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Developing Creativity Through Collaborative Problem Solving

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This paper discusses an alternative approach for developing problem solving experiences for students. The major argument is that students can develop their creativity by engaging in collaborative problem solving activities in which they apply a variety of mathematical methods creatively to solve problems. The argument is supported by: considering why developing creativity is a worthwhile adventure for students; describing why collaborative problem solving is a unique endeavor in which to engage students more fluidly and less compartmentally; and discussing how students can become skilled at thinking logically and creatively about mathematics content through their engagement in collaborative problem solving tasks.

Keywords: problem solving, creativity, collaboration, metacognitive thinking

Introduction

With the growth of an information-based economy, new technology innovations, and globalization, the need to enhance students' skills and competencies "such as the ability to communicate, collaborate, think critically, and solve problems—are considered even more valuable" than conventional thinking (Jerald, 2009, p. 46). Research on developing mathematical creativity has focused increasingly on the potential for learning mathematics content through problem solving activities that encourage collaborative discourse (Hwang, Chen, Dung & Yang, 2007; Pehkonen, 1997; Silver, 1997). Collaborative discourse has been introduced from associated fields (e.g., educational psychology and the learning sciences), suggesting that cognition and learning take place at group and individual levels.

Mathematical instruction is often approached in terms of stating and exemplifying rules: the tell, show, and do model. The assumption is that information can be presented by telling and that understanding will result from being told. Such an approach does not work because it frequently overlooks two crucial pedagogical practices: the creative element of problem solving and the benefit of collaborative discourse. Curricular reform in mathematics education calls for change in pedagogical practices, including helping *all* students develop meaningful understanding of mathematics through processes of discovering and formulating conjectures about mathematics content and its real-world applications (National Council Teachers of Mathematics, 1991; 2000; Common Core State Standards for Mathematics, 2012). One of the major assumptions outlined in these documents is that it is unacceptable to reduce learning and teaching of mathematics to practices that are merely prescriptive and recipe-oriented because of the complex nature of mathematics pedagogy.

The primary purpose of this paper is to discuss an alternative approach for developing problem solving experiences for students that will assist them in learning how to offer creative solutions to mathematical problems. In the following pages, we propose that students can develop their creativity by engaging in collaborative problem solving activities in which they apply a variety of mathematical methods to solve problems. Developing creativity through collaborative problem solving activities offers students a different context in which to construct and connect knowledge and to organize their thoughts and ideas about mathematics. This paper is organized into several sections. First, we consider why developing creativity is a worthwhile adventure for students. Second, we describe why collaborative problem solving is a unique endeavor in which to engage students more fluidly and less compartmentally. In the final section, we discuss how students can become skilled at thinking logically and creatively about mathematics content through their engagement in collaborative problem solving tasks.

What Is Creativity?

Although there are no commonly established definitions of creativity, Pehkonen (1997) argues that creativity merges both logical thinking and divergent thinking, "which is based on intuition but has a conscious aim" (p. 65). Bishop (1981) also defines creativity as relating to both divergent and logical developments. Characterized by a progressive approach, a sequential logical thinking process involves making deductions or presenting arguments. Logical thinking is "simply a matter of organizing and manipulating information. Problems or situations that involve logical thinking call for structure, for relationships between facts, and for chains of reasoning that *make sense*" (Albrecht, 1984, p. 3, emphasis in original). However, divergent thinking is "the process of generating multiple related ideas for a given topic or solutions to a problem. Divergent thinking occurs in a spontaneous, free-flowing, 'non-linear' manner" (Smith, 2012, p. 1). Therefore, divergent thinking has little to do with engaging in tasks that call for rigorous systematic or linear problem solving processes, which incorporate essential ideas and deductions and then organizing them in a series. Rather, it has to do with engaging and exploring authentic, real-world problems in innovative ways to learn and understand mathematics well. This perspective makes sense because when solving complex mathematical problems the preliminary approach to the problem may be unsystematic (e.g., spontaneous), or the approach to the problem might involve analyzing facts and ideas.

