Journal of Mathematics Education at Teachers College

Fall – Winter 2010

A Century of Leadership in Mathematics and its Teaching

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The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

This issue honors Dr. Alexander P. Karp, an Associate Professor in the Program in Mathematics at Teachers College. A native of St. Petersburg, Russia who is the author of more than one hundred publications including textbooks used throughout Russia, Professor Karp represents Teachers College at meetings and conferences throughout the world as well as through his role as managing editor of the *International Journal for the History of Mathematic Education*.

Former Teachers College Professor and Mathematics Education Chair, Howard Franklin Fehr, was among the most influential mathematics educators of his era. Through his many international contacts, he was the organizer of conferences, projects, and publications including the Congresses of Mathematics Education, a seminal conference on Needed Research in the field, and curriculum initiatives including the Secondary School Mathematics Curriculum Improvement Study.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. Themes planned for the 2011 issues are: *Mathematics Curriculum* and *Technology*. *JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,000-2,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Call for Papers

The "theme" of the spring issue of the *Journal of Mathematics Education at Teachers College* will be *Mathematics Curriculum*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2000-2500 words describing research, experiments, projects, innovations, or practices related to mathematics curriculum. Articles should be submitted to Ms. Krystle Hecker at jmetc@tc.edu by January 1, 2011. The spring issue's guest editor, Nicholas Wasserman, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2011, and final drafts of selected papers are to be submitted by March 1, 2011. Publication is expected in mid-April, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the blind manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at jmetc@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2011	Curriculum
Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids
Fall 2014	Special Students

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High Achievement in Mathematics Education in India: A Report From Mumbai

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This paper reports a study aimed at characterizing the conditions that lead to high achievement in mathematics in India. The study involved eight schools in the greater Mumbai region. The main result of the study is that the notion of high achievement itself is problematic, as reflected in the reports about mathematics achievement within and outside India as well as the complexities of a school system which, despite democratic ideals, has difficulty serving the needs of all students. This study provides snapshots of three mathematics classes that embody some of these complexities to provide a balanced account of conditions that either support or detract from the goal of high achievement in India.

This paper reports a study, the goal of which from the outset was to characterize high achievement in mathematics in India. The reason for conducting the study was that India is often mentioned as a strong producer of talent in mathematics- and science-related areas, and it seemed important to try to identify some of the features that contributed to developing this talent.

In the course of doing this research, however, which involved visiting eight schools in the greater Mumbai area, the original task changed. Digging a little beyond the surface of what seems like a successful system for producing highly talented students in mathematics and science (the focus here was on mathematics), a more complex picture emerged. That is not to say there were not examples of talented students, good schools, and excellent curricula. But the main story that emerged was that the quest to find a good education in India is not necessarily an easy one, and by looking into what schools are actually like, we can begin to appreciate both the challenges and opportunities found in Indian schools.

There are a number of qualifications to make before reporting this study, the first of which is that India is too large and diverse a nation to characterize in any simple way, especially in a paper of this length. A number of simplifications have been made—first, to focus on one city, Mumbai, which by itself has an immensely diverse school system. A second simplification is to give snapshots of the schools in the study, rather than in-depth case studies. A third simplification is to focus on reporting on these schools without delving too deeply into the many cultural, historical, and economic forces that have shaped the system. An analysis of these forces would be beneficial for understanding why mathematics education takes the forms it does today, but that is beyond the scope of this project.

The paper is organized into three sections. The first section provides some background for the question of high achievement in India. It explores why there is a perception that India is a high-achieving nation in the areas of mathematics and science, and reasons why this characterization might be, in some ways, misleading. The second section provides snapshots of how mathematics is taught in three schools at different educational levels. Each of these snapshots contains elements that both contribute to and detract from the goal of producing high-achieving students. The third section discusses the question of high achievement in a larger context of Indian education.

Background

To understand high achievement in mathematics education in India, one must begin by first untangling what might be called a "myth of high achievement." The myth of high achievement reflects the fact that despite positive reports about achievement in India according to some sources and statistics, there are deep and widespread problems in terms of quality and access to education for all students.

