use mathematics for solving problems related to daily life and work. In Cyprus, France, and Hungary, changes towards a wider inclusion of modelling and applications in the national curricula in mathematics can be observed (see García, Maaß, & Wake, 2010). Furthermore, the influence of underlying *didactical constructs* in the education systems of various countries have to be kept in mind.

The term 'didactic' is widely used in Europe, but it is not common in non-European countries. Nonetheless, terms with similar roots, which are derived from pedagogical and philosophical traditions that are shared with many European countries, also have their origins (especially relevant to mathematics education) stemming from the work of the German mathematician Felix Klein (1968). Etymologically 'didactic,' in German *Didaktik,* in French *didactique*, in Italian *didattica*, in Spanish *didáctica*, derives from an

[1] http://www.ictma.net/index.html

ancient Greek word. It describes, in a narrow sense, the theory of class instruction. In a broader sense it means the theory and practice of learning and teaching. As a result, in many European countries it is common to speak about "mathematical didactics" synonymously with "mathematics education" as is used in other parts of the world.

When looking at the theoretical perspectives on modelling, while keeping the historical background and future trends in mind[2], I can identify three perspectives as representing a stronger European view on mathematical modelling education: realistic, epistemological and educational. Despite the fact that in all three perspectives mathematical modelling is understood as the process of translating back and forth between the real world and mathematics, questions as to how modelling should be taught and learned (including decisions about which kinds of modelling problems to choose) and which activities are related to modelling, have no general consensus.

To make this diversity more transparent further (theoretical) explanations are given, and an example will be used to illustrate differences between the three named perspectives. Realistic modelling is prevalent in Germany, the United Kingdom, Netherlands, Denmark, and the Scandinavian countries; epistemological modelling is represented strongly in Roman countries such as Spain, Portugal, France, and Italy; the educational modelling perspective is described later.

The Modelling Cycle—An Example

Imagine that, in several mathematics classrooms in Europe, teachers give their students the modelling problem presented in Figure 1 and asks them to solve as a group.

This type of modelling problem is very interesting because the context is authentic[3] for many learners between 12 and 18 years of age. Referring to the modelling cycle in Figure 2 used by some (e.g., Blum & Leiss 2005; Borromeo Ferri, 2006), learners must first understand the task (i.e., build a situation model or a mental representation of the situation). Then they must simplify and structure the task to produce a real model, which requires the making of adequate assumptions. In this case, learners think about the customers: How do business people use their cell phones in contrast to older people or to themselves? Now mathematics is needed for modelling the situation and it is obvious that linear functions could provide an appropriate model for solving this problem. Several mathematical competencies are required from the learners, e.g., writing down the functional equation correctly in connection with the given numbers of the pay scales. Also knowledge about different kinds of representations is required. A graphical solution representing the three graphs could be one approach; another would be to equalize the pay scales as is shown in Figure 3. Note that this focuses only on the call charges and not the texting charges.

Interpreting and validating the results are necessary sub-competencies and steps within the modelling process: If Devil-media is compared to ZING the point of intersection is at 90. A graphical representation will show that the pay scale of ZING is profitable over that of Devil-media until 90 minutes has been used, and after this time the pay scale of Devil-media is better. Further interpretations should be made with learners in order to encourage them to think about the solution, to reflect about the solution, and to make the transition step back from mathematics to reality.

There are different ways that a teacher can structure and prepare a lesson when implementing the modelling problem presented above. One possibility would be to start with this example before the concept of linear function is developed to give meaning to the topic, and to show where one can find this fundamental concept in daily life situations. Another possibility is to use the modelling problem after the concept has been developed and reinforced through several exercises. Although a standard model is inherent in this problem, given the context, it is worthwhile for learners to develop it.

