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TABLE OF CONTENTS

Preface

Articles

TABLE OF CONTENTS

Other

Model Eliciting Activities: Fostering 21st Century Learners

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Real world mathematical modeling activities can develop needed and valuable 21st century skills. The knowledge and skills to become adept at mathematical modeling need to develop over time and students in the elementary grades should have experiences with mathematical modeling. For this to occur elementary teachers need to have positive experiences themselves with mathematical modeling. This paper describes how a four-part 21st century skills framework—innovate, synthesize, adapt, and communicate—was integrated with a mathematical modeling activity that one class of pre-service elementary teachers completed. The pre-service elementary teachers were able to generate a variety of ideas, communicate effectively, and work towards successful solutions by using their resources and prior knowledge.

Keywords: mathematical modeling, creativity, pre-service elementary teachers, 21st century skills, model-eliciting activities

Introduction

What mathematics do students need to learn and in what ways should they learn this mathematics to be productive citizens? As technological innovations increase, new jobs are created and existing jobs change. In the past decade people age 18 to 25 in the United States have held, on average, more than 6 jobs during that age range (U.S. Department of Labor, 2013). Because of this educators need to prepare students to be adaptable to the changing job market. To meet these demands we need to educate students to have robust mathematical knowledge while at the same time teaching them 21st century life skills, which include the abilities to be innovators, adaptable, good communicators, and synthesizers of information.

This paper describes how well-structured mathematical modeling problems, Model-Eliciting Activities (MEAs), can be used to have students apply mathematical content, while at the same time develop the creative skills and abilities needed to be successful in the 21st century. An example MEA will be discussed with work from pre-service elementary teachers to show how these 21st century skills can be developed with these activities.

It is important to begin mathematical modeling in the elementary grades and it has been shown to be a useful way for children to develop realistic problem solving skills (English & Watters, 2005; English, 2009). Mathematical modeling is an essential part of the Common Core State Standards Initiative (CCSSM, 2010) as one of the Standards for Mathematical Practice (SMP) to be developed throughout K–12th grade. Expertise in mathematical modeling is not an easy task; it takes thousands of hours spread over many years to master complex skills involved in creativity and innovation, which are a part of mathematical modeling (Ericsson, Krampe & Tesch-Romer, 1993). In fact, ten years has been suggested as the minimal amount of experience (Hayes, 1989). If we can begin fostering 21st century skills with elementary students, then we will enable students to be well prepared to meet the needs of future and present challenges. For pre-service elementary teachers to integrate mathematical modeling in their teaching, they need to have positive experiences themselves with this approach.

21st Century Learners Framework

Creativity and intelligence are both important topics that get at many core traits that relate to success in life (Sternberg, 1999). Aspects of both creativity and intelligence can be developed during mathematical modeling and are the key components of my 21st century learners' framework. This framework (Figure 1) builds upon the three intellectual abilities that Sternberg (1985) discussed as particularly important to creativity: (1) innovate, the ability to see problems in new ways that are outside the bounds of conventional thinking; (2) synthesize, the analytical ability to recognize which information is the most important and which ideas are worth pursuing; and (3) communicate, the practical ability to convince others of the value of one's ideas. There are different views on the relationship between intelligence and creativity (Gardner, 1993; Rubenson & Runco, 1992; Torrance, 1975). However, it has been found that people who are good at novel problem solving tend to be both intelligent and creative (Sternberg & O'Hara, 1999). To the three intellectual abilities, I have added a fourth competent, *adapt*, which is an essential life skill and feature of creative activity.

