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A Century of Leadership in Mathematics and Its Teaching

Mathematics Pre-K through 8

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My Entirely Plausible Fantasy: Early Mathematics Education in the Age of the Touchscreen Computer

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ABSTRACT This paper offers an account of what early mathematics education could look like in an age of young digital natives. Each “Tubby,” as the tablets are called, presents Nicole (our generic little child) with stimulating mathematics microworlds, from which, beginning at age 3, she can learn basic math concepts, as well as methods of calculation and number facts. Tubby presents Nicole with mathematics story books, which both she and her adult readers will enjoy. Tubby also offers guidance to parents and teachers on how to promote Nicole’s math learning. On entrance to school, Nicole, along with her peers, experience an organized curriculum that includes real world activities, mathematics microworlds, interactive storybooks, and a textbook, all residing in peace and harmony within Tubby. Tubby has enough energy to help Ben, Nicole’s teacher, to engage in multimedia pedagogy. Finally, Tubby conducts stealth assessment that provides Ben with actionable information on how to tailor his teaching to individual students and also eliminates the need for standardized achievement tests. The virtual world I have described is an attainable fantasy. But the real world stands in the way of the virtual. Without massive investments in professional development and teacher pay, and without a commitment to guarantee high quality education for *all* children, Tubby will dissolve into virtual nothingness.

KEYWORDS *early mathematics education, educational technology, touchscreen tablet, microworlds, curriculum, stealth assessment, cognitive developmental psychology, interactive storybooks*

As is well known, early mathematics education requires considerable reform and improvement (Ginsburg, Lee, & Boyd, 2008). Can computers help? Here is what I imagine could happen some time in the near future but probably will not, for reasons I will explain.

A Free Tubby

First, at age 3, every child receives a gift from the U.S. Department of Education: a solid, virtually unbreakable, touchscreen computer tablet named Tubby, which (see Figure 1) can be thrown with impunity from a highchair or into the bath, with no adverse results to either the device or to the child (from the parent). The rationale is



Figure 1. Tubby the Tablet.

that these computers can be used to enhance mathematics learning beginning with the preschool years, which we know are critical to later mathematics achievement, which in turn is essential for the modern economy (Heckman, 2000, 2004). Widespread deployment of the Tubby can therefore be thought of as an investment in the country's future. Another, not inconsiderable, benefit is that sound early mathematics education can produce a real interest in the subject and joy in learning it. We should not be concerned only with arming our little combatants to participate and prevail in the international "math wars" and economy.

The idea of 3-year-olds, and even younger children, using tablets is not a fantasy. Common observation shows that they easily learn basic touchscreen computer skills like swiping, shrinking, and tapping in order to use the tablets to play games, watch movies, and take photographs. Our authentic and very young digital natives see touchscreen devices as quotidian features of their worlds, as natural as bananas. The children seem to think of tablets as benevolent, if sometimes frustrating, friends that can provide amusement, even over long periods of time. Their parents are perhaps even more appreciative of the touchscreens than are the children, especially during preparation of dinner or long drives.

What Does Tubby Do?

Of course, swiping, shrinking, and tapping on a tablet are merely means to an end. Tubby makes them available, not for game play, but to promote early, meaningful, and coherent mathematics education that extends from age 3 through the early grades of elementary school. Tubby has several pedagogical techniques at its disposal.

Microworlds

Tubby offers a series of "microworlds," computer generated environments structured so as to afford mathematics learning (Papert, 1980) in a sequence that is both mathematically coherent and accessible to children at different points in their learning and development. For example, the MathemAntics software (Ginsburg, Jamalian, & Creighan, 2013) offers an environment, Hens Laying Eggs (see Figure 2), which is designed to promote ideas about numerical equivalence and inequality (more than and fewer than).

