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Rousing[1] Students' Minds in Postsecondary Mathematics: The Undergraduate Learning Assistant Model

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ABSTRACT This article presents several of the challenges facing postsecondary mathematics education and describes how the undergraduate Learning Assistant (LA) program has been used as a catalyst to engage faculty and students in redesigning opportunities to learn mathematics. Characteristics of the LA program that have been used to transform introductory undergraduate science courses are discussed. We then describe how the LA program was implemented in a mathematics department vis-à-vis the specific contextual features of a mathematics department at the University of Colorado Boulder.

KEYWORDS *Calculus, Learning Assistant, undergraduate mathematics, collaborative learning, recitation, mathematical reasoning*

Student success in undergraduate mathematics has significant implications with regard to whether they choose to continue into science, technology, engineering, and mathematics, or STEM, majors and future related careers. But even for those students who do not choose to major in mathematics, science, or engineering, success in entry-level undergraduate mathematics courses such as calculus can make or break their decision to persist in postsecondary education (Ferrini-Mundy & Graham, 1991; Moreno & Muller, 1999; Subramaniam, Cates, & Borislava, 2008). This article describes how the undergraduate Learning Assistant (LA) model was used as a catalyst to motivate changes in the teaching and learning of postsecondary calculus.

Calculus continues to maintain significant status in undergraduate STEM education. In a meeting in Washington in 1987, mathematicians reported that "as many as 40% of undergraduates were failing introductory calculus, and even those who passed did not appreciate the subject's relevance" (Wilson, 1997, p. A12). And yet, more than 25 years later, the same failure rate is common in many university calculus programs. At University of Colorado Boulder (CU-Boulder), for many years the percentage of students who earned a D, F or withdraw in first semester calculus had been a consistent 34%. However, after implementing some of the preliminary features of the Learning Assistant program in the calculus recitations, the percentage of students earning a D, F, or withdraw dropped to 27%. This drop was equivalent for both males and females and all ethnic groups showed corresponding improvements. This result elicited the following questions: Was this an anomaly or the beginning of a positive trend in increasing student access to postsecondary calculus? More importantly, what are the specific benefits of the LA calculus program that translated, it appears, into improved end-of-course grades for students?

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[1] The reference to Tharp and Gallimore's (1991) book *Rousing Minds to Life* is intentional as Vygotsky's theories of socially mediated learning, which are discussed extensively in their book, are an explicit part of the LA model.

The Status Quo for Student Learning of Calculus

There are few studies of undergraduate mathematics teaching (Speer, Smith, & Horvath, 2010). But conversations with other mathematics faculty, information gleaned at mathematics and mathematics education workshops and conferences, and perusal of web-based curricular materials has revealed to us quite a bit of commonality in the way in which calculus is taught at many universities and community colleges. Three to four times per week a lecture is prepared and delivered by mathematics faculty (cf. Natarajan & Bennett, 2014). For the recitation, which is usually offered once or twice a week, a doctoral student is assigned as a teaching assistant (TA). Typically, the TA either prepares a session that highlights key features of the lecture or reviews related homework assignments. With respect to course structure and student interaction, often the lecture is teacher-directed (and usually held in large lecture halls) and recitations have lower instructor-to-student ratios, but are still mostly directed by the assigned TA. That is, student-to-student interaction is usually not observed in the lecture or the recitation (Alsina, 2001).

The content of a calculus course is also quite similar across institutions (cf. Hillel, 2001). De facto national curriculum standards for the first two semesters of calculus have been recognized by the ubiquitous offerings of Advanced Placement Calculus AB and BC at the high school level, with course guidelines and assessments that crystallize key features of the calculus sequence that one might expect in the first two semesters. Quite often, however, in both the secondary and post-secondary versions of the course, to meet the pacing demands greater emphasis is placed on procedural fluency at the expense of conceptual understanding even though these goals are not in conflict.

This describes how calculus was taught at CU-Boulder as well, until recently. But in the last half-dozen years or so, we have introduced some significant changes, perhaps the most substantial of which is our undergraduate Learning Assistant program.

