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The Effects of Constraints in a Mathematics Classroom

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The dictionary definition of constraint is one-sided, solely restrictive. The problem-solving definition is two-sided. Constraints come in pairs. One retains its restrictive function, precluding something specific; the other directs search for its substitute. The paired constraint model is applied to both domain and classroom. I discuss the effects of curricular, variability, testing, cognitive, and talent constraints; demonstrate how paired constraints can be used to create a new curriculum; and close with suggestions for using constraints effectively and creatively in the classroom.

Keywords: constraints, creativity, mathematics, classroom

Introduction

Kilpatrick's image of curriculum as "a linear path through a multidimensional domain" (2011, p. 8) is a fine place to start a conversation about constraints in the mathematics classroom. Considered from a problem-solving perspective, both path and domain are constructed and defined by paired constraints that *promote* some things and some ways of doing those things, and *preclude* other things and other ways. Since I like to introduce ideas with examples, I imagined two possible pairs for a path. One, based on Simon's (Zhu & Simon, 1987; Simon, 1988) seminal work on learning by doing, would privilege procedural over conceptual knowledge. The other, following the finding that a single generic example can produce greater transfer than multiple concrete examples (Kaminsky, Sloutsky, & Heckler, 2008), would promote abstract and preclude concrete examples.

Notice—different sources, different pairs, same process.

The process, the *precluding* and *promoting,* is the core of the constraint model, which frames my thought process throughout this paper. I first (more formally) elaborate on the model, then (less formally) apply it successively to domain, path, and other constraints in the classroom.

Modeling Constraints

Mathematics involves posing, structuring, and solving problems. Constraint pairs are tools used to structure solution paths in what Newell and Simon (1972) called problem spaces. A problem space is how a given solver represents a given problem. The term "given" is important. Students, who are novices, represent problems differently from teachers, who are experts (Chi, Glaser, & Farr, 1986). Good teachers recognize the differences and use them to guide instruction.

My example of a novice problem space is shown in Table 1. My novice is a child learning to solve single digit addition problems in increasingly efficient ways.

As the table shows, a problem space has three parts: an initial state, a goal state, and between the two, a search space in which the solution path from initial to goal state is constructed. In traditional problem solving models, the path is constructed using operators. An operator is a conditional statement, an "if…then" rule that specifies an action (the "then") to be taken in a specific situation (the "if"). In the model I use operators are replaced by constraint pairs. One of each pair retains its expected, restrictive function, precluding or limiting search in some parts of a problem space. The other promotes or directs search for its substitute (Reitman, 1965; Simon, 1973; Stokes, 2005; 2007). As expertise is acquired, more elegant and efficient substitutions will be selected.

Table 1.

Parts	Description		
Initial State:	$3 + 5 = x$		
Search Space:	*Preclude*		*Promote*
	Guessing	→	Counting all
	Counting all	→	Counting on
	Counting on	→	Counting from higher addend
	Counting	→	Retrieve from memory
Goal State:	Solve for x		
Criterion:	With the most efficient strategy		

In my addition example in Table 1, the initial state is an equation, the goal is to solve for *x*. There is a criterion for mastery: solve with the most efficient strategy. The solution path that produces that strategy (retrieve) is the promote column. The constraint pairs that structure the solution path show a progression from less to more efficient strategies. I borrowed these from Siegler and Jenkins' (1989) categorization of addition strategies. In their studies, children go from guessing to counting all the digits separately (1 to 3, and 1 to 5) and then together (1 to 8); from counting all to counting on (starting with the 3); from there to counting from the higher addend (starting with the 5); and finally, to simply retrieving a known solution from memory. As the example shows, mastery of single digit addition is constraint based. Each successively more efficient strategy improves on, and substitutes for, the one preceding it.

Domain Constraints

Constraints define domains, well-developed areas of expertise that, like mathematics, have agreed upon performance/solution criteria (Abuhamedeh & Csikszentmihalyi, 2004; Chi, 1997; Simonton, 2004). In my own work (Stokes, 2010; 2013), I've considered four kinds of constraints.

- *Goal constraints* are overall criteria: agreed upon solutions in well-structured, well-defined problems; not yet specified solutions in ill-structured, incompletely defined ones. These are primary because other constraints are chosen to satisfy or specify them.
- *Source constraints* are existing elements that a solver works with (promoting) or against (precluding). These elements include subject and task constraints.
- *Subject constraints* identify content.
- *Task constraints* involve applications: materials and ways of using them (methods) to construct solution paths.

Constraints also redefine and expand domains. New paradigms are acquired within a domain in the same way that new skills are acquired by the individual. If I were to construct a problem space for a paradigm shift, my preclude column would include elements of older paradigms, perhaps "emphasize basics or concepts;" my promote column, a substitution like "emphasize applications" that defines a newer one.