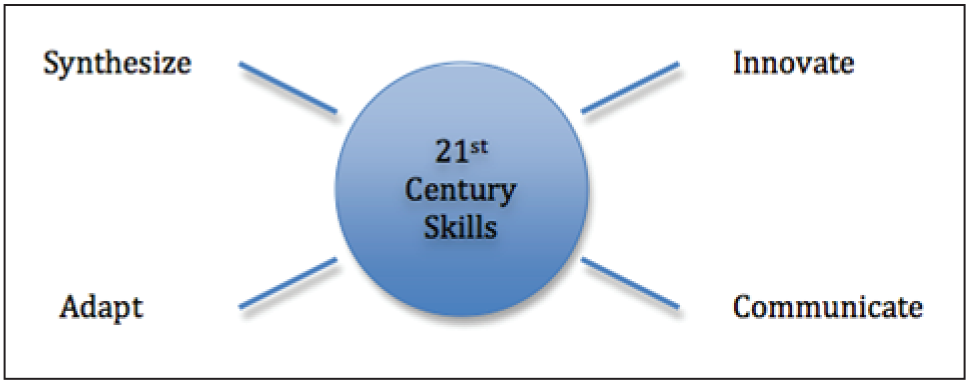

Figure 1. 21st Century Learners Framework

These four 21st century skills can be developed during mathematical modeling activities and are connected to the SMP. For this paper I will use one measure of innovation, which is idea generation (Sternberg & Lubart, 1999). This is an important part of realistic mathematical modeling as models developed are often based on multiple iterations of ideas (Lesh & Doerr, 2003). The first SMP, make sense of problems and persevere in solving them, echoes this idea (CCSSM, 2010). In mathematical modeling students often will have to work through several ideas in order to arrive at a solution that is adequate. In order to do this, new ideas need to be discussed and refined to allow this process to happen.

The ability to synthesize is needed in order to determine which ideas and information are important to pursue. Expert synthesizers not only see things differently but they *know* things differently, which enables them to focus on the most important information to solve a mathematical problem (Lesh, 2010). The second SMP, reason abstractly and quantitatively, is essential for realistic mathematical modeling (CCSSM, 2010). Expert synthesizers can work between contextualized real world mathematics and decontextualized symbolic work. An idea that is worth pursuing must work in the realistic context of the problem.

Throughout the mathematical modeling process clear communication is essential to develop and explain ideas. The third SMP, construct viable arguments and critique the reasoning of others, and sixth SMP, attend to precision, are tied to communication (CCSSM, 2010). Mathematical modeling often involves students working in groups where they need to create rationales for their decisions and also critique others ideas. Precision during mathematical modeling can involve different representations including written, verbal, symbolic, graphical, pictorial, concrete, and realistic.

Adaptation in this framework relates to the ability to use available resources in the most effective manner. These resources could include personal knowledge and abilities, the knowledge and abilities of others, and physical resources like technology and materials. The fifth SMP, use appropriate tools strategically, is integrated with adaptation (CCSSM, 2010). People that are successful in life and in the workplace are able to adapt to working with different people and making the most out of their available resources. During mathematical modeling activities students can select from the available physical resources in order to develop their solutions. These four aspects of the framework can be fostered and developed in students through the use of Model-Eliciting Activities.

Model-Eliciting Activities (MEAs)

Model-Eliciting Activities are well-structured, realistic mathematical modeling activities that enable participants to innovate, synthesize, communicate, and adapt effectively in order to develop a model, which is a solution. MEAs are carefully designed based on six principles that have been written about previously (i.e. Hjalmarson & Lesh, 2009; Lesh, Hoover, Hole, Kelly, & Post, 2000). In MEAs there are multiple solutions that can be developed that meet the needs of the problem presented. This open-ended aspect encourages innovation and idea generation. The information provided in MEAs, other students' ideas, and other groups' ideas, allows students to go through an iterative process of model development (Stohlmann, 2012). Through this process students synthesize information and ideas by deciding which are the most helpful to developing a useful solution. Throughout the MEA process students must communicate effectively with each other verbally, in writing, and through multiple representations (Stohlmann, Moore, & Cramer, in press). Materials and technology resources are made available to students so that they can select which ones will help them in their solution development.

Bigfoot Model-Eliciting Activity

The Bigfoot MEA was completed by a class of 30 pre-service elementary school teachers, enrolled in a mathematics content class, at the conclusion of a unit on functions and proportionality. The class was taught by an associate professor of mathematics education with over 20 years' experience in teacher education. The pre-service teachers had not previously completed an MEA.

The Bigfoot MEA (Stohlmann, 2012) is a modification of the Big foot MEA (Lesh & Doerr, 2003) in which participants figure out how big someone is by using a copy of their footprints. This structure of the problem was kept the same but the realistic context of the problem was changed for the Bigfoot MEA. Every MEA has the participants develop a model for a realistic client. In this MEA, the pre-service teachers work for the Northern Minnesota Bigfoot Society to determine the possible height of Bigfoot. I took field notes during the implementation of the MEA and also collected the pre-service elementary teachers' written work on the MEA, written reflections after the MEA, audio recordings of each group, and each group presentation. These data sources were analyzed using the 21st century framework as a coding system.

