

SPRING/SUMMER 2014 | VOLUME 5, ISSUE 1

JOURNAL OF  
**MATHEMATICS  
EDUCATION**  
AT TEACHERS COLLEGE

*A Century of Leadership in Mathematics and Its Teaching*

**Mathematics Pre-K through 8**

*Published Semiannually by the Program in Mathematics and Education*

**TEACHERS COLLEGE** | COLUMBIA UNIVERSITY

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## Incorporating the Common Core's Problem Solving Standard for Mathematical Practice into an Early Elementary Inclusive Classroom

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**ABSTRACT** Mathematics curriculum designers and policy decision makers are beginning to recognize the importance of problem solving, even at the earliest stages of mathematics learning. The Common Core includes sense making and perseverance in solving problems in its standards for mathematical practice for students at all grade levels. Incorporating problem solving into inclusive classrooms, in which some students may struggle with the self-regulation skills involved in problem solving, can be challenging. This article offers several problem solving procedures, as well as examples of problem solving in the classroom, that can provide young children with learning disabilities the support they need to meaningfully engage in mathematical problem solving.

**KEYWORDS** *elementary mathematics, special education, inclusive classrooms, problem solving, Common Core*

A common assumption about kindergarten mathematics is that the school curriculum at this level primarily consists of counting and learning shapes, and that mathematics in early elementary grades is only slightly more complex with the introduction of addition and subtraction. People, including many teachers and even administrators, often also assume that the only skills necessary for teachers to be able to teach early elementary level mathematics are the skills needed to *do* this level of mathematics. These types of beliefs about the seeming simplicity of early elementary mathematics are beginning to be dispelled, however, as curriculum designers and policy decision makers are coming to recognize the role of problem solving at the earliest levels in mathematics.

### Problem Solving Defined and its Role in the Common Core

Problem solving is defined by the National Council of Teachers of Mathematics (NCTM) as “engaging in a task for which the solution method is not known in advance” (2000, p. 52), and Kilpatrick defined a problem as “a situation in which a goal is to be attained and a direct route to the goal is blocked” (Kilpatrick, 1985, p. 2). Through the lens of these general definitions, it would seem that almost everything young children do in mathematics could be considered problem solving, as even the simplest mathematics situations and tasks may be novel to these young students at the start of their mathematics journey. But when one considers Pólya’s

(1945) four stages of problem solving—understanding the problem, devising a plan, carrying out the plan, and looking back—and his extensive list of heuristic strategies and questions that students are meant to ask themselves at each of the four stages, it is easy to see why many teachers may be intimidated and consider the idea of problem solving too complex for very young learners.

Nevertheless, the *Common Core State Standards for Mathematics* and NCTM have included Pólya's four stages of problem solving and many supporting heuristic strategies in practice or process standards. In the Common Core's first standard of mathematical practice, "Make sense of problems and persevere in solving them," students are expected to: explain "to themselves the meaning of a problem;" "analyze givens, constraints, relationships, and goals;" "plan a solution pathway;" "consider analogous problems, and try special cases and simpler forms of the original problem;" "monitor and evaluate their progress and change course if necessary;" "check their answers to problems using a different method;" and "continually ask themselves, 'Does this make sense?'" (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA Center & CCSSO], 2010, p. 6). It is now becoming recognized that developing the ability to solve problems in this way is important for learners at *all* levels.

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### **Problem Solving for Students with Learning Disabilities: Challenges and Possibilities**

Many children in kindergarten have a developing number sense, and with number sense, children are able to create their own mathematical procedures, identify number patterns, understand and compare quantities, and solve mathematics problems (Montague, 2005). "From early on, most students acquire the skills and strategies needed to 'read the problem' and 'decide what to do' to solve it" (Montague, 2005, p. 2). For many children with cognitive impairments such as learning disabilities, however, learning to solve mathematics problems, especially those for which the solution method is not readily apparent, can be challenging.