The Colorado Learning Assistant Program

The Colorado Learning Assistant (LA) program represents a highly researched, multi-disciplinary model of educational change (Otero, Finkelstein, McCray, & Pollock, 2006; Pollock & Finkelstein, 2008; Smith, Wood, Adams, Wieman, Knight, Guild & Su, 2009). Since its origins in a single astronomy course, it has grown to serve eleven different mathematics and science departments at CU-Boulder, and to be emulated at more than 50 institutions of higher education nationwide (Learning Assistant Program, n.d.).

The LA program uses the transformation of introductory large-enrollment courses as a mechanism to achieve four goals: a) to improve the quality of mathematics and science education for all undergraduates, b) to transform departmental cultures to value research-based teaching, c) to engage faculty in the recruitment and preparation of future teachers, and d) to recruit and prepare talented STEM majors for careers in teaching.

Since 2003, LAs have been used in science departments at CU-Boulder to support student learning in recitations[2] and support aspects of course transformation that differs across departments. LAs are undergraduate students who have had previous success with the same course and have expressed an interest in supporting the learning of subsequent cohorts of undergraduates. What is common across the LA experience at CU-Boulder (since implementation may vary at other institutions that have emulated the program), is the following:

1. LAs meet weekly with their faculty instructor to plan for the upcoming week, reflect on the previous week, and analyze formative and summative assessment data collected periodically throughout the course;
2. LAs facilitate student collaboration and small group work in weekly recitations, by formatively assessing student understanding, encouraging student interaction and discovery, and asking guiding questions; and
3. LAs from all departments attend a special School of Education seminar on Mathematics and Science Education, where they reflect on their own teaching and learning and make connections to relevant education literature (Otero, 2006).

Together with the faculty and teaching assistants who teach the targeted courses, LAs provide enhanced opportunities for enrolled students to engage actively in their learning. However, because the Learning Assistant initiative originated in CU-Boulder science departments, there were initially only limited opportunities for mathematics faculty to implement this program in ways that recognized the particular features of mathematics and mathematics courses.

[2] In general, recitations are weekly review sessions led by a teaching assistant. With large lecture calculus courses there are often multiple recitations organized to reduce the student-to-TA ratio to a small section format of roughly 30-to-1. SC

Developing and Implementing the Undergraduate LA program in Calculus I

Inspired by the culture and the success of active learning and, especially, of Learning Assistant programs in CU-Boulder science departments, the authors undertook efforts to transform our first semester Calculus course with help from Mathematics and School of Education colleagues. This endeavor commenced around 2008, and is ongoing.

Our approach has taken into account research regarding: 1) the use of cognitively demanding mathematics tasks, 2) the construction of learning environments that recognize the need to balance socially mediated learning and individual reflection, and 3) the use of undergraduate Learning Assistants to support student learning. Each of these mutually contributing factors are elaborated below, with particular attention to their implementation in undergraduate calculus.

Mathematical competencies and mathematics tasks

We do not propose changing the topics addressed in first semester calculus. Rather, we argue there is a compelling need to enhance the learning goals to engage students in tasks that promote and assess their procedural fluency as well as their understanding of essential concepts. What can often be lacking in calculus courses are opportunities for students to make sense of the fundamental concepts of calculus and build mathematical connections among topics and various representations. Although counterintuitive, the lack of cognitive demand in mathematics tasks can limit student access to higher mathematics. Student engagement can and should be promoted through conceptually meaningful tasks that are cognitively demanding for students (Stein, Grover, & Henningsen, 1996). Given the historical (Dunham, 1991; Kline, 1972) and contemporary (Lakatos, 1976; Polya, 1945) positioning of tasks in the development, teaching and learning of mathematics, it is logical to expect that increased student access can be found in the presentation of the same topics in more meaningful and conceptually revealing ways.

To support student learning, instructional tasks should elicit cognitive activity that represents a range of mathematical reasoning (Silver & Stein, 1996). To support the selection, adaptation, and design of tasks, a framework for student reasoning that respects the key topics in the content domain and the nature of how students learn and come to understand mathematics must be considered. To address the goals of this research, two somewhat broad aspects of mathematical reasoning will be used.