To begin, the child—whom I shall call Nicole—touches Tubby's welcome mat, whereupon the tablet recognizes her fingerprint and greets her fondly. With

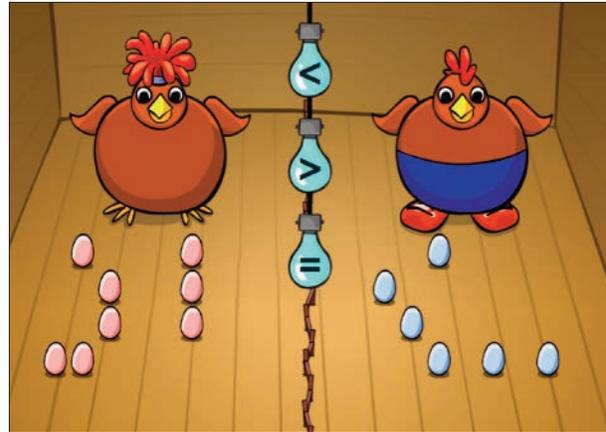


Figure 2. Hens laying eggs with $<$, $>$, and $=$ symbols.

further training, Tubby can recognize Nicole's face and voice, and can respond to her verbal commands. Nicole begins by using her finger to drag eggs into different configurations. For example, she can put the eggs in a straight line, or divide each side into two collections. Nicole can create new eggs by touching the hens' tummies, and can remove eggs by cracking them with a hammer tool that the software provides. Nicole makes many physical movements and gestures that are related to the mathematical ideas. For example, many tummy touches produce more eggs than do a small number of tummy touches. Pointing to objects one at a time can help her count them. Thus, gesture and other physical activities can help promote meaningful learning (Goldin-Meadow, Cook, & Mitchell, 2009).

After Nicole tires of exploration, Tubby gives her a problem to solve, for example to indicate which hen, Fluffy or Fancy Pants, has more or fewer eggs, or whether both have the same number by touching the symbols $<$, $>$, or $=$ in the light bulbs. The task can be made harder or easier, depending on Nicole's performance. Each hen can be made to have no more than 4 eggs, for example, or the eggs can be placed in physical arrangements that facilitate determination of same or different number. For example, Tubby can show the eggs in rows of 5 and 3 (under Fluffy on the left), and 5 and 1 (under Fancy Pants on the right). Another variant is that Fluffy produces eggs that are much smaller than Fancy Pants' eggs. Even so, 5 small Fluffy eggs are more numerous than 3 huge Fancy Pants eggs that almost fill the screen. The idea of equivalence is not limited by the objects' size.

The Hens Laying Eggs environment can offer Nicole many other affordances. For example:

- When Nicole touches the eggs, they do not break, but light up to say the number words in order;

- When Nicole touches numerals, they say their names or show their written words;
- Nicole can touch a number on the number line to indicate numerical values so that she can eventually report that Fluffy has 8 eggs and Fancy Pants 6;
- When Nicole touches the appropriate icon, a pairing tool (a 1-1 correspondence device) matches each of Fluffy's eggs with each of Fancy Pants' eggs.

Using these tools, Nicole learns basic ideas of equivalence and also is helped to understand mathematical symbolism, including $<$, $>$, $=$, and the written numerals. She also learns to say her number words in order; to read numerals and recognize written number words; to use the number line to indicate numerical answers; and perhaps incidentally learns about relative quantity on the number line. She may also learn some degree of impulse control as she assiduously engages with Tubby.

Tubby can make available to children at various stages of learning many other environments, for example a barn (see Figure 3) containing some animals. After the barn door is closed, several new animals enter or several of the original animals leave. Nicole's task is to figure out the result of the transformation in either case. This environment can teach young children basic ideas of addition and subtraction as transformations.

Environments like these can be used to present fundamental ideas of number to children beginning at age 3 or even younger. Presented in a careful sequence, the tasks within these microworlds are a virtual curriculum in early mathematics education. The curriculum can begin with simple ideas of more or less, proceed to single digit addition and subtraction involving symbols, and after that to "column" addition and subtraction, which involves 2 and 3 digit numbers and ideas of place value (which in turn are based on the idea of base ten).

How does Tubby organize the sequence? One approach is mathematical: Tubby believes that some mathematical ideas and skills must precede others. For example, a system of number words must exist before they can be applied to the enumeration of a set of objects, and arithmetic must precede algebra. Addition must be learned before multiplication. Multiplication must be learned before long division.

But Tubby also takes a psychological approach. He believes that cognitive development both imposes constraints on mathematics learning and offers opportunities for mathematics teaching. For example, research shows that young children experience difficulty with interpreting written symbolism or understanding rational number (Ginsburg, Klein, & Starkey, 1998).