Children with learning disabilities often lack the conceptual bases necessary for problem solving and have difficulty developing the same skills and strategies used by their typically developing peers in solving mathematical problems (Montague, 2005). Self-regulation, which is "the ability to regulate one's cognitive activities" (Montague, 2008, p. 1) and self-regulation strategies, such as "self-instruction, self-questioning, self-monitoring,

self-evaluation, and self-reinforcement" (Montague, 2008, p. 37), are key components of mathematical problem solving as described by Pólya (1945), Schoenfeld (1992), NCTM (2000), the Common Core Standards (NGA Center & CCSSO, 2010), and others. Students with learning disabilities are very often deficient in the ability to self-regulate (Montague, 2008) and subsequently have a difficult time engaging in the cognitive processes of problem solving.

Students with LD characteristically are poor strategic learners and problem solvers and manifest strategy deficits and differences that impede performance, particularly on tasks requiring higher level processing. These students need explicit instruction in selecting strategies appropriate to the task, applying the strategies in the context of the task, and monitoring their execution. They have difficulty abandoning and replacing ineffective strategies, adapting strategies to other similar tasks, and generalizing strategies to other situations and settings. (Montague, 2008, p. 38 – 39)

Explicit instruction in problem solving that involves structured lessons, modeling of cognitive processes, prompting and cues, guided practice, performance feedback, and many opportunities for reinforcement in order to achieve mastery is an effective method for teaching students with learning disabilities to "think and behave like good problem solvers and strategic learners" (Montague, 2008, p. 39).

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### **Problem Solving Procedures for Early Elementary Students**

Several different procedures have been developed by various researchers for the teaching and learning of mathematical problem solving in early elementary classrooms. Hohn and Frey (2002) created a problem solving method called SOLVED, a mnemonic short for **S**tate the problem, **O**ptions to use, **L**inks to the past, **V**isual aid, **E**xecute your answer, **D**o check back. "Each letter cues a concept or procedure that, if followed, would help the learner consider the necessary phases of problem solving" (Hohn and Frey, 2002, p. 374). In *Solve It! A Practical Approach to Teaching Mathematical Problem Solving Skills* (Montague, 2003), students are taught a seven step process for problem solving: read to understand the problem, paraphrase, visualize by making a picture or diagram, hypothesize a solution plan, estimate the answer, compute, and check. Along with the seven step process, students are taught a SAY, ASK, CHECK routine

for each step, where SAY refers to self-instructing, ASK refers to self-questioning, and CHECK refers to self-monitoring. Chung and Tam (2005) used a modified version of a previously developed cognitive routine for problem solving for students with mild intellectual disabilities. The five steps of this routine are: read the problem aloud, choose the important information, draw a representation of the problem, write down the steps to solve, and check the answer. A four-step procedure for solving change, group, and compare problems was designed by Jitendra et al. (2007). The steps of the procedure, known as schema-based strategy instruction, are identify the problem type (e.g. change, group, or compare), use a diagram to organize the information, make a plan to solve the problem, and solve. Students are provided with a checklist tailored to each problem type with questions that guide students through each of the four steps of the problem solving procedure.

These various problem solving methods have key commonalities—they all closely follow Pólya’s (1945) four stages of problem solving, understanding the problem, devising a plan, carrying out the plan, and looking back, and they also include various heuristic strategies and questioning techniques developed by Pólya to carry out the four problem solving stages. These procedures each create, in essence, a “problem solving algorithm”—a step-by-step routine with problem solving heuristics incorporated throughout. When used as part of a flexible, universally designed curriculum that provides multiple means of engagement (Meyer, Rose, & Gordon, 2014), these procedures give children with learning disabilities, as well as any other children needing additional support, the structure they need to engage in the cognitive processes necessary for successful problem solving.