Connections to 21st Century Framework

Innovate

Idea generation is an essential aspect of innovation that occurred in this MEA. In general it is important for participants of MEAs to have individual time to think about possible solutions before beginning to work as a group. This can enable groups to generate more ideas (Wilson, 2006) and ideas that are of greater quality (Rietzschel, Nijstad, & Stroebe, 2006). There were 7 groups of pre-service teachers that completed this MEA. The 7 groups generated an average of 5.14 solution ideas per group. The number of ideas in each group varied for groups 1 to 7 with 3, 4, 7, 6, 4, 2 and 10 ideas generated respectively. Through the groups' model development process they were able to discuss and explore which ideas would best meet the needs of their client.

Synthesize

All of the groups were able to work towards a solution that met the needs of the client. Through discussion the groups were able to move away from unproductive ideas and towards an understanding of how to come up with an estimate for the height of Bigfoot. These initial ideas included: looking at the ratio of foot length-to-foot width; using weight; using Bigfoot's foot width; using a linear regression equation with height as the independent variable and foot length as the dependent variable; using a linear regression equation with height and foot lengths that did not have much variation; using the internet to find people with large feet; and using wingspan related to foot length. Each group's final solution is presented in Table 1. All of the solutions involved either proportionality or linear regression ideas. Groups 1, 2, and 5, who used a proportion or proportional equation, did not check what their data looked like on a scatterplot. Based on the trend of the data, a linear regression equation may have been more appropriate. However, groups 3 and 7 noted that a larger sample size would be needed to be more accurate with a linear regression equation; which would be true for a proportional equation as well. Groups 3 and 4 were able to get a much larger sample size by using the ratio found on the Internet. Based on what is known about Bigfoot, all groups came up with reasonable estimates.

Adapt

The pre-service teachers were able to make use of their resources to help develop their solution to their problem. Materials were available for the groups to use in their work including rulers, meter sticks, string, scissors, graph paper, graphing calculators, and laptops. The pre-service teachers also used the knowledge of their group members in developing their solutions. An important aspect of MEAs is for participants to reflect on the mathematical

Table 1. Groups' Final Models on the Bigfoot MEA

Group	Final Idea
1, 5	Calculated ratios of height-to-foot length for each member, averaged ratios, and multiplied this by Bigfoot's foot length.
2	Calculated the ratios of height-to-foot length for each member, estimated that the ratios were all close to 6, and multiplied Bigfoot's foot length by 6.
3	Used the average human foot length-to-height ratio of 15-to-100 found on the Internet to set up a proportion with Bigfoot's foot length and noted that a larger sample size is needed to be able to use a regression line.
4	Used the average human foot length-to-height ratio of 15-to-100 found on the Internet to set up a proportion with Bigfoot's foot length and took the average of each member's foot length and divided it by the average of each member's height to get an equation. This answer was close to the Internet ratio.
6	Used each member's foot length and height, entered this into the graphing calculator to find a linear regression equation, and plugged in Bigfoot's foot length into the equation.
7	Used all but one member's foot length and height and entered this into the graphing calculator to find a linear regression equation (one member's data was not used because the group felt the data looked closer to being linear without it), plugged in Bigfoot's foot length into the equation, and noted that ideally you would want to get height and foot lengths of a sample of a few hundred people.

knowledge they used during the activity in order to make these concepts explicit. At the conclusion of the MEA the pre-service elementary teachers were asked what mathematical concepts and skills they used in developing their solution. These writings were compared to the researcher field notes, audio recordings, and pre-service teachers' written work collected in this study to ensure these concepts and skills occurred. Table 2 describes the variety of concepts and skills that the pre-service teachers used in developing their solutions.