This discussion leads us to Sternberg's (2007) conception of creativity. Sternberg contends, "*Creativity* is the ability to produce work that is novel (that is, original, unexpected), high in quality, and appropriate (that is, useful, meets task constraints)" (p. 89, emphasis in original). Furthermore, Sternberg claims that creativity requires the intellectual resources of divergent, logical, and real-world skills. Appropriate applications of these skills can empower us to view problems in original ways that are beyond the limits of conventional thinking, to acknowledge which ideas are worth following and which are not, and to know how to encourage others to value our ideas. Sternberg's framework underscores the importance that creativity can be developed in informal and formal educational contexts; the individual does not need to be a genius or have special intellectual abilities. Creativity can be subjected to "instructional and experiential influences," and the main instructional approach that is needed to develop creativity is active engagement in open-ended problem solving activities that require multiple solution strategies (Silver, 1997, p. 75).

Collaborative Problem Solving as a Mathematical Practice

Engaging students in problem solving activities to promote a conceptual understanding of mathematics is not a new idea. For decades, the National Council of Teachers of Mathematics (2000) has had at its core the perspective that "learning mathematics with understanding is essential" and applying the processes of problem solving, reasoning, and proofs are the best ways to meet this demand (p. 20). A similar notion is advocated by the Common Core State Standards for Mathematics (2012), which also emphasizes conceptual understanding of mathematics through the mathematical practices of problem solving and reasoning. Research suggests that these practices or processes are effective ways to promote conceptual and procedural understanding of mathematics content (Gagatsis & Shiakalli, 2004; Jonassen, 2000; Niemi, 1996; Rittle-Johnson, Siegler, & Alibali, 2001). Not surprisingly, then, is the logical argument that problem solving as a pedagogical practice can support students conceptual development of mathematics content. It follows that collaborative problem solving can also be a medium in which to develop mathematical creativity.

When students engage in collaborative problem solving, they create and maintain knowledge to help them make sense of the mathematics they are learning. Therefore, the development of mathematical creativity through problem solving is an intellectual process involving cognitive functions. Developing creativity does not just happen for the learner; it happens through organized instruction and activity with others. The idea is to create a learning community in which students are taught how to perform as creative thinkers who make meaning of mathematics content for themselves and with others. How the problem is solved is key to the process; learners must actively engage in the process, generating creative thoughts and ideas as the process unfolds.

Collaborative problem solving is a method of mathematics pedagogy by which students work together in small groups scaffolding each other's mathematics learning, while working towards achieving a common goal. From a Vygotskian perspective, peer interaction is crucial for facilitating conceptual understanding and complex reasoning (Vygotsky, 1978). Through social interactions, more-knowledgeable students scaffold their less-knowledgeable peers to higher levels of understanding of the content. Through both verbal and non-verbal social interactions that occur within collaborative problem solving (such as speaking, listening, and writing), students make cognitive strides in the development of mathematical knowledge. Collaborative problem solving differs from other forms of "group work" because of its highly structured nature; everything from group selection, problem solving activities, and even student behavior are carefully and explicitly structured to promote mathematical understanding and creative thinking. Over the past twenty years, mathematics education researchers have analyzed various aspects of effective collaborative problem solving programs, and the resulting body of research offers teachers many insights into collaborative problem solving and the best methods by which to implement it in their classroom. Teachers must make a pointed effort to develop an environment that promotes effective collaboration so that students may take advantage of the enormous potential social interaction has on making sense of new mathematics content.

Collaborative Problem Solving: A Pedagogical Overview

Not only is collaborative problem solving suitable for all grade levels of mathematics learning, but it is also an appropriate pedagogical strategy for teaching varying degrees of complexity in mathematical thinking. To understand collaborative problem solving as it relates to learning mathematics, it is important to focus on the interplay that exists between the level of intra-group collaboration a task requires and the task's level of mathematical complexity. Research shows that the degree of complex mathematical thinking, such as creative thinking, required in problem solving has an effect on the level of group collaboration needed to solve problems, which in turn relates to the effectiveness of collaborative problem solving (Gillies & Ashman, 1998).

Just as mathematics problem solving tasks range in the degree of complexity of the mathematical thinking required in solving them, they also range in terms of the extent of group collaboration they require. Low-level collaborative tasks are defined as those that "involve low levels of collaboration because the children are required only to share information or determine how to divide their labors so that each person's contribution can be combined to form the final product" (Gillies & Ashman, 1998, p. 747). There is no need for discussion or disagreement when solving such low-level problems. High-level collaborative tasks, on the other hand, "require students to interact about the processes and discuss planning, decision making, and the division of labor as well as substantive content" (p. 747). These usually involve ill-structured problems, correspond to higher-level thinking processes, and promote higher reasoning interactions among students (Gillies & Ashman, 1998).