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### Examples of Problem Solving in the Classroom

The following are examples of typical word problems posed to students in an early elementary inclusive classroom and the accompanying dialogue that models the cognitive process for solving the problem. The problems, methods for solution, and modeling of thought processes are based on the schema-based instruction developed by Jitendra et al. (2007), though any of the problem solving procedures described previously could be incorporated into an early elementary classroom, depending on the skills and needs of the group. The examples utilize a modified version of the dialogue provided in the structured lesson example in Montague’s *Math Problem Solving For Primary*

*Students with Disabilities* (2005). Modifications were made to make the problem and solution routine appropriate for kindergarten or first grade students. Children in this age group are learning to read, but most have not yet developed the skills to read mathematical word problems, so the problem is read aloud to the class or small group. Most are able to write numbers and spell non-conventionally (some may be able to write a close approximation for lollipop; others may only write a few letters for the sounds they hear in a word, such as *lpo* for lollipop). In the following examples, it is assumed that the children have been taught the schema-based instruction routine and are being given regular, teacher-supported opportunities to practice using the routine and accompanying diagrams for problem solving. A class checklist of the four-step routine that includes pictorial cues should be on display in the classroom to serve as a visual support for this process. A teacher’s imagined dialogue, including narration of all internal thought processes, written work, and demonstrations to the class, is provided for each example.

#### Example 1: Change Problem with Manipulatives

*Listen and watch what I think and do to solve this problem.*

**Manny had 6 lollipops. Aya gave him 5 more lollipops. How many lollipops does Manny have now?**

The first thing I need to do is Step 1, retell the problem, or say it in my own words. Manny had 6 lollipops, and Aya gave him 5 more, and I need to figure out how many lollipops Manny has now. I also need to think about what kind of problem this is. I know the problem has a “beginning”—Manny had 6 lollipops. And I know there is a “change” because Aya gave him 5 **more** lollipops. I have to figure out the end. There is a beginning, a change, and end, so I know this is a change problem.

Next I need to do Step 2, organize my information into my change diagram. I know the problem is about lollipops, so I’m going to write ‘lollipops’ in the beginning, change, and ending spots on the diagram. ‘Manny had 6 lollipops and Aya gave him 5 more.’ Ok, Manny started with 6, so I’m going to write 6 in the beginning circle. Aya gave him 5 more, so that’s the change, and I remember that “more” means adding, so I am going to write +5 in the change box. I’m not sure how many he has now, so I’m going to write a question mark in my ending circle. Do I know what my question sentence is? Yes, ‘how many lollipops does Manny have now?’

Next is Step 3, plan to solve the problem. Do I need to add or subtract? Manny got more lollipops, so I know I

need to add. Did I write the number sentence? Oh, I almost forgot to write my number sentence!  $6+5=$ \_\_. Are there tools I can use to help me? Yes, I think I'll use cubes to figure this out.

Last is Step 4, solve the problem. Manny had six lollipops, so I need to start with 6 cubes. Then Aya gave him 5 more lollipops. Ok, so now I will put out 5 more cubes to show that he got 5 more lollipops. Now I need to count all of my cubes. 1, 2, 3, 4, 5, 6...7, 8, 9, 10, 11. Manny has 11 lollipops now. I'll write down my answer in the ending circle—11 lollipops. I'll finish writing my number sentence— $6+5=11$ . Now I'll check my work; I'll count the cubes one more time. 1, 2, 3, 4, 5, 6. That's how many lollipops Manny started with. 1, 2, 3, 4, 5. That's how many lollipops Aya gave him. 1, 2, 3, 4, 5, 6...7, 8, 9, 10, 11. Yes, my answer is right,  $6+5 = 11$ . Manny has 11 lollipops now.

### Example 2: Compare Problem with Schematic Representation Drawing

*Listen and watch what I think and do to solve this problem.*

**Jeremiah has 11 stickers. Amaris has 4 fewer stickers than Jeremiah. How many stickers does Amaris have?**

The first thing I need to do is Step 1, retell the problem. Jeremiah has 11 stickers. I'm not sure how many stickers Amaris has, but she has 4 fewer stickers than Jeremiah. And I have to figure out how many stickers Amaris has. Now I need to think about what kind of problem this is. Hmm...the problem tells us how many stickers Jeremiah has, and we know Amaris has 4 fewer than Jeremiah. That sounds like comparing Jeremiah's stickers and Amaris' stickers, so this must be a "compare" problem.