Table 2. Mathematical Concepts and Skills Used in the Bigfoot MEA

Mathematical concepts	Mathematical skills
• Ratios • Measurements • Conversions between units • Operations • Proportionalities • Linear functions • Lines of best fit • Scatterplots • Sample sizes • Averages • Correlations • Interpreted tables, graphs, and data • Looked for relationships between variables • Unit rates • Solved equations • Scale factors	• Modeling • Reasoning • Functions on the graphing calculator • Critical thinking • Using measurement devices such as rulers or meter sticks • Making real world connections • Assessing the reasonableness of solutions

Communicate

Clear communication is essential for participants while completing an MEA. Participants must demonstrate to their classmates the rationale behind their reasoning and critique the reasoning of others. When groups developed a mathematical solution they used their prior knowledge to determine if the height of Bigfoot that they came up with was reasonable and defensible, as well as if the mathematical concepts they used were valid. Three groups initially found that their mathematical model produced a height for Bigfoot that was less than 6 feet tall. They believed this to be too short and moved to a new idea for solving the problem. Another group, that used a line of best fit, tried to check the reasonableness of Bigfoot's height by using a proportion with Bigfoot's foot length and data for one person's height and foot length. They came up with an answer that was different by a foot from their line of best fit prediction. They decided to then use their solution that used more data, the line of best fit. One other group relied on their personal knowledge of Bigfoot's possible height, while two groups relied on the Internet to check if their height was reasonable.

Each group had the opportunity to share their ideas to the whole class to convince other groups of the validity of their solution strategy. After their presentations, groups were given the chance to revise their solutions. Two groups changed their ideas to align with group 3—using the average human ratio of height-to-foot length from the Internet—and group 7—adding that ideally a larger sample size would be needed. For a more detailed description of each group's communication throughout the MEA, with a focus on different representations see (Stohlmann et al., in press).

Pre-service Teachers' Views on MEAs

At the conclusion of the activity the pre-service teachers were asked of their impressions of MEAs. Table 3 provides the readers with a few examples of the typical comments the pre-service teachers shared. The pre-service teachers felt that they would like to use a similar activity in their teaching because it was a real world application of math, integrated different subjects, and was engaging, interactive, fun, and open ended with no one right answer. This is a common occurrence during MEAs as many involve participants in a flow experience, which involves positive feelings and engagement that can lead to longer retention of concepts (Lesh, Carmona, & Moore, 2009). Through analysis of the audio recordings and filed notes the pre-service teachers appeared to be engaged throughout the task. Their conversations involved the context of the problem and the best way to solve the problem based on prior knowledge and experiences.

Closing

It is vital for young people today to have 21st century skills in order to be able to be successful in the changing job market. Teacher education programs need to prepare elementary teachers to develop these skills in their students. Mathematical modeling through MEAs can foster this

Table 3. Pre-service Elementary Teachers' Views on MEAs

This activity made me realize that there are many different ways for students to solve problems and it is a good idea to create problems such as this one in which there can be more than one correct answer so that students are able to apply a multitude of skills to the problem that make sense to them.
If we can find more (ways) to engage students in math, and make it relevant, then students will be more invested in learning and being successful in math.
I like that it was a mathematics lesson, but it wasn't saying, 'today we will learn about how to factor' etc. We used multiple math concepts to solve—which also helps students who are better at some things to work and share with others. I would definitely use problems like this in the classroom!
A problem I and many others faced in elementary and middle school was wondering "why does this matter?" during math class. In fact, math does really matter, but students may not realize it if it is not applied.
These MEA activities are so great and interactive.

creative activity through innovation, synthesis, adaptation, and effective communication. These skills are essential as Feist (1999) has stated that, "Universities, businesses, the arts, entertainment, and politics—in other words, all of the major institutions of modern society—are each driven by their ability to create and solve problems originally and adaptively, that is, creatively" (p. 289).

These skills need to begin to be developed in elementary school. For this to happen pre-service elementary teachers need to have positive experiences with mathematical modeling. In general, discussing and critiquing each other's ideas should be a normal practice for elementary school students and not just in mathematical modeling activities. Students often enter elementary school with great capabilities for creativity and exploration. As students progress through school, their creative thinking is often stifled and becomes more a matter of producing work that they think the teacher wants. A focus on high stakes testing may have increased this problem as creative thinking has decreased in the past decade for K–12th grade students compared to the previous four decades (Kim, 2011). In fact, when students' creative needs are not met, they often become underachievers (Kim, 2010). The CCSSM (2010) emphasis on mathematical modeling is a needed focus for students and for pre-service and in-service teaching training. Students need more opportunities for creativity to be fostered through realistic, open-ended modeling problems so that they will be empowered and prepared with mathematics knowledge and skills to meet the challenges of the 21st century.