Working to develop students' mathematical creativity through problem solving should be the goal of any form of mathematics; without developing adequate problem solving skills, learning to make mathematical calculations is a pointless endeavor because it exists outside the realm of any useful purpose. Strong problem solving ability not only allows students to make practical sense of the mathematics they learn, but also requires them to think on both cognitive and metacognitive levels that are necessary to succeed in other academic areas. For teachers that are learning to use collaborative problem solving as a means of teaching, it is important to develop a thorough understanding of the relationship between mathematical problem solving and collaborative activities. This means understanding the interrelated cognitive and metacognitive processes as they are associated with both problem solving and collaborative problem solving, as well as how the structure of problem solving tasks relates to these processes (Schoenfeld, 1985).

Cognitive and Metacognitive Processes

It is important to develop a working definition of the terms *cognitive process* and *metacognitive process*. The definitions implied by Artzt and Armour-Thomas (1992) in their cognitive-metacognitive framework for understanding mathematical problem solving are particularly useful in the context of this paper. *Cognitive processes* refer to students' strategy selection, processing efficiency (the thought processes related to implementing a problem solving strategy), and social scaffolding of peers during the problem solving process. Dees' (1991) investigation illustrates these last two categories by empirically linking the act of making an effort to understand an opponent's position and to articulate their own position to a higher mastery of the concept being learned. Beyond the cognitive realm of problem solving processes exists *metacognitive processes*, which refer to a student's action of thinking about these cognitive processes (in other words, a student's *thinking about thinking*).

Considering this understanding, Artzt and Armour-Thomas (1992) develop a framework for analyzing the roles these processes take on during mathematics problem solving in the collaborative environment. Under such conditions, these roles take the form of episodes of student behavior that exist as the following easily monitored steps to problem solving: (1) reading the problem [a cognitive process]; (2) analyzing the information presented in the problem [a metacognitive process]; (3) exploring possible problem solving strategies [both cognitive and metacognitive processes]; (4) planning to execute a particular strategy [a metacognitive process]; (5) implementing a chosen strategy [both cognitive and metacognitive processes]; and (6) verifying the results of one's chosen strategy [both cognitive and metacognitive processes]. To these we add: (7) understanding the problem [a metacognitive process] and (8) watching and listening [to which a level was not assigned since these are more abstract, and thus more difficult to monitor, examine, and code]. We found that the collaborative group that had the lowest frequency of metacognition while the group with the highest frequencies of cognition was the only one that did not finish the problem solving activities. This indicates that metacognition is an important aspect of developing creativity through problem solving because it acts as a means of monitoring and regulating the cognitive processes. It is important to note that in the problem-solving environment, an appropriate interplay between cognition and metacognition is a necessary condition for developing creativity. This interplay is best achieved in the group setting by means of communication between group members (Albert, 2000). Communication is not merely limited to speaking; rather watching and listening play an equally important part of the communication process in group problem solving (Albert & McKee, 2001; Albert, 2012). Watching and listening are not always negative events, but they must occur in the context of balanced behavior. To summarize, cognition and metacognition are two interrelated variables that affect students' problem solving performance and creativity, and can be greatly influenced by the social scaffolding that takes place in the collaborative environment. Collaborative problem solving enables students to build shared representations of the problem and exchange creative ideas.

Learning to Work in a Collaborative Learning Environment

In the collaborative classroom, a highly structured environment is not the only factor that contributes to students' mathematics performance; the degree to which students and teachers understand how to work collaboratively affects student learning just as strongly. Research shows that the most productive and efficient collaborative classrooms are ones in which both teachers and students receive preparation in how to function in collaborative problem solving contexts (Cohen, 1994; Gillies, 2004; 2006). It is not an easy task to achieve the highly structured environment necessary to help one's students' work collaboratively to improve their mathematical understanding and to develop their creative thinking. No teacher is born with an innate understanding of the specific conditions that lend themselves to productive collaborative problem solving. Furthermore, most teachers have learned mathematics in an environment that was much more teacher-centered than today's mathematics research suggests is conducive to mathematics learning. They have little experience working (let alone teaching) in a collaborative environment. While many teachers may claim that they promote "group work" within their mathematics classrooms, research reveals that such activities are not developed enough to earn the title *collaborative problem solving.* When it comes to promoting students' learning through collaborative problem solving, teachers assume a rather indirect role. In the collaborative contexts, the teachers' primary responsibility is to mediate collaboration among students. Assuming this role marks a transition for many teachers who are used to being directly responsible for promoting student learning.

