

Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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The *Journal of Mathematics Education at Teachers College* is a publication of the
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This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Roszkopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Roszkopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETS@tc.columbia.edu

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Journal of Mathematics Education at Teachers College

Call for Papers

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue’s guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the *Consortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* “cognitive category” of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a “disposition to mathematize,” that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory’s Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher’s notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a “disposition” toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

Notes from the Curriculum Laboratory begins with a brief view of the Laboratory’s interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory’s modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

NOTES FROM THE CURRICULUM LABORATORY

on a proportional basis. In this example the total claims are \$210. So we can compensate A with $(150/210)(\$140)$ and B with $(60/210)(\$140)$. This means \$100 for A and \$40 for B.

Maimonides's method of gain described above gives B all of his claim back but this would not be true for A, who is out \$70. So a different principle of fairness, also going back in part to Maimonides, says that the settlement should attempt to equalize the losses to the claimants. Here is how to solve this type of problem using the algebra involved in solving two equations in two unknowns.

Suppose we give amounts a to A and b to B so as to equalize the losses that A and B suffer. Since we have \$140 to distribute, we have the equation $a + b = 140$. Since A's loss will be $150 - a$ and B's loss will be $60 - b$, we equate these two algebraic expressions: $150 - a = 60 - b$. This simplifies to $-a + b = -90$. Now we need to solve the system $a + b = 140$ and $-a + b = -90$. The solution of the system is: $a = \$115$ and $b = \$25$. A loses \$35 and B loses \$35, equal amounts! It is not always possible to find values of a and b that equalize loss in similar situations. It is interesting to think through what the algebra "tells us" when this is the case. Note that there is much mathematical food for thought here because different fairness principles yield different answers, and all of these ideas need to be extended to situations with more claimants.

References

Young, H., *Fairness*, Princeton U. Press, 1995.

Finding Average Rainfall

Stuart Weinberg

Teachers College Columbia University



Planning by a community includes predicting the availability of water resources. In this modeling activity, students will find an approximation for average rainfall and total volume of water based on readings from rainfall gauges. Lesson outcomes include the realization that the placement of gauges must be considered and a weighted average utilized. In the course of the lesson, students will have an opportunity to make a connection between the construction of the perpendicular bisector of a line segment and finding the solution to a real world problem.

Problem: Estimate the rainfall during a one-week period of time for a rectangular-shaped territory within the state of Rajasthan. Dimensions of the territory are 16 km by 18km. The estimate will be based on the readings of three rain gauges scattered around the territory.

Assume the readings are as follows:

Gauge	Rainfall (depth)
A	12.6 mm
B	13.4 mm
C	10.8 mm

Answer the following questions:

- What is the average depth of rainfall for the territory?
- Using the estimate of average depth, what is the total volume of rainfall?
- Do we need additional information to answer either or both of the two questions?

Development

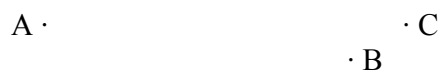
Generally, students can be expected to find the mean of 12.6, 13.4, 10.8. Therefore, the goal of the lesson is to elicit the idea that a weighted average based on the locations of the gauges is necessary.

Ask students: Suppose the readings (depths) for A, B, and C are 2, 2, and 5mm, respectively and a diagram or map is displayed showing A and B in close proximity to each other:



Again, students can be expected to find the mean and suggest that the average depth is 3.

Next, ask students to consider the following layout for rainfall gauges:



Now suppose that the readings for A and C are 2 and 5mm, respectively. What can we expect the reading for gauge B to be? Because rainfall is known to be a local phenomenon, students are likely to respond with "5." But this would give us an average rainfall of 4 mm, which is different from the earlier mean of 3.

So, we elicit from the class that the placement of the gauges matters.

Working in small groups, students will be provided with a handout (Figure 1) showing the locations of the three rainfall gauges. Groups will be asked to use the data and find a "sensible" way to approximate average rainfall—one that considers gauge locations.

Following presentations of methodology by groups and perhaps building on one or more of the methodologies, the teacher will initiate a discussion that leads to the following solution:

We divide the territory into regions. The number of regions will equal the number of gauges and defined so that each point in the region is closer

to its gauge than any other gauge. (Having students “construct” this definition of region by calling on relevant geometry is a key feature of the lesson.) We weight the rainfall measured at each region’s gauge according to the portion of the territory’s area in that region. The estimate of rainfall for the territory is the weighted average of the rainfall readings.

In finding the rainfall estimate, we have created a Voronoi diagram in which the boundaries of regions are determined from their centers of influence; in this problem, centers are the rainfall gauges. Every Voronoi boundary is a portion of the perpendicular bisector of a segment joining two centers.

This lesson can lead to a consideration of various methods for computing area of polygonal and other figures. (One possible method is to superimpose on the “map” of the territory a transparency with a coordinate grid.)

Question to pose to the class after finding the weighted average: How does the Voronoi diagram change if we add a fourth gauge D? Let us examine a couple of different locations, including 1) D in the interior of triangle ABC and 2) D in the exterior of triangle ABC.