Reproduction of procedures, concepts, and definitions. This reasoning category involves knowing facts, recognizing equivalents, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, and developing technical skills, as well as dealing and operating with statements and expressions containing symbols and formulas in "standard" form. The reasoning elicited by such tasks is almost exclusively memorization and recall, although several tasks in this category may collectively promote mathematical connections between topics or representations. Tasks in the reproduction category are often similar to those found on many standardized tests. These tasks are quite familiar to mathematics teachers and students from K-12 to postsecondary mathematics.

Connections, justification, and generalization. Tasks exemplifying this reasoning category promote conceptual connections within and between the different domains in mathematics, motivate students to integrate information to solve simple problems, and often are accessible to a range of solution strategies. As these tasks are "more open to a range of strategies" they tend to be more open-ended. Students solving problems of this type need to interpret different representations according to situation and purpose, and they need to be able to distinguish and relate a variety of statements (e.g., definitions, claims, examples, conditioned assertions, proofs).

Tasks in the connections category are sometimes situated within a context to promote connections between mathematics and modeling of realistic phenomena, requiring students to recognize and extract the mathematics embedded in the situation. Tasks that require mathematical justification and generalization, even though they are sometimes regarded as a separate category, are also included in this category for the purposes of this study since we are more concerned with identifying tasks that require more than memorization and recall.

Construction of Collaborative Learning Environments

With more cognitively demanding tasks come greater opportunities for socially mediated learning and collaborative problem solving. Lectures have a purpose and place in undergraduate STEM education. The well-crafted lecture, even though it can be interpreted as a passive learning experience, can focus students' attention on key mathematical ideas, methods, and techniques. Of course "can" is the operative qualifier. How students focus in lectures varies depending on the extent to which the instructor takes into account students' prior knowledge and the extent to which students come prepared and attend to the main points. Recitations also serve an important role in student learning and are usually offered as part of a calculus course. In contrast to the lecture, recitations can be quite varied in their approach and purpose. They can range from opportunities for mini-lectures on a specific topic, to discussions of homework problems, to an opportunity for students to collaborate in solving fundamental problems and applications of recently learned concepts.

Currently, as with most undergraduate mathematics programs, there exist many opportunities for students to engage in self-initiated, individual reflection on what they are learning in calculus. Then again, time and support for self-initiated individual work in undergraduate mathematics is not the problem. Lectures and recitations can offer interested students passive or direct feedback on their solutions to assigned problem sets. However, there are few opportunities for students to share work in progress, model and sketch related representations, and propose different solution strategies to their classmates. The proposed addition of collaborative problem solving in recitations draws upon the contributions of Vygotsky (1978) and contemporary findings in cognitive science and theories of socio-constructivist learning (Bransford, Brown, & Cocking, 2000; Donovan & Bransford, 2005), which suggest students need opportunities to test and hone how they articulate their emergent ideas among their peers. Focus and guided collaboration can support student learning in mathematics by contributing to the extension of one's zone of proximal development, described by Vygotsky (1978) as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (p. 86). How might their insights, methods, conjectures and refutations stand up to the scrutiny of other learners when they are solving nonroutine mathematics problems (e.g., Lakatos, 1976)? When students have opportunities to compare and articulate their work with others, those who have recently experienced personal advances in their own learning can often locate important representations or metaphors that "worked for them." These can serve as cognitive scaffolds that support new learning of their classmates (cf. Topping, 1996). Research on peer-supported collaborative environments has identified several important features that support student learning, including the verbal development of academic language and ongoing feedback, assistance, and explanation of solution strategies while peers are solving problems (Webb & Mastergeorge, 2003). Such small group collaboration can also result in improved achievement, dispositions, and persistence in undergraduate mathematics (Springer, Stanne, & Donovan, 1999).