The positive story about high achievement in India can be seen in various reports of Indian education from other countries. For example, Ruth Simmons (2010), president of Brown University in the United States, recently described the educational reforms now under way in India (and China) as potentially creating a "tsunami" of competitive students and workers in science, technology, and mathematics-related fields. Even as early as the 1990s there were a large number of temporary US visas granted to doctorate recipients of East Asian descent in science and engineering:

> Particularly striking is the large concentration of recipients from Asia, with 60 percent coming from four countries: the People's Republic of China (21.0%); Taiwan (13.7%); India (12.2%); and South Korea (11.1%). (Black & Stephan 2003)

Even Japan, which has traditionally scored highly on international mathematics exams such as TIMMS and PISA, has started to turn to India as a model for mathematics education. International schools that use Indian curricula, down to the detail of coloring Indian flags, are experiencing increased enrollment, and bookstores are stocked with textbooks with names like *Extreme Indian Arithmetic Drills* and *The Unknown Secrets of the Indians* (Fackler, 2008).

There are also reports from inside India that paint a picture of education in general, and mathematics education in particular, as alive and healthy. The following quote comes from the *India Science Report*, a nationwide survey of beliefs and attitudes toward science.

> Mathematics remains the most preferred subject, with a third of students in classes six to eight rating it as number one, and over 21% still feeling the same way in classes 11 and 12. (Shukla, 2005, p. ii)

A third of Indian students think mathematics is their favorite class in the lower grades, and over a fifth still say so in grades 11 and 12. Those are large proportions, given that mathematics is a subject that is considered difficult by many students. The *India Science Report* goes on to say that the students in India who study mathematics by and large tend to do so because they like the subject, not because of parental pressure or job pressure.

If one looks at a different set of statistics, however, particularly with an emphasis on elementary education, the story of high achievement in mathematics education in India looks less promising. While India and China are often touted as the up and coming countries, it seems clear by many indices that India is far behind China in terms of literacy, mathematics education, and general access to education for the masses.

> In the late 1940s, national statistics for primary school enrolment and other human development indicators were comparable between China and India. Both countries then experienced major political transitions and embraced similar economic development priorities. Half a century later, reports prepared for the 2000 World Education Forum indicate that China had far outperformed India in terms of school enrolment ratios and on indices of the efficiency of primary education. (Rao, Cheng, & Naraian, 2003, pp. 154–155)

A study supported by India's National Institute of Educational Planning and Administration in the early 2000s showed that in Hindi-medium schools, there were a large number of underachievers in mathematics.

> The achievement scores based on class IV competencies were very low as compared to class I mean achievement scores. The gap was large in mathematics as compared to language. The mean score in mathematics for Hindi medium schools was 40.46 percent as compared to 56.5 percent for the mean score in language.… In view of the low achievement scores, the underachievers are large in number. (Aggarwal, 2001)

Even in English-medium schools, which have the reputation of being of a higher standard, students did not perform well in mathematics.

> The achievement levels of children studying in English medium schools were analyzed separately. The mean score was 47.8% in mathematics…. About 38 percent of children failed to score more than 40% in mathematics…. Therefore the general impression that all is well with English medium schools is not correct. (Aggarwal, 2001)

This study covered only government schools, which tend to cater to the less wealthy students, where there are even more problems than in the English-medium schools. One example is enrollment, especially at higher grades. At the upper primary level (ages 11-14) the enrollment ratio (percentage of students that attend school) is just above 50% (District Information System for Education [DISE], 2008). While enrollment numbers have increased in recent years thanks to a number of education initiatives especially at the lower primary education, universal access is still a far-off dream. The statistics of what the top students are doing belie a sad fact that millions of children in India, especially those from poor families or low castes, still are without basic education.