Next is Step 2; I have to organize my information into my compare diagram. Since the problem is about stickers, I'm going to write "stickers" in the bigger, smaller, and difference spots on my diagram. Ok, now what goes in the bigger spot? Well, the problem says Amaris has fewer stickers. That means Jeremiah has more stickers, so Jeremiah must go in the bigger spot. I'll write Jeremiah. And now I'll write 11 stickers. Amaris has fewer stickers, so I'll write her name in the smaller spot. But I'm not sure how many stickers she has, so I'll write a question mark there. And the difference must be 4 stickers since Amaris has 4 fewer stickers than Jeremiah.

Step 3 is plan to solve the problem. Do I need to add or subtract to solve the problem? I'm not sure because the missing number is in the middle of the problem and not at the end! Oh wait, I almost forgot! The four fewer stickers is the **difference** between the number of stickers Jeremiah

has and the number of stickers Amaris has. To find the difference, I have to subtract. Let me write my number sentence.  $11-$ \_\_=4. Are there tools I can use to help me? I'll draw a picture this time.

Now I have to do Step 4, solve the problem. I'll draw a row of 11 circles. Those will be Jeremiah's stickers. Now I'll draw circles for Amaris' stickers, one-by-one, right underneath Jeremiah's. I'm not sure how many to draw for Amaris yet, but I know Amaris has 4 fewer stickers than Jeremiah, so Amaris' row should be 4 stickers shorter than Jeremiah's row. To make sure they line up, I'll draw a line between each one of Jeremiah's stickers and a sticker of Amaris'. That way it will be easy to see once Amaris' row is 4 stickers shorter than Jeremiah's. Ok, I see if Amaris has 1 sticker, that's 10 fewer than Jeremiah, so I have to draw more. If Amaris has 2 stickers, that's 9 fewer than Jeremiah...If Amaris has 6 stickers, that's 5 fewer stickers than Jeremiah. Oh, we're almost there! If Amaris has 7 stickers, that's 4 fewer stickers than Jeremiah. I can see that Amaris' row is 4 stickers shorter than Jeremiah's; Jeremiah's row is 4 stickers longer than Amaris'. So Amaris has 7 stickers. I'll write my answer in my number sentence:  $11-7=4$ . And I'll check my work. Jeremiah's row has 1, 2, 3...10, 11 stickers. I see Amaris' row has 4 fewer stickers than Jeremiah's row, and Amaris' row has 1, 2, 3, 4, 5, 6, 7 stickers, so my answer is right. Amaris has 7 stickers.

Following the problem demonstration, the class would practice aloud the four steps and accompanying questions of the schema-based instruction routine. The children would then be given a similar problem, and the teacher would guide the group's practice in thinking aloud and working towards a solution. As an alternative, a student could be asked to demonstrate the routine with a similar problem and explain his or her thinking at each step. Then, students would practice the process by solving another problem individually or in pairs (Montague, 2005).

As children becoming increasingly comfortable with this routine, the demonstrated visual representation of the problem is changed from manipulatives to a schematic representation drawing. "Students who have difficulty solving math word problems usually do not construct a representation of the problem that considers the relationship among the components and, as a result, they do not understand the problem and have no clue about a plan to solve it" (Montague, 2005, p. 3). It is imperative that students with learning disabilities, and other students who exhibit difficulties solving mathematical word problems, are not only provided with a structured routine for problem solving but are

also provided with instruction, explicit modeling, and teacher support in creating schematic representations that represent the relationships and other important parts presented in problems (Montague, 2005).

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### **Assessing Young Students' Problem Solving Skills**

A variety of assessments can be used to evaluate problem solving performance for early elementary students. Informal observation and questioning is a common classroom practice implemented during independent or small group work time. This form of assessment is flexible, can focus on a specific feature of student activity or behavior during problem solving, and can provide insight into areas of engagement and attitude not easily obtained through other assessment methods (Charles, Lester, and O'Daffer, 1987). Some of the disadvantages of informal assessment, such as interference with classroom management or the time and effort required to consistently assess all students and maintain records (Charles, Lester, and O'Daffer, 1987), can be overcome when strong classroom routines and structures, along with an organized record keeping schedule and system, are put into place.