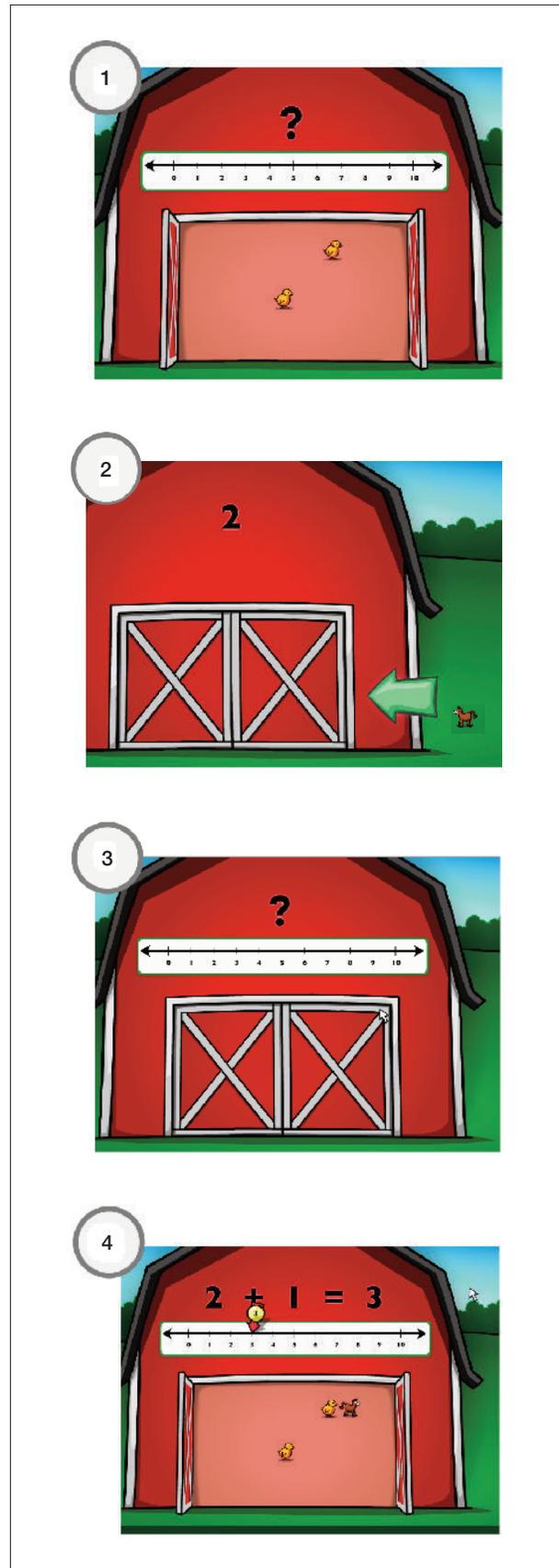


Figure 3. Barn that teaches addition.

Hence Tubby introduces informal mathematical ideas before their symbolic representations and whole number before rational number. At the same time, because research shows that children develop their own “everyday mathematics” involving addition and subtraction, Tubby uses these informal ideas to introduce formal mathematics (as in the case of the barn problem shown in Figure 3).

Although believing that sequence is important for mathematics learning, Tubby is wisely skeptical about developmental progressions. His reasoning is simple. For example, according to psychological research, children have difficulty with understanding the = sign as referring to the idea of equivalence. Instead, they believe that it indicates an operation: “get the answer” or “makes” (Seo & Ginsburg, 2003). Thus children say that the = sign in $4 + 2 = 6$ means that $4 + 2$ makes or results in that sum. Tubby is familiar with this research, but at the same time he conjectures that if children were taught differently they might indeed learn that = can indicate equivalence. In other words, psychological research shows what children know under current conditions of development and teaching, but usually—in my reading of the literature—does not investigate what children *could* know under different circumstances. Another way to put it is that innovative microworlds and teaching might tap Nicole’s learning *potential* that has gone unrecognized. So Tubby encourages educators not to feel bound by current psychological norms. Be adventurous, Tubby says, because you may be surprised by what children are capable of learning. Nicole may be able to learn something if only you knew how to teach it (and may not know something only because you do not know how to teach it).