Lessons in European Classrooms

The cell phone task is used only as a vehicle to illustrate fictive classroom scenes in Europe. The Roman speaking countries, primarily, have a special view on the teaching and learning of mathematical modelling. These countries start from a theory-related background described as epistemological modelling. They refer to the Anthropological Theory of Didactics (ATD) of Chevallard (1999; 2001) or to approaches of the theory of the 'contract didactique' of Brousseau (1997). García & Higueras (2005), García, Gascón, Higueras, & Bosch, (2006) and Dorier (2005) brought these views into the discussion of modelling and applications. The ATD-approach proposes a general epistemological model of mathematical knowledge, conceived as a human activity (see García et al., 2006). In Chevallard's sense the notion of *praxeology* (or mathematical organization) is structured in two levels, first the *praxis* or "know-how" (including various problems and techniques to solve them) and second the *logos* or "knowledge" (including the "discourses" that describe, explain, and justify the techniques used and at times produce new techniques). Within the ATD-approach, mathematics is seen as any other human activity, as something that is taught, learned, or practiced in social institutions. This is named *mathematics praxeology*. ATD, can be viewed as one central underlying theoretical and didactical concept for mathematics teachers in the Roman speaking countries; however, when using the cell phone task in school, the focus

[2] See also the last three ICTMA proceedings

[3] For theoretical thoughts concerning "authenticity" see the works of Vos (2005; 2011)

Pay Scales for Cell Phones

There are three different offers of pay scales for cell phones. For which type of customer would each of the offers be most profitable? Please give arguments for your decisions.

	Devil-Media	*ZING*	*CityPhone*
Basic Charge	4,50€[1]	No basic charge	25€ flat rate per month
Call Charges	0,15€/min.	0,20€/min.	No charge for calls or texts beyond flat rate
Text Charges	0,10€/SMS	0,20€/SMS	

[1] 1, 00€ = $1.33 (Exchange rate 10.08.2013)

Figure 1.

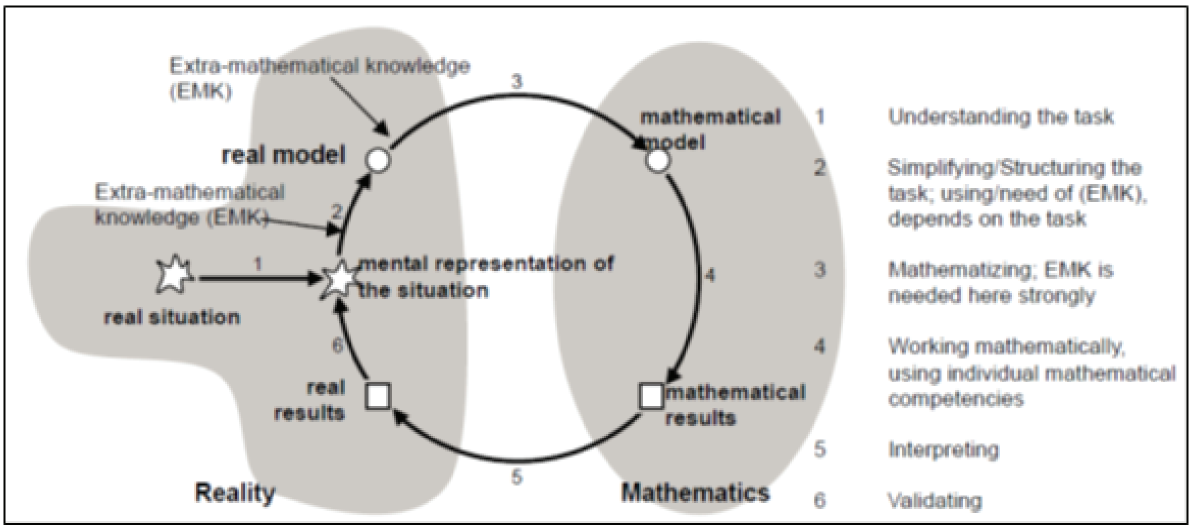

Figure 2. Modelling cycle (Borromeo Ferri, 2006)

Devil-Media and Zing: $0{,}15x + 4{,}5 = 0{,}20x \rightarrow 90 = x$

Devil-Media and CityPhone: $0{,}15x + 4{,}5 = 25 \rightarrow 143{,}3 = x$

CityPhone and Zing: $0{,}20x = 25 \rightarrow 125 = x$

Figure 3.

will not specifically lie on the transition processes from reality to mathematics and vice versa, but will be placed on the inner mathematical-structures of the problem.