Constraints in the Classroom

Caveat magister.[1] There are multiple constraints in the mathematics classroom. In addition to domain constraints, there are *curricular* constraints, determining how and in

what order domain specific skills are taught; *variability* constraints, specifying how differently skills are applied; *testing* constraints, affecting and reflecting teaching; *cognitive* constraints, limiting information processing; and *talent* constraints, directing interest to brain areas with the greatest neural plasticity; (Stokes, 2010). I consider each in turn.

Curricular Constraints

Becoming an expert means mastering the constraints that define one's domain (Ericsson, 1996; 2007). The Common Core State Standards for grades K–12 specify the mathematical constraints to be mastered and the order in which that mastery is to be attained. What they do *not* standardize are curricula designed to meet these learning criteria. New curricula must be created, adopted (by a school), and adapted (by a teacher) in the classroom. Paired constraints, including some things and some ways of doing things and precluding some other things and other ways, offer a model for their creation as well as their adaptation. To demonstrate the process, in later sections I discuss (first) how a new kindergarten math curriculum was created, and (second) how curricula and lesson plans can be re-created in the classroom.

Variability Constraints

Variability constraints determine how diversely, in how many different ways, something should be done. They are critically important early in learning, when children (or adults) are first introduced to a domain. This is because learning *how* to do something involves learning *how differently* to do it (Stokes, 1999; Stokes & Harrison, 2002). The how is the skill; the how differently is what I call a *learned variability level*. I think of it as a preferred, habitual range, within which responses differ from each other (Stokes & Balsam, 2001).

High habitual levels are desirable because they facilitate further learning and transfer in the domain of their acquisition (Stokes, Lai, Holtz, Rigsbee & Cherrick, 2008). For example, young children who initially use more strategies while developing their mathematical skills (Carpenter & Moser, 1982; Siegler, 1996) acquire new strategies faster. Table 1 shows five simple addition strategies. To start, my novice simply guesses. With experience, she retrieves the sum from memory. In between, she counts all, counts on, or counts from the higher addend. This last is called the "min" strategy. Children who switched between strategies more often prior to mastering the min strategy acquired it sooner than those who used fewer (Siegler & Jenkins, 1989). Importantly, less efficient (but still sufficient) addition strategies do not disappear (Fuson, 1990). With mastery, more efficient strategies are used more often than less efficient ones, but

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[1] Let the teacher beware.

variability—measured as the number of different strategies used on a problem set—remains stable (Siegler, 1996). The number is the product of the child's acquired, habitual variability level.

I suggest ways to help children acquire desirably high habitual levels in the section on using constraints in the classroom.

Testing Constraints

Standards lead to standardized tests. Test preparation improves performance by making students familiar with content and form. Two other benefits might make test preparation less odious to teachers. "Teaching to test," means doing math differently. To reiterate, the benefit of doing things differently is the high variability, which facilitates learning and transfer. Taking the constraint pair perspective, precluding the usual/promoting the different, should make both student and teacher more variable. This is not a bad thing.

The other benefit is called "test-enhanced" (Roediger, McDermott, & McDaniel, 2011) or "test-potentiated" (Arnold & McDermott, 2013) learning. The argument is that retrieving information enhances long-term retention and subsequent performance. The evidence comes from studies showing that students who repeatedly retrieve information (via re-testing) retain it better than students who spend an equal amount of time re-studying the same information (Schwartz, Son, Kornell, & Finn, 2011). Short quizzes have the same effect as longer tests. This too is not a bad thing.

I also talk about testing for teaching (as opposed to teaching for testing) in the section on using constraints in the classroom.

Cognitive Constraints

Cognitive constraints are based on a child's brain capacity, particularly that of the prefrontal cortex (PFC). The PFC, considered critical to working memory and problem solving, is underdeveloped in K–12 children. A child's current capacity determines both the complexity and the speed with which problems can be solved. Complexity is affected by memory span: older children can hold more items in working memory than younger ones (Henry & Millar, 1991; Huizinga, Dolan, & van der Molen, 2006; Siegler, 1996). Processing speed too increases with development. Older children problem solve faster than younger ones (Bjorklund & Green, 1992; Kail, 1986), and find it easier to inhibit off-task or inefficient responding (Williams, Ponesse, Schachar, Logan, & Tannock, 1999).

Some children's brains mature sooner than others. These will be your faster learners. However, the subjects in in which they are quickest will depend on what I call talent constraints.