Tubby’s Secret Weapon

Tubby knows that children enjoy mathematics but teachers and parents often do not. In fact, they may be frightened of mathematics and pass on their fear to their children and students (Beilock, Gunderson, Ramirez, & Levine, 2010). Given this unfortunate situation, Tubby deploys a stealth weapon, namely the storybook. Tubby knows that children like to read storybooks and that parents and teachers like storybooks, too. Tubby also knows that many storybooks, although not intending to do so, entail significant mathematical ideas. *Goldilocks* and *Ten Little Monkeys Jumping on the Bed* both include informal mathematical ideas like more, less, how many, adding, subtracting, dividing up to be fair, near, far, up, and down. Tubby believes that storybooks of this type can benefit from computer affordances—they can

become interactive and happily reside in Tubby, along with the microworlds.

Imagine Nicole with her Tubby, reading *Ten Little Monkeys Jumping on the Bed*. As noted, stories can entail mathematical ideas; in this case, a subtraction narrative in which one monkey at a time falls off the bed to leave $n - 1$ as the result. Tubby goes beyond the existing story to present Nicole with interesting *Little Monkey* problems. For example, Tubby stops the narrative to ask her to figure out how many monkeys would be left on the bed if two jumped off. To solve the problem, Nicole could move the virtual monkeys off the bed to leave $n - 2$ monkeys on the bed. Tubby then encourages Nicole to return the two monkeys to the bed so that she can begin to learn the inverse relation between subtraction and addition (not in so many words). Tubby can also relate the *Little Monkey* problems to the *Barn* environment shown in Figure 3; indeed, the Bed and Barn problems are essentially the same. The storybook also has a small dictionary of definitions suitable for young children. If the child drags a mathematics word to the dictionary, the latter can offer an informative and amusing explanation of the concept in question. In these ways, Nicole can learn a good deal of mathematics through the interactive storybook curriculum, in which are imbedded mathematical microworlds and tools. She thanks Tubby profusely.

Unbeknownst to Nicole, controls in the electronic innards of the storybook allow the parent or teacher to enable or disable various reading tools. For example, the book can speak to the child as the appropriate word lights. Or the parent or teacher can read to the child. Or the child can read to the parent or teacher, and if the child does not know a word, he or she can touch it to learn what it says. These features can, I conjecture, help Nicole to read.

Will Nicole Become a Captive of Her Virtual World?

Tubby is aware of the possibility that Nicole may be overly involved in her virtual world and may even see it as real. Tubby disapproves. Knowing that a child should not sit in front of electronic devices all day, Tubby shuts down at appropriate intervals and suggests—nay, orders!—Nicole to find something to play with. Tubby knows that although its virtual objects can do things that real objects cannot do (for example, squeezing into a small barn 1,047 large elephants), real objects can do things that virtual objects cannot do (for

example, form a block structure into which Nicole can enter and take a nap). Also, the real world allows Nicole, if she wants, to do nothing except exercise her imagination to create fantasies.

Another benefit of a real world is language. Tubby recognizes that his verbal skills are limited. He can give instructions and provide verbal feedback, but he lacks the verbal fluency and flexibility of even a 3-year-old. He understands that live persons are necessary to promote Nicole's language. Further, Tubby understands that although Nicole is fond of him, she may benefit from what humans call a "loving relationship" with parents and other humans. Tubby heartily endorses the real world.

Does Nicole Need Parents?

Tubby also knows that sometimes parents add value beyond their "loving relationship" with Nicole. Parents can help Nicole focus on Tubby and his activities, guide her interaction with them, set the controls that determine the reading process, ask questions to determine what Nicole has learned and to stimulate new learning, point Nicole in new, non-Tubby directions, and soothe her hurt feelings when Tubby points out the occasional mistake. Tubby not only understands the value of parents, but embraces it and provides parents with needed assistance. By touching the Emergency Mathematics Services (EMS) icon, parents can learn about reading to children, explore the thinking that underlies children's performance, undertake home mathematics activities, and finally even learn something that is classified (don't tell anyone).

First, EMS offers a video depicting "dialogic reading," which helps the parent learn how to read to and with young children (Whitehurst, Arnold, Epstein, Angell, Smith, & Fischel, 1994). The basic idea is that parents should ask questions that help the child engage with the written material, for example, "Why did Fluffy break those eggs? Why does Fancy Pants have more eggs than Fluffy?" These questions not only raise important issues for Nicole to consider but also encourage her to speak, to think about her own thinking, and to put her "everyday mathematics" into words. Dialogic reading thus promotes metacognition (awareness of thinking), communication, and justification of thinking. Dialogic reading helps Nicole to learn fundamental aspects of mathematics—much more than memorized number facts!