Uri Treismann (1992), in his research on the use of study groups to increase access of African-American and Latino students in UC-Berkeley's undergraduate calculus, drew from this same theory of socially mediated learning. In addition, the Berkeley study group model strongly suggests the use of challenging problems that move students beyond simply reviewing exercises that reinforce procedural knowledge to support sense-making and application of their knowledge in new ways. Getting students together to review answers to problem sets is not what we are suggesting here. Rather, the need to collaborate in problem solving should be authentic. To promote discussion and exchange of ideas the mathematics problems used in recitations need to be "group worthy" (Boaler, 2006).

The engagement in recitations is key to students' deeper understanding of the material, which we hope to leverage into their pursuing further mathematics and mathematics classes. Instructor and student activity in recitations used to be focused primarily on answering homework questions. Now, students participate in engaged inquiry that encourages a greater focus on conceptual understanding and fosters connections among distinct ideas.

Use of Learning Assistants

Like any department at CU-Boulder, The Department of Mathematics applies for Learning Assistants through "LA Central," the umbrella campus unit that oversees the LA program. In their application, faculty must describe their commitment to course improvement and transformation, and to active, student-centered teaching and learning. Student applications to be hired as a Learning Assistant are also accepted through LA Central.

From those students who have applied to be Mathematics LAs, the Mathematics Department selects perhaps twenty or so to interview. (We typically use ten to a dozen LAs each semester.) The interviews are conducted by mathematics faculty who will be teaching calculus. Typically, selection will be based on fluency with and appreciation for the subject material, and, at least as importantly, on enthusiasm for communicating mathematics to others.

The responsibilities of a CU-Boulder Mathematics Department LA are similar to those of other LAs in other science departments on campus. A Mathematics Department LA assignment entails ten hours of work a week, with that time divided up as follows:

- Three hours a week facilitating small group work in calculus recitations;
- One hour a week tutoring in the Department's Undergraduate Mathematics Resource Center, or "help lab;"
- One hour a week in meetings with Calculus instructors, focusing on content;
- Two hours a week attending a School of Education seminar on Mathematics and Science Education;
- Three hours a week of preparation.

As is the case with all LAs at CU-Boulder, Mathematics LAs attend a several-hour-long training session during the first week of classes. Also like other LAs, Mathematics LAs earn a stipend of $1500 for the semester, and receive course credit for the School of Education seminar.

Results from Initial Implementation of the LA Model

Coincident with the introduction of LAs into CU-Boulder's Calculus I recitations was the redesign of the recitations themselves, toward a more student-centered, discovery-based model wherein students work together in small groups. Given the greater attention to group activity in recitations, we reasoned that improved student performance might have resulted not only from the addition of an extra facilitator, but also from changes in the nature of the instructional tasks that were being facilitated. Perhaps the tasks selected were more conducive to sustained group discussions and open to a range of solution strategies.

Trends in Task Use

Using the criteria for student reasoning described previously under Mathematical Competencies, two mathematics graduate students who completed a graduate level course in assessment design were asked to independently analyze all instructional tasks that were used in Calculus I recitations during the first three semesters of program implementation. The initial interrater agreement regarding the type of reasoning elicited was 71%. After discussing several tasks exemplifying recall/memorization vs. connections/justification the inter-rater agreement was 93%. The results from the second round of their task analysis are summarized in Table 1.

This summary reveals two trends in the use of tasks over the first three semesters: a modest increase in the use of application of derivative problems and a significant increase in the percentage of tasks that promote mathematical connections and justification. These trends do not reflect intentional planning on the part of the calculus supervisor or the TAs. Instead, it is more appropriate to attribute the trend to a shift in perceived needs of calculus recitations.

Table 1

Percentage of task types used in Math 1300 recitations

	Spring '08 n = 97	Fall '08 n = 103	Spring '09 n = 106
Content Domain			
Precalculus	43.3	40.8	42.5
Limits and continuity	18.6	19.4	19.8
Derivatives	15.5	13.6	13.2
Application of derivatives	6.2	10.7	9.4
Integrals	16.5	15.5	15.1
Reasoning Elicited			
Recall/Memorization	68.0	63.1	57.5
Connections/Justification	32.0	36.9	42.5

These data suggest the need for a deeper analysis of student reasoning that is elicited by these tasks so that we have actual evidence of student reasoning elicited by such tasks, for those students who are making satisfactory progress in calculus and those who are not. The following sections describe specific design features of tasks identified as eliciting "reasoning beyond recall."