References

Common Core State Standards Initiative (2010). *Common Core Standards for Mathematics*. Retrieved from http://www.corestandards.org/assets/CCSI_Math%20 Standards.pdf

English, L. (2009). Promoting interdisciplinarity through mathematical modeling. *ZDM: The International Journal on Mathematics Education, 41,* 161–181.

English, L. & Watters, J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal, 16*(3), 58–79.

Ericsson, K. A., Krampe, R. T., & C. Tesch-Romer. (1993). The role of deliberate practice in the acquisition of expert performance. *Pyschological Review, 103*, 363–406.

Feist, G. J. (1999). The influence of personality on artistic and scientific creativity. In R. J. Sternberg (Ed.) *Handbook of Creativity.* (273–296). New York: University of Cambridge Press.

Gardner, H. (1993). *Creating Minds.* New York: Basic.

Hayes, J. (1989). Cognitive processes in creativity. In J. A. Glover, R. R. Ronning, & C. R. Reynolds (Eds.). *Handbook of Creativity* (135–145). New York: Plenum.

Hjalmarson, M. & Lesh, R. (2009). Design research: Engineering, systems, products, and processes for innovation. In L. English (Ed.) *International Handbook of Research in Mathematics Education (2nd Ed.)* Hillsdale, NJ: Lawrence Erlbaum Associates.

Kim, K. H. (2010). Measurement, causes, and effects of creativity. *Psychology of Aesthetics, Creativity, and the Arts, 4*, 131–135.

Kim, K. H. (2011). The creativity crisis: The decrease in creative thinking scores on the Torrance tests of creative thinking. *Creativity Research Journal (23)*4, 285–295.

Lesh, R. (2010) Tools, researchable issues and conjectures for investigating what it means to understand statistics (or other topics) meaningfully. *Journal of Mathematical Modeling and Application, 1*(2), 16–48.

Lesh, R., Carmona, G., & Moore, T. J. (2009) Six sigma learning gains and long term retention of understanding and attitudes related to models and modeling, *Mediterranean Journal for Research in Mathematics*

Education: An International Journal (Special Issue – A Tribute to the Work of Gerald Goldin), *9*(1), 19–54.

Lesh, R., & Doerr, H. (2003). Foundations of models and modeling perspective on mathematics teaching, learning, and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (3–33). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for developing thought-revealing activities. In A. Kelly & R. Lesh (Eds.) *The Handbook of Research Design in Mathematics and Science Education.* Hillsdale, NJ: Lawrence Erlbaum Associates.

U.S. Department of Labor (2013). *America's youth at 25: school enrollment, number of jobs held and labor market activity: results from a longitudinal survey*. USDL-13-0339. Bureau of Labor Statistics.

Rietzschel, E., Nijstad B., & Stroebe, W. (2006). Productivity is not enough: A comparison of interactive and nominal brainstorming groups on idea generation and selection. *Journal of Experimental Social Psychology, 42*, 244–251.

Rubenson, D. & Runco, M. (1992). The psychoeconomic approach to creativity. *New Ideas in Psychology, 10,* 131–147.

Sternberg, R. J. (Ed.). (1999). *Handbook of creativity.* Cambridge, England: University of Cambridge Press.

Sternberg, R. J. (1985). *Beyond IO: A triarchic theory of human intelligence.* Cambridge, England: Cambridge University Press.

Sternberg, R. J., & Lubart, T.I. (1996). Investing in creativity. *American Psychologist, 51*, 677–688

Stohlmann, M. (2012). YouTube incorporated with mathematical modeling activities: benefits, concerns, and future research opportunities. *International Journal of Technology in Mathematics Education, 19*(3), 117–124.

Stohlmann, M., Moore, T., & Cramer, K. (In press). Pre-service elementary teachers' mathematical content knowledge from an integrated STEM modeling activity. *Journal of Mathematical Modelling and Application.*

Torrance, E. (1975). Creativity research in education: Still alive. In I. A. Taylor & J. W. Getzels (Eds.) *Perspectives in Creativity* (pp.278–296). Chicago, IL: Aldine.

Wilson, C. (2006). Brainstorming pitfalls and best practices. *Interactions, 13*(5), 50–63.