In a Spanish classroom, the modelling activity is made more concrete by proceeding as follows: After the learners of grade eight have read the problem, the teacher will identify a mathematics praxeology around linear functions with the students. In the sense of praxis, this means *knowing how* to write the linear function equation when investigating the mathematics. Furthermore, the learners figure out which techniques are needed to successfully solve the problem. Finally, in the sense of *logos*, the teacher and students start a technological-theoretical discourse (perhaps within a plenary context) that explains and justifies the mathematical activity performed (i.e., defining what a linear function is, and how it can be represented differently). In this modelling activity, both, the extra-mathematical and intra-mathematical aspects of the cell phone task were taken into account. Nevertheless, while preparing and enacting this modelling activity, the importance for the teacher is clearly on the latter aspect.

Teachers from England, Germany, or the Netherlands may follow another approach when using the cell phone task, by focusing on the real situation itself at the beginning of the lesson; then, later, focusing on transition processes and on several phases of the modelling process, in particular, interpreting and validating the mathematical results. The central characteristic, formulated by Haines & Crouch (2005), of the realistic perspective is that modelling is primarily an activity for solving authentic problems and not for developing mathematical theory. This does not mean that learners solving the cell phone task do not talk about the concept of linear function as in the Spanish classroom, but they use the linear function as one possible mathematical model and as an important concept for solving the problem. During the modelling activity, for example, in England the teacher may be motivating the students to refine their developed models or to reduce the number of generated assumptions or variables while building a real model when they start to work on the task. When learners present their results in a plenary session, discussion is about the mathematical model, not about building new theories.

Comparing the fictive scenes of the modelling activity in Spain and England it becomes apparent that the respective epistemological and realistic perspectives seem to represent opposite poles within the classification (Kaiser & Sriraman, 2006). Supposing the approach of praxeology becomes the main focus, then many mathematical activities are identified as using real context for mathematisation not as modelling activities. If there would be a strong orientation only concerning the realistic approach, then a main part of theory development in mathematics while modelling should only be completed in parts. Both perspectives are important and are implemented in the teaching culture of these countries. Therefore, mathematical modelling, when continuously embedded in mathematics lessons, fulfills the justifications formulated by Blum (1996; 2011). Independent of other variables not mentioned in this paper concerning further factors influencing teaching in general, modelling is taught diversely, as seen from the various historical influences and current directions of research

The educational modelling perspective can be seen as an approach between epistemological and realistic modelling as Kaiser & Sriraman (2006) already proposed. On the one hand, learning processes are structures, and on the other hand the understanding of mathematical concepts is promoted. Hence, from this perspective teachers would ensure that learners undertake all the phases of the modelling cycle, and then think about the inner-mathematical structure of the task (e.g., linear function in the cell phone example). Further, as an aim of educational modelling, the teachers would move the activity to the meta-level in order to discuss the steps of the modelling process. While the majority of research studies developed in the area of modelling can be classified under the perspective of educational modelling, it is unclear how many teachers in Europe follow these integrative ideas when they teach modelling.

In summarizing this fictive view of European classrooms, the diversity of theoretical approaches or didactical concepts that influence the teaching and learning of mathematical modelling can be illustrated. From these various views have emerged very fruitful discussions within the area of research and practice, with strong implications for teaching modelling in Europe[4]. As is the basis for the discussion so far, one can say that, based on historical roots, the teaching and learning of mathematical modelling has a different emphasis in Europe than in the United States, South America, or Asian countries. In the next section an example of a design-based research study across Europe is presented.