The notion of high achievement in mathematics in India, then, is a bit like the story of the blind men and the elephant. You get a different picture of the elephant depending on which man you ask. India is indeed a nation with deeply rooted educational traditions that on one hand produce an impressive number of world-class scientists and engineers, and on the other hand struggle to provide a proper education for a large proportion of the population.

Snapshots

Below, snapshots are provided of three schools that in some way capture the complexity embodied in the notion of high achievement discussed above. Each of these schools displays some elements that could contribute to high achievement and some elements that could detract. The examples were chosen from a sample of eight schools ranging from Pre-Kindergarten (age 3-4) to Junior College (age 16-17).

To understand the context in which these schools function, we begin with some background information about the Mumbai school system. The greater metropolitan area of Mumbai contains approximately 18 million inhabitants, which if taken as a country would rank as the fiftieth largest (Mehta, 2005). Taken as a city, it is the world's second largest, after Shanghai. By the end of the 1990s there were over two thousand primary schools serving over a million students (Juneja, 2001). Half of the population of Mumbai is homeless or lives in slums (Juneja, 2001), and many of these children do not attend

Figure 1. Teacher presenting lesson on conditional probability

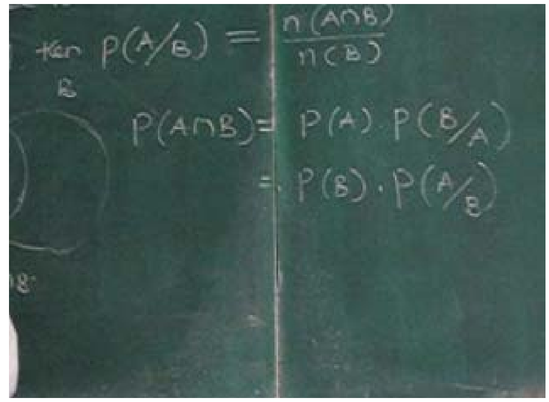

Figure 2. The basic formulas for conditional probability

schools (see Banerji, 2000 for an explanation of why). Most schools in Mumbai, as in the rest of India, follow a 10 + 2 + 3 system, with the first ten years[1] focusing on general education, the next two years[2] with more specialized education, and the last three years as degree programs. After the tenth and twelfth grades, students take high stakes exams, which determine their placement in Junior College and university, respectively. The most sought after universities are highly competitive. For instance, India Institute of Technology accepts only 7,500 of 400,000 applicants annually.

Because of the competition for top universities, there is high pressure on students to do well on the tenth and twelfth grade exams. As an example of this pressure, no school visits for this study were allowed in classrooms at these grade levels. The situation is so competitive that most students, even from poor families, get outside tutoring to prepare for the high stakes exams. In one school visited during this study, tutoring played such a strong role in students' lives that many students came to school only to eat lunch, which is provided free by the government.

The schools discussed below are among the better schools in the study. In the better schools, there were good facilities for students, trained teachers, and a fairly high standard of education. In some of the other schools, there were buildings without proper plumbing and high rates of teacher absenteeism.

Example 1

The first example comes from a classroom in an urban Junior College. It was a somewhat unusual visit because there was a major cricket match that day and few students were in the school. The teacher, who was considered the best mathematics teacher in the school, had decided not to teach that day, but he offered to give a lesson as part of this study. He went down to the class without any notes[3] and lectured to the handful of students who had shown up to class.

The lesson was on conditional probability, and the teacher stood at the front of the room in front of a small board to provide the basic facts about the topic. Talking to the class, asking rhetorical questions, and then writing on the board, he gave the basic formula for conditional probability and a Venn diagram to show how the formula was derived (see Figures 1 and 2). He explained when to use the formula (in cases where one event has happened and another has yet to happen). And finally, he gave an example using the formula (see Figure 3). The students either watched the teacher or wrote down what he was writing on the board in their notebooks.