Teachers need to learn language and questioning skills that are appropriate for the group environment so that they may help students develop group interdependence. Teachers must learn to strike a balance between working with individual students and working with collective groups. Teachers need to know how to use their role as a moderator to take away their students' dependence on them and to focus students' dependence on one another. Without undergoing appropriate professional development, most teachers will not be prepared to effectively manage a collaborative problem-solving environment. Developing an understanding of collaborative learning can help teachers make the transition from a traditional classroom to a collaborative environment.

Just as teachers participate in professional development to learn how to implement collaborative problem solving activities, their students must receive instruction in how to engage as active and effective members in collaborative groups. The premise is: If students know how to function as effective group members, then the path is paved for them to apply creative solutions to complex problems. For the purpose of this paper, two specific problem-solving models are highlighted to illustrate how students might collaborate to become skilled at thinking logically and creatively about mathematical content through their engagement in problem solving tasks. Each of these models is based on Polya's (1985) problem solving framework in which emphasis is placed on helping students understand the problem they are to solve and consider creative ways for approaching the problems as illustrated in Figure 1. Polya argued, "The teacher who wishes to develop his students' ability to do problems must instill some interest for problem solving into their minds and give them plenty of opportunity for imitation and practice" (p. 5). The two problem-solving models presented here embrace this assumption put forth by Polya.

The models illustrated in Figures 1 and 2 are developed to encourage students to consolidate their knowledge, to use a variety of procedures or strategies, and to focus on processes instead of the answer; i.e., they are designed to give attention to students' thinking. They also emphasize the importance of communicating thoughts that are mathematical representations and connections to students' ideas and understanding. Therefore, students need to explain their thinking to others and write explanations about approaches or strategies used to solve the problem. In addition, each of the models consists of three strands that are related cognitively. The inter-related parts are: (1) understanding the problem [things to do before solving the problem]; (2) choosing and implementing a solution strategy [things to do while solving the problem]; and (3) evaluating the solution [things to do after solving the problem].

For students to use these models effectively, the teacher needs to provide initial guidance. In other words, when students are at the beginning stages of learning how to collaborate, the teacher takes the lead as a model and guide. For example, a problem is presented to the class and the first step is to help students understand the information presented in the problem. The teacher leads a discussion that encourages students to explore and think through the underlying mathematical structure of the problem instead of focusing on extraneous, surface details of the problem. The focus of the discussion is on process, modeled by the teacher, which illustrates systematic thinking. Thus, the teacher shows students how to approach, plan, and think about the problem. The teacher continues the modeling by demonstrating to students how to develop written explanations that describe what they know about the problem and what they need to find out about the problem. This part of the instruction and discussion highlights the process and makes explicit the steps that students might think about or go through in constructing the problem in their own words. Once the teacher is assured that students understand the problem, students are encouraged to explore solution strategies. The teacher continues instruction as a facilitator of students' thinking by asking probing questions and by giving helpful hints that scaffold student learning. Once students have solved the problem, the teacher engages them in a discussion in which students review the solution strategy they applied to the problem and evaluate the reasonableness of the solution obtained to see if it makes sense in meeting the goal of the problem. This discussion is also about how the answer was obtained and why a particular strategy worked or did not work.

USE: A Collaborative Problem Solving Approach

UNDERSTAND THE QUESTION

Ask yourself:

What are we trying to find?
Have we solved a problem like this one before?

Restate the problem in your own words.

Read the problem aloud.
Inflect your voice as you read the question.

Write a statement that shows what the group knows about the problem.

Draw a picture.
Organize the information in a table or chart.

Look up information you do not know.

SELECT A STRATEGY

Use what you already know.
Draw a picture.
Look for a pattern.
Check for errors and monitor your progress as you go along (step-by-step).

EVALUATE YOUR ANSWER

Ask yourself:

Does the answer or solution make sense?
Did we use all of the important information?