Second, EMS provides parents with access to videotaped interviews that illustrate children's mathematical thinking. The videos show that children

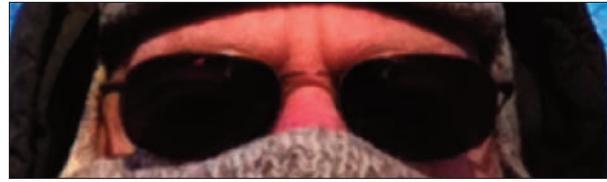


Figure 4. An EMS secret agent.

often think differently from adults and have an "everyday mathematics" of which adults are often unaware. The videos describe challenges that children face in dealing with certain kinds of problems. For example, children may not initially understand that counting the monkeys requires numbering them one at a time while not counting any twice, and also that the last number reached does not refer only to the last object but instead indicates the numerical value of the set as a whole. The videos are dramatic, amusing, and informative and are linked to key mathematical issues that arise in the course of the storybook.

Third, EMS provides suggestions, some in text and some in videos, about useful activities designed to promote Nicole's mathematics learning at home. These include storybook reading, board games, cooking, photography, and many others.

Fourth, at some peril to my person I can report that EMS engages in stealth assessment. Aided by a grant from the National Security Agency, an EMS secret agent (see Figure 4) in every Tubby records and reports on all of Nicole's interactions with Tubby, from the micro-worlds to the storybooks. The EMS agent not only records accuracy over time in response to all of the tasks, but also Nicole's choice of strategies (like use of a line-up tool that neatly lines up objects in straight horizontal lines of 5 or 10 with leftovers in another line) and the number of times she reads each storybook or works in a microworld. The agent can easily obtain this information from a simple scan of Tubby's log files (the information stored in Tubby's memory). The EMS agent is now experimenting with new software that will recognize and understand the speech of all the little Nicoles playing with their Tubbies, as well as their parents' speech. This kind of voice recognition and interpretation allows EMS to record and decipher children's questions to parents and responses to parents' questions. Although still in a crude stage of development, the EMS agent's stealth assessment is more informative than we are supposed to know. Once perfected, stealth assessment of this type can provide specific suggestions for home mathematics activities especially appropriate for the child.

What Happens When Nicole Goes to Preschool?

Nicole goes to preschool,¹ which, in my fantasy, is universally available for 3-year-old and older children in the United States and offered by the local public school system. Each of the preschool teachers has graduated with a degree in early education from an institution of higher learning, at least at the undergraduate level. Their college education in teaching was coordinated with the local school system. Practicing teachers who themselves were graduates of the institution of higher learning, along with professors of education, mentor and coach the college students—the prospective teachers—learning to teach in the local schools. Once matriculated and placed in the well-equipped and attractive local schools, where they are extremely well paid, the new teachers receive frequent professional development opportunities in which the master teachers and the college professors collaborate to offer coaching, lesson/Tubby study, and other forms of training.

In the classroom, Nicole, as well as the other little Nicoles and Nicks and the preschool teacher (whom I refer to as “Ben”), all have Tubbies, each of which is linked to the others through the school’s wireless system. This allows the teacher to monitor the children’s Tubbies, to send messages to individual children, to assign activities in which children have to work cooperatively with their Tubbies, and so on. Inter-Tubby-communication is flawless because bandwidth is extremely broad and wireless transmission is always amazingly speedy. In fact, Internet access is free for all U.S. residents, as specified in a recent amendment (28.0) to the U.S. Constitution.

Nicole Learns From an Explicit Curriculum

Tubby provides Ben with an extended curriculum entailing a carefully developed sequence of mathematics learning activities for his little Nicoles and Nicks. One component of the curriculum describes various classroom activities involving physical manipulatives, games, whole group, small group, and individual activities, as in the case of several curricula designed for 4- and 5-year-olds, for example, *Big Math for Little Kids* (Ginsburg, Greenes, & Balfanz, 2003) and *Building Blocks* (Sarama & Clements, 2004). A second curriculum component is a sequence of microworld activities in which Ben’s students engage, under his guidance, on

their own Tubbies. The third component is the story-books, which are synchronized with the live classroom activities and with the sequence of microworld activities. For example, EMS notifies Ben that a useful way to introduce addition is to begin with the addition story-book that he displays on the interactive whiteboard. Following this, he is advised to do an activity in which children learn to add by placing toy bears into containers, after which he directs his little Nicoles and Nicks to the Tubby barn activities.