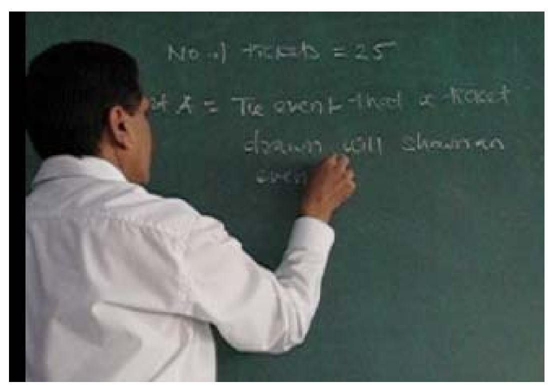

Figure 3. Working an example

This lesson exemplifies one of the two types of lessons observed in the schools in this study. Either the

[1] Corresponding to primary and secondary school.
[2] This is upper secondary school, called Junior College in Mumbai and High School in some other states.

[3] This was typical—all the teachers from Kindergarten on seemed to have their lessons memorized or needed only brief references to the book.

Figure 4. An innovative elementary school outside Mumbai

Figure 5. Finding integers that sum to ten

teacher introduced a new concept, like in this case, or he or she worked out problems (called sums) similar to ones in the book to apply these concepts. The students themselves seemed to understand that this was the standard pedagogy. One student said that the way mathematics was taught was to learn many techniques and apply them in increasingly complex situations.

Many aspects of teaching are standardized, guided by national and state syllabi, including the write-up of answers. A local researcher provided a sense of the kind of detail required: "If you use the quadratic equation you must write the solution set in open bracket minus one comma one close bracket. You can't write 'solution set is equal to.'" This might explain the attention students paid to this lesson, even with most of their peers gone to a cricket match. It also might reflect the careful diction and exposition of a teacher with an entire lesson very easily stored in his head. The reason given for this type of regularity has to do with the grading of tenth and twelfth grade exams, which are done by hand.

What seems striking about this example, besides the lack of student attendance, is the general atmosphere created by the teacher, even at short notice. The teacher is prepared and, in presenting the material, gives a sense of orderliness, discipline, and competence. Even with few students in the classroom, the class time among those attending was taken seriously. No mistakes were made, and the lesson was given with textbook clarity.

Example 2

The next example comes from an urban ninth grade classroom learning geometry (the curriculum is integrated). The school was housed inside a complex that was the residence for scientists working at a particular scientific institution, so most of the children in this school came from households with at least one highly educated parent.

In this class, the teacher was proving the claim that the parallelogram has the same area as the rectangle. She drew the following picture:

She proved the claim by showing the two triangles were congruent. Her proof was very clear and easy to follow. Afterwards the researcher asked the students how many felt they could reproduce this proof if asked to do so later. All of them raised their hands. Then she asked how many of them believed that the two triangles were congruent *before* the teacher showed them the proof. Few raised their hands. They were then asked how many would have thought of adding the extra triangle to make the link between the parallelogram and rectangle? Again few raised their hands.

This example illustrates what was evident in a number of schools in this study. Students were very good at solving problems they had seen before. They were not very good at the creative thinking needed to answer problems that deviated only slightly from the familiar. However, students were quite eager for new questions and information when they were presented. In this case, students had not thought about the relationship between the rectangle and parallelogram before, but they seemed genuinely happy to learn about the connection when the opportunity arose. The teacher asked the researcher to go to the board to give another presentation of the material,[4] and students were nearly falling out of their chairs to take part in the discussion.

What seems striking about this example is the behavior of the students. They were curious, engaged, and

[4] This was also typical—many teachers saw the presence of a researcher as a resource for the class. The teaching schedule was fairly flexible to allow for this kind of spontaneous lesson.

hungry. While the teacher herself did not exploit this energy during her presentation, it took very little from a visitor to spark good, rich mathematical discussion.