Empirical Studies in Europe

Research within mathematical modelling in Europe is broadly based, and encompasses studies dealing with cognitive processes of learners and teachers (e.g., Matos & Carreira, 1997; Borromeo Ferri, 2010; Borromeo Ferri & Blum, 2009b); diagnosis and feedback (Besser, Blum & Klimczak, in press); measurement of modelling competencies (see, e.g., Grünewald, in press); and questions concerning pre- and in-service teacher education in modelling (e.g., Blomhoj & Hoff Kjeldsen, 2006; Borromeo Ferri & Blum, 2012). From the many studies available, the LEMA Project (Learning and Education in and through Modelling and Applications[5]; leader: Katja Maass[6]), funded by the European Union, is briefly presented as an example—not only because six countries from Europe[7] participated actively as partners, but also because the overall aim of the project was to facilitate a change in teachers' classroom practices so as to include mathematical modelling activities.

The six partner nations developed a course of professional development that also provided an opportunity to find out more about mathematics teaching across Europe. During the three-year period of the project, the LEMA team analyzed teachers' needs by looking at their beliefs about mathematics, the mathematics tasks they used, and their opinions about modelling tasks. The central results showed that teachers are aware of applications of mathematics, but that they prefer not to use open (modelling) tasks because these tasks are too difficult for their students and time consuming. On the basis of this analysis, materials were developed in a modular structure so that they could be used in different national contexts and at both the primary and the secondary levels. The materials

[4] See, e.g., Proceedings of CERME-8 and CERME-9: http://www.mathematik.uni-dortmund.de/~erme/index.php?slab=proceedings
[5] Project period: 2006-2009
[6] University of Education, Freiburg, Germany
[7] Germany, Spain, Hungary, Greece, France, United Kingdom

provide a professional development course of up to 5 days. The material was piloted in six European countries during 2008 with more than 120 teachers from Cyprus, England, France, Germany, Hungary, and Spain attending courses organized by the LEMA team. The results of the evaluation of this course, acquired from a questionnaire (Maaß & Gurlitt, 2009), show that a significant increase in pedagogical content knowledge and self-efficacy in relation to modelling could be claimed, and that teachers' intentions to include modeling in their lessons was very high (Maaß & Gurlitt, 2011). Last, the materials were optimized, in order to be provided to teachers in the named countries and other countries in Europe. The LEMA Project shows that there are strong efforts needed to motivate teachers to use modelling problems in mathematics lessons across Europe—although curriculum requirements provide additional motivation to do so.

Discussion and Implications

The aim of this article is to provide insight into the current discussion of mathematical modelling education in Europe. A deeper look into the current theoretical perspectives on mathematical modelling illustrated two perspectives which represent European views on modelling—the underlying didactical approaches of ATD and of the realistic approach. These concepts influence the teaching and learning of mathematical modelling because they are implemented in the teacher education programs of the various countries. The comparison of teaching sequences in Spain and England, representing the two didactical poles, clarifies different foci on modelling, namely a more theoretically based inner-mathematical approach and a more contextual reality based approach that includes the whole modelling cycle. The educational perspective provides a balance between these poles. Many empirical studies (not only European ones) can be summed up under this approach. The LEMA Project was a means for obtaining a better overview about teachers' views on modelling across Europe, as well as the perceived status of curricula of which modelling was already a part, and that marked for future inclusion. The trend in Europe is to promote mathematical modelling as an important part of the learning of mathematics and accordingly, as a component of general education. The starting point should be in-service mathematics teacher education at the university level, with an obligatory course on the teaching and learning of mathematical modelling for both undergraduate and graduate students (Borromeo Ferri & Blum, 2009a). Of great importance within in-service teacher training programs will be the provision of materials and tasks that teachers can use immediately in school.

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