I save for last the curriculum’s fourth feature because it may alarm progressive educators, especially constructivists, whom I adore. Hoping they remain patient and calm, I must report that Tubby offers Nicole a textbook.

A Post-Textbook Textbook

“A textbook?” you say. “Outrageous! Obviously textbooks are developmentally inappropriate for toddlers and young children, probably through the first several grades. Young children do not learn well from boring textbooks.”

It is true that children may not learn well from current textbooks and that it would be absurd to give toddlers the latest edition of even the very rare very good textbook. But consider the post-textbook textbook. You will love it.

One reason is that the post-textbook textbook is Tubby(!) in one of his many incarnations. The post-textbook textbook Tubby undergoes developmentally appropriate growth in harmony with Nicole’s growth. When Nicole is 3, so is Tubby; when Nicole is 4, so is Tubby. The readers of this journal will no doubt figure out the rest.

When post-textbook textbook Tubby—let’s call him Tex—is 3, he offers Nicole very few printed words or mathematical symbols. Tex occasionally asks Nicole to touch the numerals from 1 to 5, whereupon each lights up, dances around, and says its name as the appropriate number of objects appears. Occasionally Tex asks Nicole to touch some shapes, or to draw some with her finger, but that is about the extent of Tex’s efforts. After all, he is only 3.

But at 4, Tex undergoes a remarkable growth spurt. He now looks a little like a page, the top right hand corner of which shows a numeral that when touched says its name (“I am page 3!”). Some pages issue invitations to activities from various microworlds: “Let’s

¹ This is a short hand term for a setting that offers some form of organized education for young children. Some “daycare” centers are included if they offer at least some education.

play the barn game,” or “Can you think of other ways to line these up?” Some pages introduce the mathematics storybooks. Other pages help Nicole to write the numerals by tracing them on dotted outlines. Some pages become “Nicole’s very own math book” in which she can write or draw or photograph anything that she thinks belongs in it. Other children can send her “math messages” and pictures for her book (Tex is an e-mail device, too), as can the teacher or parents.

When Tex is 5, he makes greater use of written words, and therefore looks a little more like a standard textbook. But of course, Tex can say his words and, in fact, helps Nicole to read them. Tex links to Ben’s interactive whiteboard and to various classroom activities. Tubby allows Nicole to spend more time with Tex than when she was 4. Tex includes some practice problems and also some “homework” that Nicole can do with a parent (whom Tubby has prepared for this event).

You get the picture: as Nicole grows older, Tex can offer more microworld activities, mathematical tools, written work, practice problems, homework, and communication with teachers and parents. Moreover, I want to assure you that Tex does not intend to replicate boring and incoherent textbooks at any level. Tex’s goal is to become a rich, stimulating textbook that constructivists adore and that helps children learn important mathematical ideas and methods and share them with others.

Ben’s Tubby Provides Important EMS

At first, Ben needs a lot of guidance in implementing all components of the curriculum, from the microworlds to the storybooks to Tex. But, never fear, EMS is near. As in the case of parents, the EMS provides Ben with guidelines for reading stories to individual children and the group, particularly at the interactive whiteboard. He learns how stories, presented in a dialogic reading context (Arnold, Lonigan, Whitehurst, & Epstein, 1994), can introduce a new mathematics topic and strengthen the understanding of all the little Nicoles and Nicks already engaged in working on a topic.

EMS helps Ben to practice a multi-media pedagogy that involves fruitful integration of microworld computer activities; storybooks; live large, small, and individual classroom activities; little Nicoles and Nicks collaborating on Tubby and on live math investigations; and Tex. EMS provides Ben with video examples of teachers using multi-media pedagogy. He learns how to analyze these different aspects of teaching and learning.

He makes and analyzes videos of his own teaching. Indeed, Ben attends a bi-weekly “child study/lesson study” group (Mast & Ginsburg, 2009) in which he and his colleagues examine their own teaching, Tubby and Tex’s efforts, and children’s learning.