Example 3

The third example comes from a rural school that, without much funding, provides a Montessori-like education with simple tools such as chalk and bottle caps. Children in the countryside, as young as five years old, are sometimes charged with taking care of their younger siblings. At this school, the younger siblings were invited so that the older ones could get an education. Activities here included counting with base ten sticks (where sticks were literally sticks), finding all combinations of positive integers that sum to ten (Figure 5), and a game with bottle caps where a certain number of them were thrown and children counted how many landed with their tops up. The open-air campus allowed students to go from activity to activity at their own pace, and there were also whole class activities coordinated by a teacher.

What was most striking about this school was the relative peace and freedom to explore different activities. The school day was structured with a balance of whole group activities and individual time. Students took the individual time seriously and were eager to discuss what they were doing with the visiting adults. Even though it was sometimes difficult to handle the social context, for instance the presence of younger siblings, there was a sense of order and balance, where children had the opportunity to learn based on very simple, but effective, educational tools.

Summary of Examples

We have seen above a number of challenges that can affect the mathematical education in Mumbai schools. A cricket match can drain the student population, opportunities for creative thinking might be limited, and the poverty of rural areas might seem like a threat to providing education to children, especially when they are taxed with taking care of younger siblings. However, in these cases, it seems a number of positive characteristics dominate over the negative ones. These characteristics are summarized in the following table:

	Positive Characteristics		
Example 1	Discipline	Orderliness	Competence
Example 2	Curiosity	Engagement	Hunger
Example 3	Balance	Freedom	Simple innovation

These words describe the atmosphere created in each of the classrooms, which combined the work of the teacher, the abilities of the students, and the classroom context, which included the structure of the space, availability of materials, and curriculum. These characteristics resonate with the more positive descriptions of high achievement that reflect what appears to be working well in Indian mathematics education.

The Larger Context

We have now seen some examples of the complexity in describing high achievement in Indian mathematics education. Next, we look at some of the factors that have contributed to this complexity. According to Krishna Kumar, the head of the National Center for Educational Research and Training (NCERT), two of the biggest factors are the opposing forces of mass examination and early selection. Mass education, as shown in a fairly good light in the examples above, is supposed to provide opportunity for all students, even if some of those students are out watching a cricket game or have their younger siblings with them. Early selection, which allows some students to get a better education than others, puts pressure both on schools and individuals for high achievement. Kumar (1991) writes:

> While private schools, featuring early selection, ensure that the elite sections of society have a means to provide privileged treatment to their children, mass examination, featuring strict secrecy and parity among examinees, keep the confidence and aspirations of the masses alive…. Early selection and mass examination are two conflicting characteristics that together endorse the legitimacy of the education system. (p. 32)

This tension between early selection and mass examination seemed to play out in all the schools in this study. There were three-year-olds eagerly lined up at desks and in matching uniforms, practicing the numbers one to five, so that they would enter school knowing how to sit at their desks. There were second graders falling off their chairs to answer questions. There were ninth graders coming to a voluntary mathematics class after school to practice problem solving. There were slum children from a middle school who said their favorite subject was mathematics. Why? these students were asked. Because it makes us think, they said. In some ways the promise of high achievement in mathematics dominates over the threats.

India is now moving toward a child-centered curriculum. NCERT (2005) has created new curriculum guidelines including: connecting knowledge to life outside school, ensuring that learning is shifted away from rote methods, and enriching the curriculum to provide for overall development of children rather than remain textbook-centric (p. 5). Mathematics will have more visualization and problem solving (p. 43). It will take some time before the effect of these changes on classroom teaching or students' understanding of mathematics is known, but the fact that the government is even considering such major changes seems like a strong commitment toward providing high quality education for all students.

High achievement in India may in some ways be a myth, in the sense that there are still large inequities in the country, but it still might be possible to learn from the good examples and the good practices found there. Mathematics education in India provides an example of trying to provide quality education in the largest democracy in the world. This task is difficult and complex, but perhaps because of that complexity, we have an unusually large window through which we can study the ways in which different educational situations play out. Through the examples, we have tried to present a balanced account of the ways in which Indian mathematics education currently supports or hinders high achievement. Myth or no myth, this kind of balanced account seems essential for really understanding high achievement, not only in India, but in other countries as well.

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