Figure 1. A Three-Step Problem Solving Model

The model shown in Figure 2 provides a level of scaffolding designed to take students through a four-step problem solving process. In this case, the scaffolding questions or hints are situated to the left, and students are given space to record their explanations, providing information about their thinking and understanding of the problem. It has been determined that when students are in the initial stage of learning how to solve problems in collaborative groups, these types of scaffolds are helpful in keeping them focused, interested, and learning (Albert, 2000; 2012; Albert, Bowen & Tansey, 2009). Over time, if these types of scaffolds are used consistently, students will internalize this process and will not always need it to be explicitly stated. While working individually, students do not need to develop an "individual dynamic" like they need to develop a group dynamic with their collaborative group, because they have spent their entire academic careers developing their own "individual dynamic."

Concluding Comments

It is important to note that some parts of mathematics curricula lend themselves more readily to collaborative problem solving than others do. Specifically, mathematics problems that require some form of problem solving are more applicable to the collaborative learning environment than those that only require application of a skill. Using a variety of instructional techniques—such as in-class and out-of-class assessments, and long-range projects—should enable teachers to integrate collaborative problem solving into their pedagogy without allowing curriculum barriers to hold them back. "If [teachers] challenge the curiosity of students by setting them problems proportionate to their knowledge, and help them to solve their problems with stimulating questions, [they] may give them a taste for, and some means of, independent thinking" (Polya, 1985, p. v). Specifically, it is important that teachers find a balance of activities that promote both skill development and practice (independently) and upper-level problem solving (collaboratively). This balance allows collaborative problem solving to become an important, though not burdensome, aspect of one's mathematics pedagogy. In the pursuit of mathematical knowledge and understanding, the goal is to provide experiential contexts that promote the development of creativity for this generation of learners and the next generation.

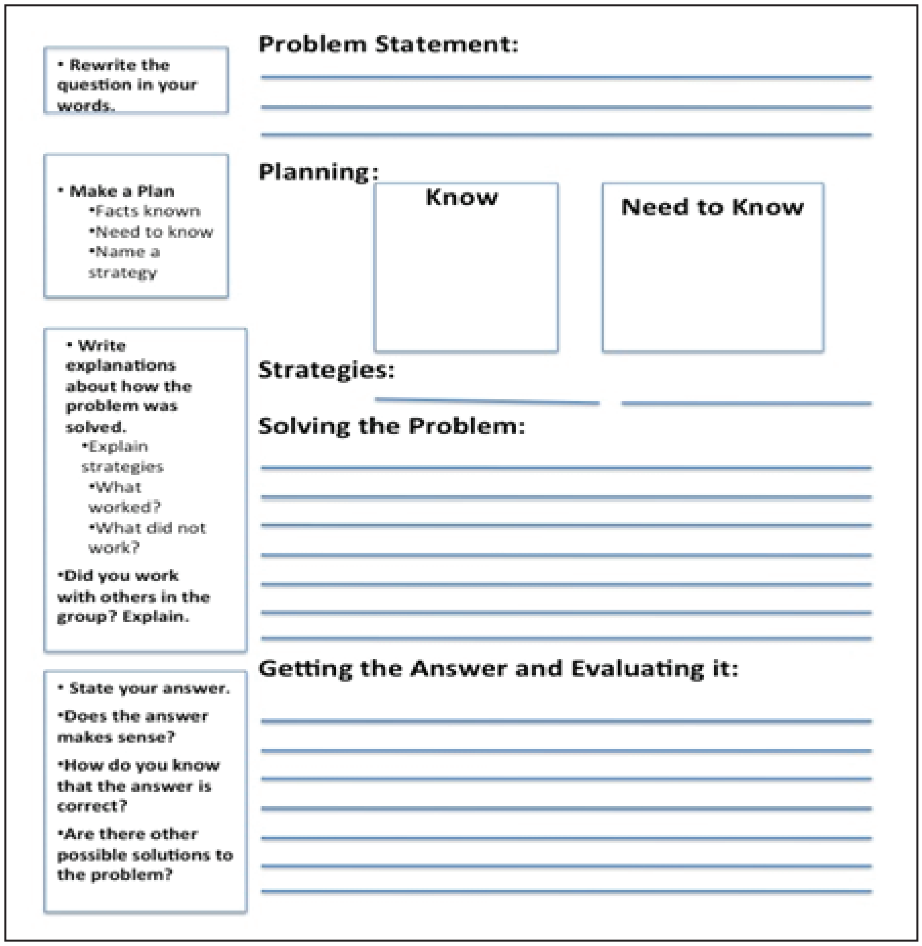

Figure 2. A Four-step Problem Solving Model with Built-in Scaffolds

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