Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A Century of Leadership in Mathematics and its Teaching

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The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology.*

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Journal of Mathematics Education at Teachers College

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the C*onsortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* "cognitive category" of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a "disposition to mathematize," that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory's Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher's notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a "disposition" toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

Notes from the Curriculum Laboratory begins with a brief view of the Laboratory's interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory's modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

where to lower a diver so that he can salvage sunken treasure from various sites. Although the context is markedly different from that of going shopping, students should be aware of the flexibility of their mathematical models.

Modeling provides a unique opportunity to identify real-life problems and questions and investigate them mathematically. At present, the dearth of materials for mathematical modeling amplifies the need for capable instructors and curriculum designers. Through a combination of creativity and flexibility, teachers can work alongside students to explore and expand upon issues encountered in daily life, illuminating underlying mathematical structures in the process. In this way, we hope to instill in students the belief that they can use formal mathematics to great effect well beyond the classroom walls.

References

Common Core State Standards Initiative (2010). *Common Core State Standards for Mathematics.* Retrieved from http://www.corestandards.org/the-standards. (30 March 2011).

de Villiers, M. (2009). *From the Fermat point to the De Villiers points of a triangle.* Retrieved from http://math.kennesaw.edu/~mdevilli/devillierspoints.pdf. (30 March 2011).

Meteorology: Describing and Predicting the Weather—An Activity in Mathematical Modeling

Heather Gould
Stone Ridge, New York

Websites such as weather.com usually don't give the actual current temperature at your location—it's an educated estimate! This modeling activity begins with students exploring the distribution of temperatures across a "map" and ends with the students using mathematical models to estimate the temperature at any given point on that map.

On the first day of the two-day modeling activity intended for algebra classes, students are asked to consider a map with points at which the temperature is known (known-temperature points) and use the map and their experience with temperature to determine how temperature changes over distance from known-temperature points. They should conclude that the temperature changes linearly. The challenge arises when they must find a way to estimate temperature at a point *not* collinear with any two other points at which the temperature is known (Figure 1). Creative students will solve this problem employing a variety of methods, but each must be based on the assumption of linearity. To determine students' understanding, they are asked to use the model to check if there will be freezing rain along a given route.

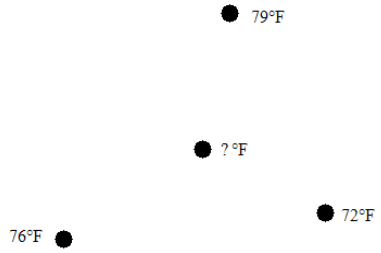

Figure 1

On the second day, students are encouraged to use the graph of a linear function to model the same situations as in the previous day. By doing this, students learn about the properties of linear functions and their graphs. Students who chose to use this method will deepen their understanding and students who chose to use different models are able to make connections between their model and linear functions.

Teachers who employ modeling activities will be surprised at the variety of methods and models that students create, as well as the wealth of discussions that they cause. For example, in determining how the temperature changes with respect to distance, students use their experience to determine that temperature change must be continuous: it is unreasonable to assume that there might be a sudden "jump" in temperature or there might be a "hole" in which no temperature exists. This helps students understand the definition of mathematical continuity. Debate may result during the process of identifying the rate at which temperature changes over a distance: such discussion gives students the opportunity to explore characteristics of various functions. Students should conclude that a function with a constant rate of change—a linear function, as they will learn—is the *most reasonable* function to begin modeling temperature change. This may change once the model has been constructed and tested, as students may find it necessary to refine the linear model, particularly when applying it to the real world; in doing so, students learn to consider which variables to ignore for the sake of ease and the particular constraints of their model. They learn that models can be refined to include these variables and constraints once a preliminary model is produced and tested.

Further discussion should address the different methods used when the point at which the temperature needing to be estimated is not collinear with two known-temperature points. Two examples of models students may create are shown in Figure 2. On the left, a student uses the concept of linearity to approximate the temperature twice:

the first approximation is the point of intersection of the line through two known-temperature points and the second uses the line through the remaining known-temperature point and the point in question. On the right, the student approximates the temperature at the midpoints of the triangle. Another triangle is created and the student continues to use midpoint approximations for the triangle containing the point in question until a reasonably accurate estimate is found. Both methods result in similar answers (depending on the students' attention to accuracy), are valid, and use the concept of linearity. This gives the students a chance to experience the idea that there is usually no one "right" way to solve a mathematics problem and each has its own merits and demerits.

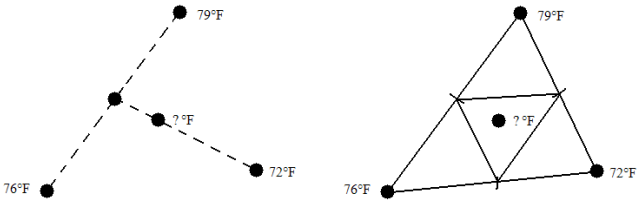

Figure 2

Mathematical modeling should be used in a classroom to teach students that mathematics can be found in many real-world situations; the mathematics necessary to solve a problem within that situation already may be known or new methods may need to be invented. Mathematical modeling activities like the one described here will help a student discover how mathematics is developed and how innovations occur.