Talent Constraints

Talent constraints are related to plasticity in areas other than the PFC. Plastic means moldable, pliable, variable. Neural plasticity refers to the relative ease with which the child's brain adapts to different kinds of environmental stimuli. Adaptation results in the establishment or reorganization of associative networks in ways that facilitate further expansion and adaptation (Garlick, 2002; Nelson, 1999) in specific brain areas (Trainor, 2005; Werker & Tees, 2005). The most plastic areas in the child's brain, the ones most readily reorganized, are the basis of what we call talents or gifts.

Talents, like other constraints, are two-sided. They simultaneously promote and preclude interest and skill development in different domains. How enthusiastically and how easily children acquire specific skills depends on the brains they were born with. For example, the child predisposed to notice, recognize, and remember patterns (spatial, aural, numeric) will be motivated to pursue pattern making in the relevant domain (architecture, music, mathematics). Conversely, the plasticity that promotes earlier entry and mastery of one domain (Winner, 1996), will preclude equal interest in another, one in which the brain is not as adaptable and adept.[2]

Using Constraints to Create a Curriculum

At last, here comes application. To illustrate how paired constraints can be used to create a curriculum, I'll follow my own thinking process in developing a math program for kindergarten. Since I study problem solving, I started with what I knew.

Substitution One

I knew that experts represent and solve problems using large meaningful patterns in their areas of expertise (Ericsson, 1996; Newell & Simon, 1972). For mathematicians, these patterns involve relationships between numbers and symbols. This was my question: with practice, and practice primarily with numbers, symbols, and the relationships between them, can children learn to think and problem solve like mathematicians? This became my goal criterion—*thinking in numbers, symbols, and relationships*—and produced my first *subject* constraint pair (on content):

Preclude words → *Promote numeric, symbolic patterns.*

By "words" I meant videos with cartoon characters giving directions, as well as work sheets with word problems related to stories or situations.

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[2] Lower plasticity does not, however, preclude competency or—with enough time and effort—mastery (Ericsson, 1996). The important thing for all children is early exposure to, and immersion in, a domain.

Substitution Two

The next step was figuring out *how* I wanted young children to think in and about numbers. Again, a good place for me to start was with a problem, place-value. Many Asian children have no problem with place-value. Their count makes place-value obvious. American children may have a problem because the English count obscures it (Fuson, 1990). Table 2 shows an English language version of the Asian (Chinese, Japanese, Korean) counts that make the base-10 structure of our shared number system explicit.

The count shown goes from 1 through 39, which is called three-ten-nine. Notice that every number name is *quantitative*. Calling 11 "eleven" does not readily identify its *placement in the count*, or its *place value*. Calling it "ten-one" places it after ten and before ten-two. It also immediately identifies the 1 in the tens place value as a ten and the second 1 in the ones place value as a one.

Thinking that children should learn this in kindergarten, I had a second *subject* constraint:

> Preclude the Western count → Promote an explicit base-10 count.

Substitution Three

Precluding words simplified content. Could I simplify materials as well? Could a single manipulative—like the abacus, but much simpler—make base-10 patterns visible and concrete? This became a *task constraint* pair:

> *Preclude multiples → Promote a single manipulative.*

Figure 1 presents the manipulative, a grid with moveable numbers, number names, symbols, and colored "blocks" representing tens and ones. This grid shows numbers from 1 through 5. Children interact visually (seeing the patterns), verbally (reciting each row aloud) and tactilely (moving the parts to re-create the rows or to create addition problems) with the grid. Yes, there are some words, but they all point to numbers, symbols, and patterns and not to stories or objects other than the "blocks." For example, the top row is read "number 1 same as word one equals one block." Figure 2 shows another grid with the numbers 10 to 15 (ten-five). Notice that it uses explicit base-10 names and combinations of "ten" blocks with an appropriate number of unmarked "one" blocks. Notice too, how the block patterns reiterate the numeric patterns in the base-10 count.

Am I done or just begun?

Table 3 shows my current problem space. There are spaces for additional constraint pairs. Do I need them? Have I been creative enough? What did I leave in that should be left out? Alternatively, have I been too creative, too controversial?

What did I take out that should be left in? I can't answer all those questions here and now. Their purpose, and the purpose of the example, is to show how paired constraints can be used to re-think and re-create a curriculum—or a lesson plan. What substitutions would you like to see in yours? Think about them.

Whose expertise counts?

That exercise wasn't too difficult, was it? Or do you think it was easy because of my expertise? Let me make this very clear. I may be an expert when it comes to constraints and problem solving in general, but teachers are the experts in the classroom. I can come up with constraint pairs, but how a promote column gets translated into lesson plans and implemented in the classroom is quite outside my expertise. In fact, I could not have even begun developing a radical, and surprisingly successful,[3] early math curriculum (*Only the NUMBERS Count*)—based on the constraints in the exercise—without a fantastically skilled and enthusiastic kindergarten teacher as my partner. (Thank you, Mrs. Tronza.)

