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A CENTURY OF LEADERSHIP IN
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A Primer for Mathematical Modeling

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With the implementation of the National Council of Teachers of Mathematics recommendations and the adoption of the Common Core State Standards for Mathematics, modeling has moved to the forefront of K–12 education. Modeling activities not only reinforce purposeful problem-solving skills, they also connect the mathematics students learn in school with the mathematics they will use outside of school. Instructors have found mathematical modeling difficult to teach. To successfully incorporate modeling activities I believe that curricular changes should be accompanied by professional development for curriculum developers, classroom teachers, and higher education professionals. This article serves as an introduction to modeling by defining mathematical modeling, outlining the steps to construct a model, and providing an example that illustrates the iterative non-linear process. Key to teaching modeling is the ability to understand how the modeling process differs from problem-solving activities, which this article discusses. This article describes the benefits and challenges of incorporating mathematical models. The overarching aim of this article is to serve as a primer to aid with the implementation of curricular reforms that call for an increased focus on modeling activities.

Keywords: Common Core State Standards for Mathematics, curricular reform, mathematical modeling, professional development, problem solving, teachers

Introduction

In 2010, the Common Core State Standards for Mathematics (CCSSM) were released, which called for an increased emphasis on mathematics modeling and the ability to use statistics to organize and analyze data and make decisions based on the analysis (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGACBP & CCSSO], 2010). The Common Core State Standards for Mathematics is a state-led initiative developed to set uniform unambiguous standards to guide K–12 education. Given that, to date, 45 states have adopted the Common Core, there will be increased attention and time spent developing students' abilities to: synthesize and organize incomplete or ambiguous real-world data, analyze empirical situations, construct and evaluate mathematical models, and verify that the results make sense within the applied context (NGACBP & CCSSO, 2010). The goal is to be able to make and communicate informed decisions using the model. Prior to the release of CCSSM, as a result of adopting the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* and educators calls to revise the mathematics curriculum, teachers had increasingly focused on developing students' abilities to pose problems, reason with data, quantify information, and construct mathematical models (Lesh, Middleton, Caylor & Gupta, 2008; Moore, 1998; NCTM, 1989, 1995, 2000). Consequently, mathematical modeling

has moved, and will continue to move, to the forefront of K–12 mathematics education.

Implementing the CCSSM will require a "...concerted effort by all stakeholders in the education process" (Cogan, Schmidt & Houang, 2003, p. 1). These types of non-routine modeling activities have often not been part of teacher-training programs or of mathematicians' prior education. Teaching modeling is a demanding task (Blum & Niss, 1991). One reason is because modeling activities use multiple approaches and modeling problems are less predictable (Blum & Niss, 1991). Therefore, it is not surprising that instructors have had difficulty teaching modeling activities (Blum & Ferri, 2009; Blum & Niss, 1991). According to Blum and Niss (1991), these difficulties can be overcome by giving pre-service and in-service teachers the opportunity to work with and learn about models. I concur and believe that professional development activities that provide current and prospective teachers with the opportunity to learn about, create and evaluate models are warranted. Recognizing the important role professional development can play, this article provides an introduction to mathematical modeling and, in so doing, can aid in ensuring smooth curricular transitions.

This article begins by defining what a mathematical model is, and then outlines the steps one would follow to construct a model. One essential feature of modeling activities is that their development, unlike typical textbook exercises, follows a cyclic path (Abrams, 2001; Blum & Niss, 1991; NGACBP & CCSSO, 2010; Lesh & Doerr, 2003; Niss, Blum, & Galbraith, 2007; Pollak, 2003; Zbiek & Conner,

2006). To shed light on why modeling activities may need to go through a process of revising or rejecting an answer that is mathematically correct, an example is given. Modeling differs from problem solving and typical textbook exercises in ways that may not be immediately apparent. Therefore, to help ensure that teaching methods and curricular materials developed are aligned with curricular goals for modeling activities, these types of activities are compared. After synthesizing the literature on models, the article concludes by discussing the value and challenges of incorporating modeling exercises. The overarching aim of this article is to serve as a primer for curriculum developers, teacher-education programs, and classroom teachers, who will be incorporating more mathematics modeling activities as the CCSSM's recommendations are implemented. Additionally, this article can also serve as an introduction to modeling for higher education faculty who may want to include more advanced mathematical modeling activities to build upon the new set of skills students will be acquiring in high school.