Another kind of problem arises when we are given a Voronoi diagram and wish to determine the centers of the Voronoi regions, something to be considered in subsequent lessons.

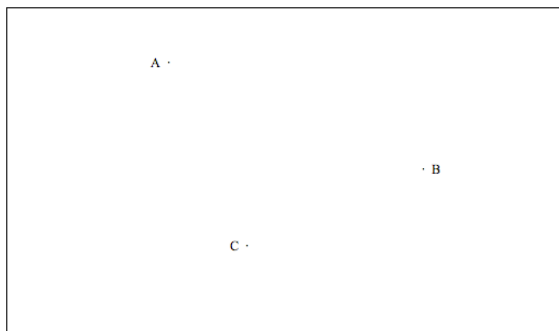


Figure 1. Territory map with rain gauges

References

The Consortium for Mathematics and Its Application (COMAP) (1998). *Mathematics: Modeling Our World*. Cincinnati, OH: South-Western Educational Publishing.

The Buckyball Has Relatives: A Classroom Approach to Polyhedra



Anahu Guzman
LIM College



The construction of fullerene models using the following group activity is an opportunity to foster creativity and collaboration and to introduce some basic graph theory definitions in an enjoyable way. Fullerenes are a family of carbon-cage molecules in the form of a hollow sphere, ellipsoid, or tube. Spherical fullerenes are also called buckyballs. From a mathematical point of view, a fullerene can be thought of as a convex 3-dimensional polyhedron with 3 edges at each vertex, and faces consisting of 12 pentagons and k hexagons ($k \neq 1$).

Number of Group Members: 3

Required materials:

- 1 pack of 2.5 inch assorted color rubber bands
- 1 12 inch or larger balloon
- 1 small pack of dental floss

Begin the discussion by examining a soccer ball; notice the pentagonal and hexagonal faces. Two faces share each edge and every 3 faces share a common point (vertex). A soccer ball resembles the “Buckyball,” one of the most famous fullerenes (C60). Since fullerene graphs are 3-valent (3 edges at every vertex), any circuit (begins and ends at the same vertex, alternating between different vertices and edges) will use two of the edges at every vertex of the circuit, which allows use of the missing edge to code the circuit using binary numbers. Code circuits using 0 for edges that point in and 1 for edges that point out of the circuit. A shell diagram associated with the code $(0011)^3(01) = 00110011001101$ is shown in Figures 1 and 2. The outer part of the circuit can be thought of as the “icing” with faces and the inner as the “filling” with faces (Figure 3).

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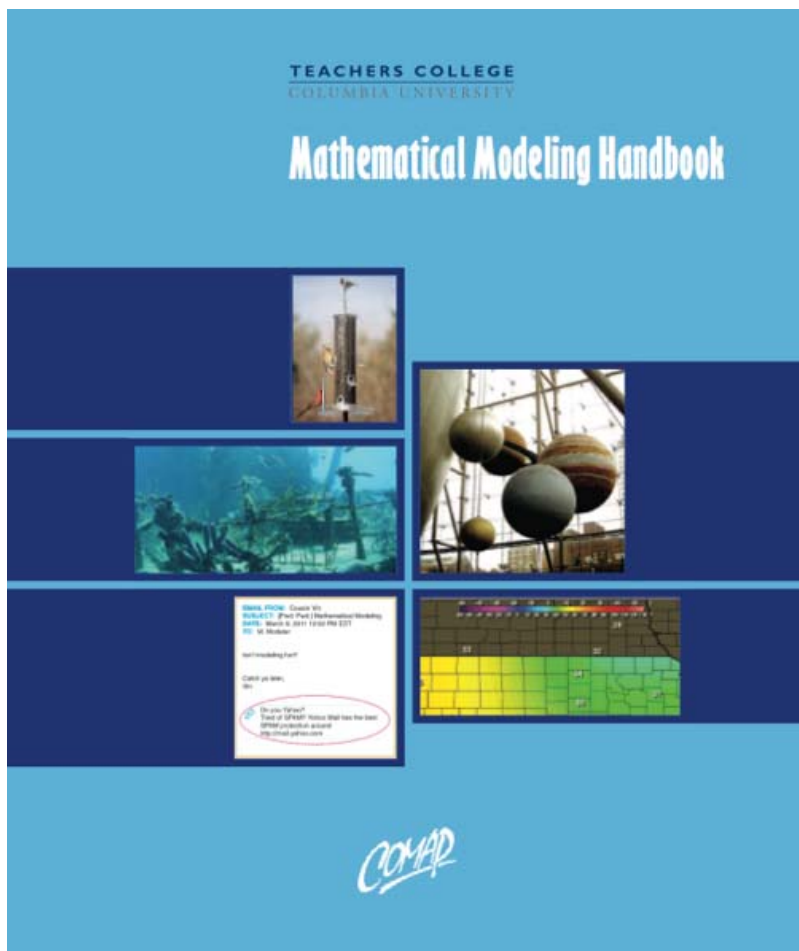
Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

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