Reasoning Beyond Recall

The problem in Figure 1 is selected from the set of instructional tasks used in fall and spring 2008 recitations. Part (a) is an example of a typical recall/memorization problem, using basic theorems and trigonometric identities to solve for the derivative of a given function. However, part (b) asks the student to explain how two completely different solutions can exist. Most calculus students run into this situation frequently (checking answers to homework problems with other students, the solutions manual, etc), but rarely do they understand why both answers are different, yet both are correct.

> **Spring '08 – Fall '08, Worksheet 11, problem 4:**
>
> (a) Confirm that $\frac{d}{dx}(\sin^2 x) = 2 \sin x \cos x = \sin 2x$
>
> (b) Your work in (a) verifies that one antiderivative of $\sin 2x$ is $\sin^2 x$. Find an antiderivative of $\sin 2x$ that involves the function $\cos 2x$ and explain how $\sin 2x$ can have these two different antiderivatives.

Figure 1. Reconciling different solutions to the same problem.

Using Mathematics in Problem Contexts

Without a context, students tend to be less motivated, thinking mathematics is meaningless (Yusof & Tall, 1999). Figure 2 is an example of a problem that suggests that calculus is not just about plugging numbers into formulas. Students must reason with one another to conceptualize what happens if one portion of the problem is changed. They are being asked to fully understand how mathematics relates to situations outside of the classroom.

> **Spring '08 – Fall '08 – Spring '09, Worksheet 3, problem 3b:**
>
> Discuss quantitiatively (i.e., without actually solving the problem) whether the lighthouse would be nearer to or farther from the shore, relative to your answer in 3(a), if the only change to the above information is:
>
> 1. The beam moves 500 feet down the shore in 4 seconds
> 2. The beam rotates once every 90 seconds
> 3. The beam moves 300 feet down the shore in 6 seconds
>
> As a reference, here's part (a): A lighthouse sits on a rock offshore and its beam rotates once every 48 seconds. Starting from the point on the shore nearest the lighthouse, the beam moves 300 feet down the shore in 4 seconds. How far is the lighthouse from the shore?

Figure 2. Calculus in context.

Building Mathematical Connections

In calculus, too often students are only concerned with finding the derivative. They are given a function and asked to perform familiar calculus procedures. However, ask them why they would want to take the derivative, and they are often stumped. Figure 3 describes a function, but does not define it with symbols. The students are asked to connect their understanding of functions, inverses, and derivatives to the average number of emails sent per day in a given year.

> **Spring '09, Worksheet 6, problem 1:**
>
> Let $g(t)$ denote the average number of email messages, in millions per day, sent in Colorado in year t. What are the units and interpretations of the following quantities? (Write your answer next to the item.)
>
> (a) $g(2007)$
>
> (b) $g'(2007)$
>
> (c) $g^{-1}(2)$
>
> (d) Do you think $g'(2007)$ would be negative, zero, or positive (explain your reasoning).

Figure 3. Connections task.

Anecdotal Reports of Student Engagement and Learning

In a traditional recitation session, a TA will work homework problems at the board, taking derivatives of functions, for example, requiring little to no effort or engagement from the students themselves. In an LA-and-TA-led recitation, all students participate actively in solving problems they have not seen before—problems that are related to material seen previously, but foster a deeper or broader understanding of that material. For example, students are asked to sketch derivatives visually, and compare their results with the algebraic formulas. The visual derivatives give students a tool for self-correction, encourage mathematical intuition, and provide a resource for visual learners.

The recitation activities also get Calculus students to learn from, and with, their peers—the LAs, as well as the group mates with whom they discuss and complete their worksheets. From our observations of recitations, their relatively unstructured and "casual" atmosphere as well as the increased level of contact between the students and the "teachers" (LAs and TAs) provides an environment in which the Calculus students are more interactive and in which not only the expertise but also the enthusiasm of the LAs and TAs towards mathematics is better communicated to students.

