# Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A Century of Leadership in Mathematics and its Teaching

# TABLE OF CONTENTS

## Foreword

**iv** **Honoring the Past—Anticipating the Future**
*J. Philip Smith, Bruce R. Vogeli, Erica Walker*

## Preface

**v.** **Mathematics Curricula: Standards and Implementation**
*Nicholas H. Wasserman*

## Editorial Point-Counterpoint

**6** **Will Common Core State Standards facilitate consistency and choice or lead to unexpected outcomes?**
*Nicholas H. Wasserman and Jacob Koelher*

## Articles

**8** **Slouching Toward a National Curriculum**
*Jeremy Kilpatrick, University of Georgia*

**18** **The Common Core State Standards: Comparisons of Access and Quality**
*Nicholas H. Wasserman, Marymount School of New York*

**28** **Modeling in the Common Core State Standards**
*Kai Chung Tam, Macau, PRC*

**34** **Reformed Curriculum Framework: Insights from Teachers' Perspectives**
*Shikha Takker, Homi Bhabha Centre for Science Education, TIFR*

**40** **From Curriculum Guides to Classroom Enactment: Examining Early Career Elementary Teachers' Orientations Toward Standards-Based Mathematics Curriculum Implementation**
*Joan Gujarati, Manhattanville College*

**47** **Design Research in the Netherlands: Introducing Logarithms Using Realistic Mathematics Education**
*David C. Webb, University of Colorado at Boulder*
*Henk van der Kooij, Freudenthal Institute for Science and Mathematics Education University of Utrecht, The Netherlands*
*Monica R. Geist, Front Range Community College Westminster, Colorado*

**53** **Using Simplified Sudoku to Promote and Improve Pattern Discovery Skills Among School Children**
*Khairul A. Tengah, Universiti Brunei Darussalam*

# TABLE OF CONTENTS


**63 NOTES FROM THE CURRICULUM LABORATORY**
*Bruce R. Vogeli*

**What is Mathematical Modeling?**
*Henry O. Pollak*

**Modeling Lessons and the Common Core State Standards**
*Benjamin Dickman, Brookline, Massachusetts*

**Meteorology: Describing and Predicting the Weather—An Activity in Mathematical Modeling**
*Heather Gould, Stone Ridge, New York*

**Packing Oranges**
*Kai Chung Tam, Macau, PRC*

**Arithmetic and Algebra to Solve Fairness Problems**
*Joseph Malkevitch, York College*

**Finding Average Rainfall**
*Stuart Weinberg, Teachers College Columbia University*

**The Buckyball Has Relatives: A Classroom Approach to Polyhedra**
*Anahu Guzman, LIM College*

**Other**

**72 ABOUT THE AUTHORS**

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.



This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology.*

*JMETC* readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

**Manuscript Submission**

*JMETC* seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu



**Library of Congress Cataloging-in-Publication Data**

Journal of mathematics education at Teachers College
p. cm.
Includes bibliographical references.
ISSN 2156-1397
EISSN 2156-1400
1. Mathematics—Study and teaching—United States—Periodicals
QA11.A1 J963

**More Information is available online:** www.tc.edu/jmetc

### Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

### Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### Looking Ahead

Anticipated themes for future issues are:

| | |
|---|---|
| Fall 2011 | Technology |
| Spring 2012 | Evaluation |
| Fall 2012 | Equity |
| Spring 2013 | Leadership |
| Fall 2013 | Modeling |
| Spring 2014 | Teaching Aids |

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To obtain additional copies of *JMETC*, please visit the *Journal*’s website www.tc.edu/jmetc. The cost per copy delivered nationally by first class mail is $5.00. Payment should be sent by check to *JMETC*, Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

# NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the C*onsortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* "cognitive category" of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a "disposition to mathematize," that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory's Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher's notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a "disposition" toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

*Notes from the Curriculum Laboratory* begins with a brief view of the Laboratory's interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory's modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

conceptual thinking. Another experiment based on Scenario 2 can be done by weighing real objects, which is not hard to achieve if a digital scale is available.