Packing Oranges

Kai Chung Tam
Macau, PRC

We know how storekeepers stack oranges in a nice way (Figure 1), but Kepler asked if this is indeed the most efficient way and if there are no other competitive methods[1]? Being inspired by Kepler's problem of sphere packing, we will compare the efficiency of different ways of packing by looking at how many oranges are contained in a box of a certain size. How do we count the number of oranges? Theoretically, someone counts one-by-one until there are no more oranges. But when the amount is large, other techniques are needed to pursue the answer. Here is an important aspect of mathematical modeling: real-world situation comes first, the mathematics follows naturally. When Archimedes wanted to figure out the number of grains of sand that would fit in the universe, he came up with the idea of using repeated multiplication as a simpler way to represent very large numbers[2].

Figure 1. Stack of oranges leads to sphere packing problem

In the design of *Packing Oranges*, I use several scenarios in which there is potential mathematical content about packing. Here are the scenarios:

> **Scenario 1:** Sam shows to the guests a full box of randomly arranged oranges, stating that anyone who is able to figure out the right number of oranges can take away as many oranges as s/he would like. Well, is it just a gimmick? Or by what means can we estimate quickly?
>
> **Scenario 2:** Wait. How does Sam know the exact number of oranges? If not, can he be cheating? Sam is glad to tell us how he determined the number. He basically knows the total weight of the box of oranges (although the guests don't know) and so he can divide the total by the weight of one orange.
>
> **Scenario 3:** Suppose that we have successfully won the prize. How can we take away as many oranges as possible? Sam offers some (other, not huge) boxes that you can use, and the problem becomes not only counting, but also what kind of arrangement allows the most oranges in a certain container.

A variety of mathematical tasks go along with these scenarios. For example, using Scenario 1, students can begin by thinking about a simpler problem. Two-dimensional experiments can be used to verify a method of efficient counting, based on a division of the total area by the area of one circle. This method has an obvious flaw in that it does not count the spaces in between the circles; students are meant to discover this flaw and suggest an improvement based on the experimental results and

[1] George Szpiro (2003) wrote on this topic a monograph for the general audience. *Kepler's Conjecture*. John Wiley & Sons, Inc., Hoboken.

[2] Heather Hasan (2006), *Archimedes: The Father of Mathematics*. Rosen, New York.

SEARCH EXTENDED

Teachers College Columbia University
Department of Mathematics, Science, and Technology

MATHEMATICS EDUCATION FACULTY VACANCY

Teachers College invites applicants for a faculty position in mathematics education. A successful candidate will have an earned doctorate in mathematics or mathematics education at the time of appointment and a demonstrated ability to pursue an active research agenda in a significant area of mathematics education. Applicants should be qualified to teach graduate-level courses in at least two areas of mathematics and in two or more areas of mathematics education. Preference will be given to candidates with prior experience teaching school mathematics. All candidates are expected to demonstrate an ability to establish a research program and a potential to obtain external funding. Candidates also are expected to establish a record of continuous, scholarly productivity and a record of leadership. Minority candidates are strongly encouraged to apply.

Rank: Open Rank, Tenure Track

Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

Teachers College as an institution is committed to a policy of equal opportunity in employment. In offering education, psychology, and health studies, the College is committed to providing expanding employment opportunities to persons of color, women, and persons with disabilities in its own activities and in society.

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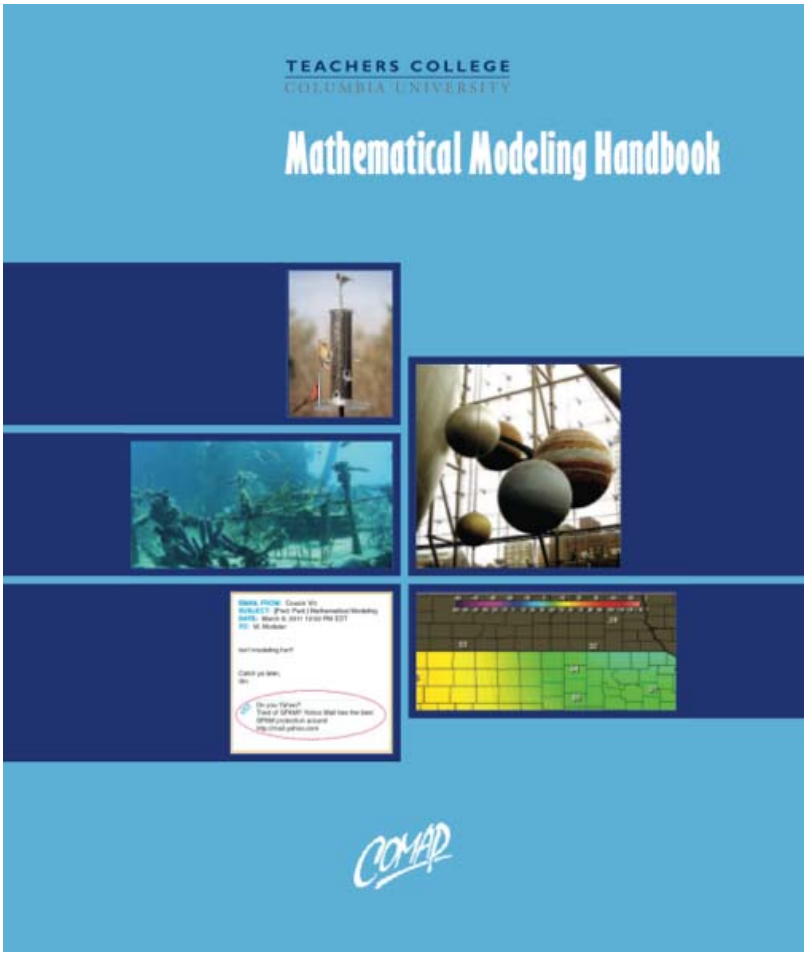

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