Responses of LAs

We asked LAs a number of questions about their own experiences. Most of these questions regarded the effect of the LA program on the LAs, but we also asked one question that addressed the LAs perceptions of their impact on student learning: What effects do you believe LAs have on the students in the courses using LAs?

Some of the responses that we received to this question included:

- I think it helps them out more than they know. It gives them someone to ask questions to, someone they can relate to, and it helps relieve tensions for the class.
- I think it is nice to have an undergraduate in the classroom. The LA program provides for that here at CU-Boulder, which I think is awesome.
- LAs basically motivate students to learn because they were sitting in the same class just like the students a year or two years ago. That tells the students that LAs were students just like them.
- I received a lot of feedback from students who said that it was interesting being taught by someone their own age who was really energetic about math. I also think that a class with LAs also allows students to be able to communicate with teachers who are their own age. The LA experience is a great experience both for students and the LAs.
- Our experiences are much more tangible than those of the TAs because we've taken the same classes recently, we know the professors, and we understand the 'life of the CU-Boulder undergrad.'

Toward Transformation of Department Culture

When the Mathematics Department Chair and Associate Chair for Undergraduate Studies learned that some of our best majors were working as LAs for other STEM departments because we did not have our own LA program, we decided it would be important to establish one. The recent addition of LAs in the mathematics department has gradually, but very noticeably, helped transform the culture and the "life of the mind" in our department. A number of mathematics faculty members who are not typically inclined towards thinking about pedagogy or issues in mathematics education have noticed that the ranks of mathematics LAs include some of the very best students from their upper-division "pure math" courses. When faculty learn more about the LA program, and about what LAs learn and teach, their respect for teacher training and education research grows.

Also, the very apparent enthusiasm of the Mathematics LAs, as well as the LA-fueled improvements to our Calculus I course, have piqued the interest of a number of faculty, so that we now have more faculty interested in using LAs in their own courses. Of course, one has to find a place for these LAs, and the directors of the CU-Boulder LA program encourage faculty applicants for LAs to use them in innovative ways that truly enhance the learning experiences of the students in the affected courses. Thus, more departmental faculty have become interested in course transformation, curriculum development, and, more generally, in thinking about what and how we are teaching.

The LAs are also an intellectual resource for the department—graduate students as well as faculty. Indeed, the LAs provide pedagogical perspectives and ideas—many of which they glean through their School of Education seminar—that are useful and novel to the more "traditionally" trained contingency of the Mathematics Department.

Summary

The authors, the mathematics faculty who have worked with LAs (and many who haven't), and a variety of others associated with the department's LA program believe strongly that Learning Assistants have improved the teaching, learning, culture, and atmosphere of the department in substantial ways. This belief is supported by the improvement in D/F/Withdraw rates, the task analysis, the LA responses, and the other evidence cited above. It is also consistent with documented evidence of LA effectiveness in other science departments (e.g., Otero, Finkelstein, McCray, & Pollock, 2006; Pollock & Finkelstein, 2008; Smith, Wood, Adams, Wieman, Knight, Guild, & Su, 2009).

Further analyses, entailing deeper, broader studies of student understanding, student success, departmental attitudes, and the like, are warranted, and are currently being completed by CU-Boulder Department of Mathematics and School of Education faculty.

In learning, some six or seven years ago, of LA programs in other units on campus, we asked ourselves, "Why isn't our Math Department doing this?" Now that we are, we encourage college and university mathematics departments nationwide to ask the same question.

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References

Alsina, C. (2001). Why the professor must be a stimulating teacher: Towards a new paradigm of teaching mathematics at university level. In D. A. Holton (Ed.), *The teaching and learning of mathematics at university level: an ICMI study* (pp. 3 – 12). Dordrecht: Kluwer.

Boaler, J. (2006). "Opening our ideas": How a detracked mathematics approach promoted respect, responsibility, and high achievement. *Theory into Practice, 45*(1), 40 – 46.

Bransford, J. D., Brown, A., & Cocking, R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school.* Washington, DC: National Academy Press.