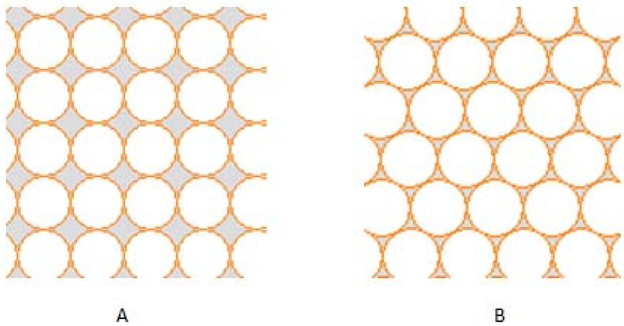


Figure 2. Two regular arrangements in 2-d

In Scenario 3, a winner can take as many oranges as possible, so it is natural to ask: what is the maximum number? Such a question provokes the thinking about efficient ways of packing oranges, and also the definition of packing density. They can be randomly or regularly arranged in the box. This lesson focuses mainly on the ways of regular arrangements (Figure 2), for which the counting becomes a geometry exercise suitable for high school students[3]. However, an interesting relationship can be explored between if carefully packed oranges are significantly more efficient than randomly packed oranges. In particular, do the "benefits" for packing outweigh the "costs" in time? Regular arrangements can be defined as those that repeat periodically in every direction, causing the density of the arrangement to be well-defined also. This is where important mathematization takes place. In reality, boundaries of containers prevent any perfectly regular arrangements, but students might realize that the differences are small. One observation for this lesson may be that students notice that the difference is even more negligible for larger containers, which can serve as an introduction or reinforcement to the concept of limit. The difficulties involved in packing oranges can produce interesting discussions of significant mathematics based on the various mathematical models used.

*Note*: This lesson is designed to match four CCSS modeling standards in Geometry (G-GMD.3, and G-MG.1,2,3), although the concept of modeling could be broadened so that Geometry itself becomes a set of modeling activities. Indeed, as will be exemplified in the lesson, Geometry can be seen as a model containing objects and their spatial relationships idealized from the physical world.

[3] If we neglect the mathematical difficulties of probability theory, we can use simulation programs to demonstrate the packing density. Just as there is not a unique way to arrange oranges regularly, there are also different ways to randomize packing, and the resulting density depends quite strongly on the randomization (cf. *Science Magazine* **303**, page 968). This idea could be developed as a "follow-up" modeling lesson.

# Arithmetic and Algebra to Solve Fairness Problems

Joseph Malkevitch
York College

One big concern all of us have is being treated fairly. Not only do fairness issues arise in our daily lives, but they are a big aspect of the news—ranging from fair tax systems to whether disaster relief money is distributed fairly after an emergency such as the current earthquake/nuclear crisis in Japan.

Here is an applied problem which leads to a system of linear equations and provides students with the opportunity to connect mathematics with something that may arise in their daily lives or which they may see in the news. The problem may help motivate students in wanting to master the algebra skills needed to solve systems of equations.

Suppose that there are disaster relief funds available amounting to \$140. (I will use relatively small numbers but the problem can be scaled so that one talks about \$140,000 or \$1,400,000 instead.) For simplicity assume there are exactly two claimants, A and B, whose claims have been verified to be \$150 and \$60, respectively. We will assume that there is a "judge" who must distribute the \$140 whose work is paid for with money other than the \$140, that all the money is going to be distributed to the claimants, and that these are the honest amounts that are due them. How much money should be given to A and B?