Defining a Mathematical Model

What is a mathematical model? Blum and Niss (1991) defined modeling or “model building” as “...the entire process leading from the original real problem situation to a mathematical model” (p. 39). For example, imagine planning a future financial goal, such as paying for college, buying a car, purchasing a home, or saving for retirement. Developing a plan to reach any one of these goals requires estimating how much one can save, deciding how the savings will be invested, and approximating the rate of return. This is how a modeling activity begins. An authentic situation, from outside the field of mathematics, prompts the construction of a mathematical model (Abrams, 2001). Researchers have noted the importance of transitioning from authentic problems, typically found outside of the field of mathematics, to mathematical models (Blum & Niss, 1991; Zbiek & Conner, 2006). In the examples above, although the amount invested and rate of return may vary from year to year, estimates of numerical values will be needed to construct the model. Models are not designed to be exact replications of real-world situations; nor can they be. They are idealized representations of real-world situations (Blum & Niss, 1991; Pollak, 2003). Once constructed, the model is then used to interpret the real-world situation (English, Fox & Watters, 2005). After performing mathematical operations, the results are translated back to the real-world situation to be confirmed (Pollak, 2011). Model construction is a multistep complex process, which warrants an in-depth description.

Model Construction

Reviewing the literature, several researchers have provided descriptions and/or diagrams that explain how a model is constructed (Abrams, 2001; Blum & Niss, 1991; NGACBP & CCSSO, 2010; Doerr, 1995; Ellington, 2005; Pollak, 2003; Warwick, 2007; Zbiek & Conner, 2006). A description of steps that need to be taken to construct a model is provided.

- Select an authentic problem typically from another academic discipline or real-life. The questions posed arise from the real-world context that needs to be understood.
- Identify variables and classify them as essential or superfluous. Which of the variables prove to be essential to setting up the model depends on the questions being asked.
- At this stage, the problem is unstructured. To impose a structure, seek out relationships among the essential variables. The situation is idealized with the construction of a mathematical model.
- The model is now a mathematical entity, which can momentarily be stripped of its context. An array of problem solving techniques can be employed to answer the questions posed in the first step.
- Return to the real-world scenario to verify, interpret, and communicate the results. As the results are translated from the idealized mathematical problem to the authentic real-world situation, it is important to confirm that the results make sense within the context of the problem.
- If the results either do not make sense within the real-world context, cannot be validated, or are not practical, return to the first step. This necessitates checking the assumptions, variables selected as essential, model construction, and problem-solving techniques applied. The model will need to be modified or reconstructed. If the results are valid, it is still worthwhile to investigate whether the model could be further refined to more precisely or completely answer the questions posed. Additionally, it is worthwhile to investigate whether the model could be generalized and applied to other similar situations.

The most worthwhile modeling experiences occur when students set up models for authentic situations (Blum & Niss, 1991; Zbiek & Conner, 2006). I agree and have found the process to be the most meaningful when authentic tasks intersect with students' interests. There are usually multiple models that could be constructed to answer a question (Doerr & English, 2003). For example, students could construct graphs or other visual representations, tables, flowcharts,

set up algebraic equations, differential equations, or use statistics to answer the questions posed. The modeler's experience, knowledge base, and style of learning can all influence the type of model constructed. At the final stage, results are validated and communicated or rejected, which leads to revisions. An example is provided to illustrate this process. The table in Figure 1 gives the number of people living with HIV in South Africa. Students could use the data to construct models and then use their models to make future predictions. This activity highlights the need to possibly modify or redesign models. This activity also illustrates the importance of validating, revising, correcting, and refining mathematical models.

Modeling, Models, and Problem Solving: A Comparison

Potentially, mathematics educators and curriculum developers could have some difficulty distinguishing between modeling, problem-solving, and typical textbook exercises. Part of the difficulty may be related to the fact that modeling activities make use of problem-solving techniques. Additionally, problem-solving and textbook exercises, like modeling activities, can make use of real-world data. To develop and implement a curriculum that is in line with the CCSSM's recommendations, it is important that practitioners understand the differences between these types of activities. Six key differences are: (1) the starting point of the activity; (2) the need to differentiate between essential and non-essential variables; (3) the need to idealize the problem, and the opportunity to select the mathematical techniques applied; (4) the variety of approaches; (5) the need to transfer between the real-world and the model at the start and end of the process; and (6) the work advancing in a cyclic, rather than a linear, manner. Each of the differences is discussed in more depth here.