Donovan, M. S., & Bransford, J. D. (Eds.). (2005). *How students learn: Mathematics in the classroom.* Washington, DC: The National Academies Press.

Dunham, W. (1991). *Journey through genius: The great theorems of mathematics.* NY: Penguin Books.

Ferrini-Mundy, J., & Graham, K. G. (1991). An overview of the calculus curriculum reform effort: Issues for learning, teaching, and curriculum development. *American Mathematical Monthly, 98*(7), 627 – 635.

Hillel, J. (2001). Trends in curriculum: a working group report. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 59 – 70). Dordrecht: Kluwer.

Kline, M. (1972). *Mathematical thought: From ancient to modern times.* Oxford, UK: Oxford University Press.

Lakatos, I. (1976/1987). *Proofs and refutations: The logic of mathematical discovery.* Cambridge, UK: Cambridge University Press.

Learning Assistant Program. (n.d.). Retrieved November 2, 2014, from laprogram.colorado.edu

Moreno, S. E., & Muller, C. (1999). Success and diversity: The transition through first-year calculus in the university. *American Journal of Education, 108*(1), 30 – 57.

Natarajan, R., & Bennett, A. (2014). Improving student learning of calculus topics via modified just-in-time teaching methods. *PRIMUS, 24*(2), 149 – 159.

Otero, V. (2006). The learning assistant model for teacher education in science and technology. *Forum on Education of the American Physical Society,* 31.

Otero, V., Finkelstein, N. D., McCray, R., & Pollock, S. (2006). Who is responsible for preparing science teachers? *Science, 313*(5786), 445 – 446.

Pollock, S.J., & Finkelstein, N.D. (2008). Sustaining educational reforms in introductory physics. *PhysRev: ST Phys Ed. Rsrch.,* 4, 010 – 110.

Polya, G. (1945/2004). *How to solve it: A new aspect of mathematical method.* Princeton, NJ: Princeton University Press.

Silver, E. A., & Stein, M. K. (1996). The quasar project: The "revolution of the possible" in mathematics instructional reform in urban middle schools. *Urban Education, 30*(4), 476 – 521.

Smith, M. K., Wood, W. B., Adams, W. K., Wieman, C., Knight, J. K., Guild, N., & Su, T. T. (2009). Why peer discussion improves student performance on in-class concept questions. *Science, 323*(5910), 122 – 124.

Speer, N. M., Smith III, J. P., & Horvath, A. (2010). Collegiate mathematics teaching: An unexamined practice. *The Journal of Mathematical Behavior, 29*(2), 99 – 114.

Springer, L., Stanne, M. E., & Donovan, S. S. (1999). Effects of small-group learning on undergraduates in science, mathematics, engineering, and technology: A meta-analysis. *Review of educational research, 69*(1), 21 – 51.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33, 455 – 488.

Subramaniam, P. K., Cates, M., & Borislava, G. (2008). Improving success rates in calculus. *MAA Focus, 28*(5), 20-21.

Tharp, R. G., & Gallimore, R. (1991). *Rousing minds to life: Teaching, learning, and schooling in social context.* New York, NY: Cambridge University Press.

Topping, K. J. (1996). The effectiveness of peer tutoring in further and higher education: A typology and review of the literature. *Higher Education, 32*(3), 321 – 345.

Treisman, U. (1992). Studying students studying calculus: A look at the lives of minority mathematics students in college. *College Mathematics Journal, 23*(5), 362 – 372.

Vygotsky, L. S. (1978). *Mind in society: Development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Webb, N. M., & Mastergeorge, A. M. (2003). The development of students' helping behavior and learning in peer-directed small groups. *Cognition and instruction, 21*(4), 361 – 428.

Wilson, R. (1997). A decade of teaching reform calculus' has been a disaster, critics charge. *Chronicle of Higher Education, 43*(22), A12 – A13.

Yusof, Y. M., & Tall, D. O. (1999). Changing attitudes to university mathematics through problem solving. *Educational Studies in Mathematics, 37*(1), 67 – 82.