To answer a question of this kind one has to apply some principles of what is "fair." One notion of being fair is that everyone be treated equally. It is this reasoning that is behind "one person, one vote." Rich people, middle income people, poorer people, highly educated and uneducated people are all treated equally. However, in this example if we merely give half of the amount available to each claimant, \$70, we give B more than he requested (was entitled to). In light of this, back in the Middle Ages, the scholar Maimonides suggested the idea that "gains" be distributed to claimants as close to equal as possible but not exceeding the claim. This would mean giving each claimant \$60, but using the remaining \$20 (\$140 – \$120) to add to what is given to A. The final settlement would be \$80 to A and \$60 to B.

Another notion is that claimants owed larger amounts should be compensated more than claimants with smaller claims, which will not typically happen when all claimants are treated equally. This leads to the idea of settling claims on a proportional basis. In this example the total claims are \$210. So we can compensate A with (150/210)(\$140) and B with (60/210)(\$140). This means \$100 for A and \$40 for B.

Maimonides's method of gain described above gives B all of his claim back but this would not be true for A, who is out \$70. So a different principle of fairness, also going back in part to Maimonides, says that the settlement should attempt to equalize the losses to the claimants. Here is how to solve this type of problem using the algebra involved in solving two equations in two unknowns.

Suppose we give amounts $a$ to A and $b$ to B so as to equalize the losses that A and B suffer. Since we have \$140 to distribute, we have the equation $a + b = 140$. Since A's loss will be $150 - a$ and B's loss will $60 - b$, we equate these two algebraic expressions: $150 – a = 60 - b$. This simplifies to $-a + b = -90$. Now we need to solve the system $a + b = 140$ and $-a + b = -90$. The solution of the system is: $a = \$115$ and $b = \$25$. A loses \$35 and B loses \$35, equal amounts! It is not always possible to find values of $a$ and $b$ that equalize loss in similar situations. It is interesting to think through what the algebra "tells us" when this is the case. Note that there is much mathematical food for thought here because different fairness principles yield different answers, and all of these ideas need to be extended to situations with more claimants.

*References*

Young, H., Fairness, Princeton U. Press, 1995.

# Finding Average Rainfall

Stuart Weinberg
Teachers College Columbia University

Planning by a community includes predicting the availability of water resources. In this modeling activity, students will find an approximation for average rainfall and total volume of water based on readings from rainfall gauges. Lesson outcomes include the realization that the placement of gauges must be considered and a weighted average utilized. In the course of the lesson, students will have an opportunity to make a connection between the construction of the perpendicular bisector of a line segment and finding the solution to a real world problem.

Problem: Estimate the rainfall during a one-week period of time for a rectangular-shaped territory within the state of Rajasthan. Dimensions of the territory are 16 km by 18km. The estimate will be based on the readings of three rain gauges scattered around the territory.

Assume the readings are as follows:

| Gauge | Rainfall (depth) |
|---|---|
| A | 12.6 mm |
| B | 13.4 mm |
| C | 10.8 mm |

Answer the following questions:

- What is the average depth of rainfall for the territory?
- Using the estimate of average depth, what is the total volume of rainfall?
- Do we need additional information to answer either or both of the two questions?

Development

Generally, students can be expected to find the mean of 12.6, 13.4, 10.8. Therefore, the goal of the lesson is to elicit the idea that a weighted average based on the locations of the gauges is necessary.

Ask students: Suppose the readings (depths) for A, B, and C are 2, 2, and 5mm, respectively and a diagram or map is displayed showing A and B in close proximity to each other:

A ·                      · C
B ·

Again, students can be expected to find the mean and suggest that the average depth is 3.

Next, ask students to consider the following layout for rainfall gauges:

A ·                      · C
· B

Now suppose that the readings for A and C are 2 and 5mm, respectively. What can we expect the reading for gauge B to be? Because rainfall is known to be a local phenomenon, students are likely to respond with "5." But this would give us an average rainfall of 4 mm, which is different from the earlier mean of 3.

So, we elicit from the class that the placement of the gauges matters.

Working in small groups, students will be provided with a handout (Figure 1) showing the locations of the three rainfall gauges. Groups will be asked to use the data and find a "sensible" way to approximate average rainfall—one that considers gauge locations.