1. One of the most important differences that distinguish modeling activities from problem solving is the starting point of the activity. Modeling activities begin by finding a problem (Pollak, 2011). According to Doerr, Arleback and O'Neil (2013), the construction of a mathematical model begins with "model eliciting activities," during which students' ideas about problems that are both meaningful and realistic are drawn out. Problem-solving and textbook exercises usually bypass this first step, which is essential to modeling activities.
2. Once the question is posed in a modeling activity, the next step in the process of constructing a model requires one to determine which information is important and which is unimportant. According to Zawojewski (2010), problem solving consists of "...a search for a means (i.e., procedures, steps) to solve the problem, where the goal is to find the correct way to get from the given information to the goal(s) set forth" (p. 238). The fact that the starting point of a problem-solving activity is to select a procedure suggests that non-essential information is omitted. By comparison, modeling activities begin by identifying which of the variables under consideration are critical to setting up the model.
3. According to Blum and Niss (1991), "the real model has to be mathematized, i.e. its data, concepts, relations, conditions, and assumptions are to be translated into mathematics. Thus, a *mathematical model* of the original situation results" (p. 38–39). When the problem is translated into the language of mathematics, the real-world situation is idealized. Because all models require translating problems, a wider variety of techniques can be selected to be applied. By contrast, textbook problems are already written using the language of mathematics (Köhler, 2002). This may limit the number of techniques that could be applied.
4. Modeling activities give rise to a more diverse set of approaches (Doerr, 1995; Dominguez, 2010; Doerr & English, 2003), when compared to classroom problem-solving exercises which often focus on applying a narrow set of skills that match a particular curricular objective. This is to be expected since students must make several decisions in the modeling process.
5. Modeling activities arise from real-world questions and therefore the solution must be able to be understood in terms of the real-world context. This means that in modeling activities, even if the mathematical steps are correct, the results need to be examined to ensure that they are reasonable within the external real-world context (Pollak, 2011). Modeling activities require one to "...interpret information in the task and interpret the required outcome" (Zawojewski, 2010, p. 239). By contrast, when students apply problem-solving techniques, they may disregard the context of a problem (Köhler, 2002).
6. Finally, as we have seen in the previous example, modeling activities typically follow a cyclic or non-linear path (Blum & Niss, 1991; Lesh & Doerr, 2003; Niss et al., 2007; Pollak, 2003; Zbiek & Conner, 2006), which often goes through a process of revision and refinement. By contrast, in problem solving, the beginning and end stage or "givens" and "goals" are fixed (Zawojewski, 2010). Hence, there is not the dynamic changing process that exists in modeling. Because of this, it may be that textbook exercises are more apt to be solvable with the application of steps that follow an organized linear path aligned with learning a specific curricular objective.

Acquired Immune Deficiency Syndrome, or AIDS, is a growing worldwide epidemic with devastating consequences. Imagine a scientist is investigating the rate at which the number of people living with HIV, the virus that causes AIDS, is growing in South Africa. Table 1 gives the approximate number of people in South Africa living with HIV by year (United Nations Aids Info Database, n.d.).

Assuming the scientist started working on this problem in 1995, how might he or she have modeled the growth rate of people living with HIV? Using only the data from 1990 to 1995, describe the increase in words and also use mathematical symbols or a graph to create a model that estimates the number of people living with HIV in South Africa. Does it appear that the virus is growing linearly or exponentially? Explain the difference between linear and exponential growth.

Do you believe that the model will continue to accurately predict the number of people living with HIV in future years? Use the model to make an educated guess as to how many people will be living with HIV in South Africa by the years 2000, 2005, and 2010? Create two new models, one using the data from 1990 to 2000 and the other using the data from 1990 to 2011. If the original model did not accurately predict the number of people in South Africa living with HIV, investigate why and provide a written explanation. Use the United Nations Aids Info Database to investigate whether South Africa took measures to prevent the spread of HIV. Explain how these measures may have influenced the model.

Use data from the United Nations Aids Info Database website to repeat the exercise above, investigating the spread of HIV in Madagascar. In 1990, the percent of the population living with HIV in both Madagascar and South Africa was 0.2%. However, by 2012, while only 0.5% of the people in Madagascar were living with HIV, 17.9% of the people in South Africa were living with HIV. Did your models predict these differences? Did one model work better at making predictions than the other? If so, can you explain why?

<u>Year</u>	<u>Number of People</u>
1990	49.4 thousand
1991	108.2 thousand
1992	220.5 thousand
1993	415.3 thousand
1994	718.4 thousand
1995	1.1 million
1996	1.7 million
1997	2.2 million
1998	2.8 million
1999	3.4 million
2000	3.9 million
2001	4.3 million
2002	4.7 million
2003	4.9 million
2004	5.2 million
2005	5.3 million
2006	5.5 million
2007	5.6 million
2008	5.7 million
2009	5.9 million
2010	6.0 million
2011	6.1 million

Figure 1. Example of Modeling the Prevalence of HIV in South Africa

Why Teach Modeling?