Using Constraints Effectively in Your Classroom

There's not much even the best teacher can do about cognitive or talent constraints. There's a great deal that can be done with variability, testing, and curricular constraints. There's even more you can do by using the curriculum example to create or re-create lesson plans. Let's talk about applying what you've learned.

High variability facilitates learning and transfer

This suggests precluding problems that are too easy, are solved without having to try different things/use multiple steps, and thus reinforce low variability. Promote, instead, problems that are *just hard enough*. This means determining the solution involves trying/doing different things. In this case, because high variability is associated with success, it will become habitual. Cognitive and talent constraints, which separate students into faster and slower learners in different domains, come into play here. In the curriculum I (partially) described, slower students can be asked to come up with three different addition combinations for the number 5 while faster students can be challenged to come up with six or more. Your task as a teacher is to create tasks that will be just hard enough for each group.

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[3] For example, on place-value, single and double digit addition and subtraction, and number line estimation, kindergarten children in the pilot outperformed those in a comparison class using the district's standard curriculum. On number line estimation, they performed as well as Chinese students of the same ages.

Table 2. Explicit Base-10 Count

Ones	*Tens*	*Twenties*	*Thirties*
	10 *ten*	20 two-*ten*	30 three-*ten*
1 one	11 *ten*-one	21 two-*ten*-one	31 three-*ten*-one
2 two	12 *ten*-two	22 two-*ten*-two	32 three-*ten*-two
3 three	13 *ten*-three	23 two-*ten*-three	33 three-*ten*-three
4 four	14 *ten*-four	24 two-*ten*-four	34 three-*ten*-four
5 five	15 *ten*-five	25 two-*ten*-five	35 three-*ten*-five
6 six	16 *ten*-six	26 two-*ten*-six	36 three-*ten*-six
7 seven	17 *ten*-seven	27 two-*ten*-seven	37 three-*ten*-seven
8 eight	18 *ten*-eight	28 two-*ten*-eight	38 three-*ten*-eight
9 nine	19 *ten*-nine	29 two-*ten*-nine	39 three-*ten*-nine

Figure 1. Grid with moveable parts representing numbers 1 to 5.

Figure 2. Grid with moveable parts representing numbers 10 to 15 (ten-five).

Table 3.

Parts	*Description*		
Initial State:	Current curricula		
Search Space:	*Preclude*		*Promote*
	Words	→	Numbers, symbols, patterns
	Western Count	→	Explicit base-10 count
	Multiple Manipulatives	→	Single manipulative
	_____	→	_____
	_____	→	_____
Goal State:	New curriculum		
Criterion:	Thinking in numbers, symbols, and patterns		

Testing facilitates retention and further learning

This is hard. I'm asking you to preclude your aversion to teaching for testing, and instead promote being positive about using *testing for teaching*. The more practical thing is this: Preclude doing all the test preparation at once, right before the tests; instead promote *integrating* it into the lessons where the same topics (say, addition or fractions) are being taught. This will make your teaching and their learning both more variable and, via retrieval practice, more effective.

Curricula are re-created in every classroom

Curricula do preclude some things and some ways of doing things while promoting others. They do not, in fact cannot, preclude your teaching style and the needs/levels of your current students. What you can preclude is teaching the curriculum *exactly* as given. What you can promote is varying *just enough*: just enough to surprise your students (and get their attention); just enough to challenge faster students and foster slower ones; just enough to do what you do best; just enough for them to do better.

Lesson plans should also be re-created

Every year, teachers learn more about teaching, about reaching different students, differently. Go back to the section on creating a curriculum. Draw a problem space. Put the elements from a current lesson plan in the preclude column. You may not want to change everything, but you probably should change some things. Substitutes go in the promote column. You'll be surprised at how often one substitution suggests another. With constraint pairs, creativity happens step-by-step.

Closing Thoughts

Good ideas are generative; they lead to other ideas and other applications. I hope the constraint pair idea will be generative in your classroom. Think about it. (The spaces are for your thoughts).
- Think of paired constraints as ways to be creative in your classroom.
- Think about problems as opportunities for new solutions.
- Think of paired constraints as ways to construct new solution paths.
- Think about a specific thing that could/should be precluded. This will be the start of your solution path.
- Think about its opposite. This will be what you promote in its place.

- Think again about an existing lesson plan. Preclude some part and promote its opposite.
- Think about another lesson plan. Reiterate.
- Think about this with another teacher.
- Think about this with your students.
- Think about _____
- Think about _____
- Keep thinking.

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