A student report, or a "student's written or tape-recorded retrospective report on a problem-solving experience" (Charles, Lester, and O'Daffer, 1987, p. 24), is another form of problem solving assessment. It may be difficult to obtain useful information from early elementary students through this type of evaluation because talking about thought processes is often difficult for young children, especially those with cognitive delays; however, student reports can be helpful in starting to develop students' awareness of their thought processes and their ability to talk about their meta-cognitive strategies.

Young children can complete attitude inventories by selecting an emoticon to represent their feeling or attitude towards specific problems or aspects of the problem solving process. This type of assessment requires minimal time or effort to implement while offering students an opportunity to participate in the evaluation process (Charles, Lester, and O'Daffer, 1987). Though an attitude inventory may function primarily as an affective assessment, it can also give insight into students' strengths and difficulties in the problem solving process. If a student selects a happy emoticon related to a particular problem or aspect of the problem solving process, he or she likely experienced success with this problem or stage—a potential indicator of a

problem solving strength. If a student selects a frowning emoticon related to a particular problem or aspect of the process, he or she likely struggled with this problem or stage, indicating a potential problem solving weakness.

Clinical interviews, a method utilizing "intensive interaction with the individual child, an extended dialog between adult and child, careful observation of the child's work with 'concrete' intellectual objects, and flexible questioning tailored to the individual child's distinctive characteristics" (Ginsburg, 1997, p. 2), can allow a teacher to delve deeply into students' problem solving skills on a one-on-one basis. This form of assessment requires a great deal of time and advanced planning of interview problems and questions, but has the benefit of providing insights into a child's problem solving skills not observable through other means of assessment.

When a more formal, systematic, and consistent form assessment method is desired, analytic scoring or focused holistic scoring can be used. Analytic scoring, an assessment in which a point system is developed for stages of the problem-solving process, and focused holistic scoring, where a number is assigned to the entire solution "according to specific criteria related to the thinking processes involved in solving problems," can both be very useful forms of problem solving assessment in an early elementary classroom (Charles, Lester, and O'Daffer, 1987). Both assessment methods produce data that can be compared within and across classrooms, and both methods assign points for children's processes and use of problem solving routines rather than for a correct final answer. But for either of these methods to be effective, scoring criteria needs to be carefully developed, and "anchor papers" that "exemplify the criteria for a point category" should be identified (Charles, Lester, and O'Daffer, 1987, p. 38). Early elementary teachers, many of whom have limited experience assessing problem solving, should work as a team, ideally along with a mathematics specialist or fellow teacher experienced in teaching and assessing problem solving, to develop scoring criteria and identify anchor papers in order to increase the validity and consistency of analytic and focused holistic scoring assessments.

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### **Conclusion**

Though the ideas of problem solving and heuristic strategies as described by Pólya may not initially seem applicable to an early elementary inclusive classroom, they are not only applicable but an integral part of mathematics curriculum for all young children. When

exposed to problem solving routines and self-questioning techniques, many children are able to develop a natural ability to solve mathematical problems. Students with learning disabilities, however, often lack the self-regulation strategies needed to naturally develop the same problem solving abilities. These students may benefit from a “problem solving algorithm,” or routinized heuristics, integrated into a rich problem solving curriculum that allows for multiple means of engagement. This curriculum should incorporate explicit instruction, modeling, visual representation of problems, verbal and visual cues, segmented activities, repeated and teacher-supported practice, frequent and immediate feedback, and many opportunities for repetition in order to achieve mastery (Montague, 2005).

The call to recognize problem solving as a key activity in mathematical education, and the appeal for this education to be organized on the basis of problem solving, is effectively a call to give students a much more active role in the learning process, and to see them actually doing mathematics, rather than simply reproducing what they have learned. (Karp, 2008, p. 48)

When problem solving structures and routines such as those presented in this article are incorporated into the inclusive classroom, students who have difficulties with self-regulation, including those with learning disabilities, are given the support they need to do mathematics—to actively and successfully engage in mathematical problem solving.

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