Knowing what a mathematical model is and understanding how to construct models, it is still worthwhile to cover the benefits of including modeling activities. The first benefit of teaching modeling activities is that modeling connects the mathematics students learn in school with the types of authentic real-world problems they are likely to encounter outside of school. According to Pollak (2011), there is a sense of disconnect between the mathematics students see in school and the mathematics they need to use in their lives. Following the recommendations of NCTM (1989, 1995, 2000), there has been an increased focus on real-world applications, in an attempt to bridge this gap. Modeling activities close this gap

by connecting the field of mathematics to other disciplines (Zbiek & Conner, 2006).

By contrast, many textbook exercises fall short of achieving this goal because they do not teach students one essential skill—how to pose questions. Generating questions is a skill to which students have limited exposure (Silver, 1994). Even complex authentic word problems from textbooks often only reinforce the skills learned in a particular section of the book (Pollak, 2011). Modeling activities provide students with the opportunity to grapple with data where the structure of the problem is not predetermined. In the process of constructing models, students learn to organize information in meaningful ways to make decisions, which is an essential skill (Lesh, Middleton, Caylor, & Gupta, 2008).

Modeling activities give students a chance to reinforce purposeful problem-solving skills; however, by making additional requirements, mathematical modeling demands more of students.

The CCSSM's recommendations, which include a focus on modeling, were also undertaken to better prepare high school graduates to enter the workforce in positions that demand a degree of mathematical literacy. According to the Programme for International Student Assessment (PISA), to be mathematically literate includes the ability to "... analyze, reason, and communicate ideas effectively ... pose, formulate, solve and interpret mathematical problems in a variety of situations" (PISA, 2009, p. 19). PISA stressed that mathematic literacy includes the ability to understand the types of problems citizens face whether they need to analyze information or need to understand a political position where mathematical literacy and competency come into play. Modeling activities prepares students to meet these goals.

Obstacles to Incorporating Models

From the perspective of some instructors, teaching mathematical modeling is difficult (Blum & Niss, 1991). One challenge is that teachers may not be familiar with the subject matter that was used in the modeling activity, as models are typically created from real-world scenarios or non-mathematical subjects (Blum & Niss, 1991). Another potential difficulty is that teachers may need or want to supplement the curricular materials, but they may not have enough time to create or to update examples (Blum & Niss, 1991). Planning lessons that involve modeling activities can be difficult for teachers because of the time required to teach modeling. Students need sufficient time to try different models (Köhler, 2002) or to reflect on the process (Warwick, 2007).

Another potential difficulty is that some teachers may be uncomfortable with the instructional techniques used to teach modeling. According to Blum and Ferri (2009), quality mathematical modeling instruction occurs when students act independently, with little input from their teachers. Ideally, instructors teaching modeling should act as guides and encourage students to take responsibility for creating the model (Blum & Ferri, 2009). However, if teachers hold traditional beliefs about the nature of the field of mathematics, even if they hold non-traditional beliefs about how mathematics should be taught, these contradictory beliefs have the potential to increase the likelihood that the teachers adopt a more traditional pedagogical approach (Raymond, 1997). Last, some teachers struggle to assess students' work, as models have been reported to be difficult to evaluate (Blum & Niss, 1991).

Can Modeling Be Taught?

Steps can be taken to help teachers overcome these obstacles. To assist teachers who are unfamiliar with the subject used to create the model, schools could encourage cross discipline conversations and cooperation (Blum & Niss, 1991). To help instructors teach modeling, pre-service and in-service teachers need knowledge of and ample experience working with models (Blum & Niss, 1991). In addition, teachers need to have knowledge of a range of pedagogical techniques available to them. Both pre-service and in-service teachers need to be equipped with the "... attitudes to cope with the demands of teaching mathematics in the desired way" (Blum & Niss, 1991, p. 5). To assist teachers who have found assessing students models challenging, a rubric could be used. Researchers have suggested that open-ended questions could be evaluated with a rubric (Asturias, 1994; Thompson & Senk, 1998). Galbraith and Clatworthy (1990) designed a rubric to assess models. The assessment tool evaluates students' work using the following four criteria: clear specification of the problem, formulation of the model, mathematical solution, and communication of results (Galbraith & Clatworthy, 1990). In each of the four categories, students' work would be rated as meeting one of three standards depending on the completeness and quality of the work. Thus, modeling can be taught and, with continued support and training, these obstacles can be overcome.

Conclusion

This primer outlines how models are constructed and illustrates how these activities can benefit students by connecting the classroom content they see with the types of real-world scenarios students are likely to encounter outside of school. Modeling activities give students the opportunity to practice purposeful problem-solving skills, but are more demanding. I discussed the need for professional development to supplement and support curricula changes, as classroom teachers, teacher-training programs, higher education faculty, and curricular developers may have had little experience working with models. Incorporating modeling could enhance students' understanding of application. Equally important, the combination of mathematics and meaningful situations highlights the fact that the field of mathematics is both relevant and important.

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