EMS helps Ben to understand the thinking that underlies children’s performance. He learns that children’s strategies and concepts develop with time and instruction. He learns that Nick may get a correct answer even though he understands nothing, and that because of a minor error of computation, Nicole may get a wrong answer even if she understands the concept. But the most exciting lesson Ben learns is how to engage in sound clinical interviewing (Ginsburg, 1997), the most powerful and practical method of formative assessment—that is, assessment designed to help teachers adjust instruction for individual children (Heritage, 2010). Ben sits engrossed in his Tubby-based clinical interview exercises for as long as his little Nicoles and Nicks are engaged in their Tubby math activities. He discovers that mathematical minds are as interesting as the mathematics they are attempting to learn.

Finally, the EMS secret assessment agent gives Ben reports on all of his students. He learns that Nicole is struggling because she is grappling with a new, but difficult strategy, and that Nick is struggling because he is not paying attention (he simply clicks on answers randomly and quickly, apparently without thought), which in turn results in his failure to learn a useful strategy and to check his work. Given reports of this type, Ben can help each child—“differentiating” their instruction. He is grateful, but even more delighted at the secret assessment agent’s next contribution.

Why You Should Love Stealth Assessment

As described earlier, the EMS secret assessment agent keeps records of everything Nicole and Nick do as they work with Tubby—on the microworlds, the textbook, and the interactions with other students. The agent records their language (and soon will be able to transform it into written English), their facial expressions, their hesitations, their right and wrong answers to textbook exercises, their speed of solution on key problems, their requests for help, their use of software tools (like the line-up tool), and the like. To say the least, these records are comprehensive. Indeed they contain more information than necessary—more than the agent can completely exploit. At the same time, the records have at least two useful purposes. The first is that, as

mentioned above, they can provide Ben with useful formative assessment so that he can adjust teaching to the needs of the individual.

The second contribution is to provide “summative assessment.” The extensive records can be used to provide evaluations, at different times during the year, of Nicole’s and Nick’s performance. The records may show that at the beginning of the year, Nicole used strategies a and b to solve subtraction problems whereas by the end of the year she had transitioned to strategies c and d. The records may show that Nick got 30% of the multiplication problems right at the beginning of year, but 70% at the end. The records may show that Nicole procrastinated on multiplication problems but was enthused about geometry. Moreover, the secret assessment agent can almost instantly compare Nicole’s and Nick’s records with those of all children—children of different ages, from different classrooms and schools, from different social groups and countries.

If the secret agent is well educated (by his programmers) in mathematics education and cognitive development, specifically regarding children’s thinking and learning, then he will rapidly sift through the huge amounts of data collected on Tubbies worldwide to produce useful reports.

I can understand that this scenario is a bit spooky. But consider the enormous benefit: the virtually complete elimination of standardized achievement testing. The logic is simple. If Nicole and Nick engage in rich mathematics learning activities as guided by Tubby, and if Tubby keeps good records of what Nicole and Nick have done over the course of the year, and if Tubby is sufficiently well educated by cognitive developmental psychologists and mathematics educators to analyze the data and derive from them useful accounts of Nicole’s and Nick’s performance, strategies, concepts, and motivations, and if Tubby can compare Nicole’s and Nick’s performance, strategies, concepts, and motivations with those of other children in other classrooms, schools, districts, states, and countries, what could you possibly learn from a standardized achievement test that you do not already know from your Tubby assessment? Good-bye and good riddance, standardized achievement tests!

Conclusion

Everything I have described is possible from a technical point of view. Current computer technology can produce Tubby, Tex, and all the other wonderful possibilities. The virtual world I have described is an attainable fantasy, which will become even more

fantastic if we train talented researchers and developers qualified to develop the next generation of Tubbies and Texes. But the real world stands in the way of the virtual. Tubby and Tex cannot work their magic until we attract talented people to the teaching profession and pay them a decent wage; educate them properly in our institutions of higher learning; provide them with effective, frequent, and meaningful professional development; provide children and schools with tablets and fast wireless service; and finally, guarantee high quality education for *all* children, not only the affluent. There is much more I can list. But let’s start with these for now. Unless we do so, Tubby will dissolve into virtual nothingness.

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