Following presentations of methodology by groups and perhaps building on one or more of the methodologies, the teacher will initiate a discussion that leads to the following solution:

> We divide the territory into regions. The number of regions will equal the number of gauges and defined so that each point in the region is closer

SEARCH EXTENDED

**Teachers College Columbia University**
**Department of Mathematics, Science, and Technology**

# MATHEMATICS EDUCATION FACULTY VACANCY

Teachers College invites applicants for a faculty position in mathematics education. A successful candidate will have an earned doctorate in mathematics or mathematics education at the time of appointment and a demonstrated ability to pursue an active research agenda in a significant area of mathematics education. Applicants should be qualified to teach graduate-level courses in at least two areas of mathematics and in two or more areas of mathematics education. Preference will be given to candidates with prior experience teaching school mathematics. All candidates are expected to demonstrate an ability to establish a research program and a potential to obtain external funding. Candidates also are expected to establish a record of continuous, scholarly productivity and a record of leadership. Minority candidates are strongly encouraged to apply.

**Rank:** Open Rank, Tenure Track

**Send** CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

*Teachers College as an institution is committed to a policy of equal opportunity in employment. In offering education, psychology, and health studies, the College is committed to providing expanding employment opportunities to persons of color, women, and persons with disabilities in its own activities and in society.*

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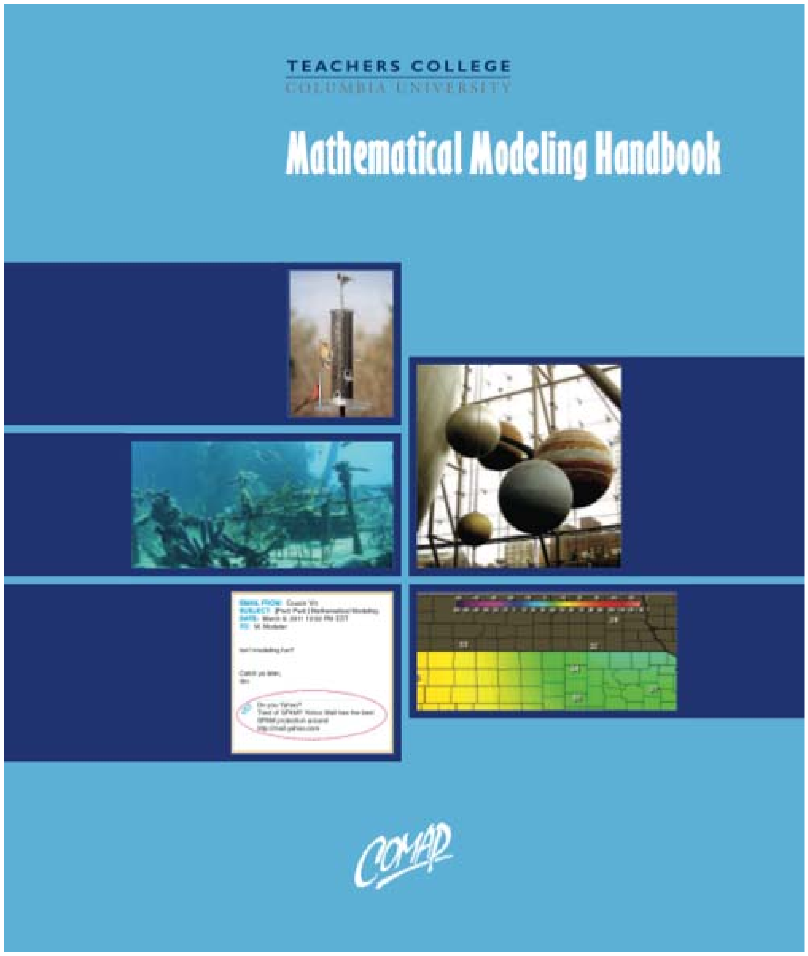


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