Journal of Mathematics Education at Teachers College

Spring – Summer 2011

A Century of Leadership in Mathematics and its Teaching

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The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

This issue honors Clifford B Upton who was a senior member of the Teachers College faculty from 1907 until his retirement in 1942. Professor Upton was among the Nation's most prolific mathematics authors. He served on the Board of Directors of the American Book Company enabling him to endow the Clifford Brewster Chair of Mathematics Education. The first professor to hold the Upton Chair was Dr. Myron Rosskopf.

Bruce R. Vogeli has completed 47 years as a member of the faculty of the Program in Mathematics, forty-five as a Full Professor. He assumed the Clifford Brewster Chair in 1975 upon the death of Myron Rosskopf. Like Professor Upton, Dr. Vogeli is a prolific author who has written, co-authored or edited more than two hundred texts and reference books, many of which have been translated into other languages.

This issue's cover and those of future issues will honor past and current contributors to the Teachers College Program in Mathematics. Photographs are drawn from the Teachers College archives and personal collections.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The theme planned for the 2011 Fall-Winter issue is: *Technology*.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Technology*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to technology in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2011. The fall issue's guest editor, Ms. Diane Murray, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2011, and final drafts of selected papers are to be submitted by November 1, 2011. Publication is expected in late November, 2011.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2011 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2011	Technology
Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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NOTES FROM THE CURRICULUM LABORATORY

The Curriculum Laboratory associated with the Teachers College course MSTM 6022: *Mathematics Curriculum Development* joined with the C*onsortium on Mathematics and its Applications* (COMAP) to address the *Mathematical Modeling* "cognitive category" of the *Common Core State Standards* (CCSS). While many of the CCSS recommendations addressed familiar cognitive categories such as Number and Quantity, Algebra, and Geometry, the category of *Mathematical Modeling* is unfamiliar to many educators. Indeed, mathematicians differ in the interpretations of mathematical modeling and mathematics educators are unsure of how to teach the modeling process, often confusing it with problem solving.

Participants in the 2010-2011 Curriculum Laboratory interpret mathematical modeling as a "disposition to mathematize," that is, the recognition of opportunities to portray real world events and situations in mathematical form. To actualize this interpretation for schools and teachers, Laboratory participants prepared the thirty mathematical modeling lessons that comprise the *Teachers College Mathematical Modeling Handbook* published by COMAP.

The Laboratory's Board of Editors, Heather Gould, Diane Murray, and Andrew Sanfratello, guided the preparation of these notes from the Curriculum Laboratory. While the actual lessons that appear in the COMAP publication are complete with teacher's notes, black-line masters, answers and extensions, the JMETC Notes are abbreviated descriptions that focus upon the goal of creating a "disposition" toward mathematization. These notes illustrate how a mathematical disposition can be achieved utilizing everyday real-world artifacts such as weather maps, parking, rainfall estimates, fairness, and packing oranges.

Notes from the Curriculum Laboratory begins with a brief view of the Laboratory's interpretation of mathematical modeling contributed by Dr. Henry O. Pollack, followed by descriptions of some of the Laboratory's modeling lessons. For complete details and teaching materials for all thirty (30) Handbook lessons, please consult the COMAP publication or visit the online version at www.comap.com/NCTM.html.

Bruce R. Vogeli

the first approximation is the point of intersection of the line through two known-temperature points and the second uses the line through the remaining known-temperature point and the point in question. On the right, the student approximates the temperature at the midpoints of the triangle. Another triangle is created and the student continues to use midpoint approximations for the triangle containing the point in question until a reasonably accurate estimate is found. Both methods result in similar answers (depending on the students' attention to accuracy), are valid, and use the concept of linearity. This gives the students a chance to experience the idea that there is usually no one "right" way to solve a mathematics problem and each has its own merits and demerits.

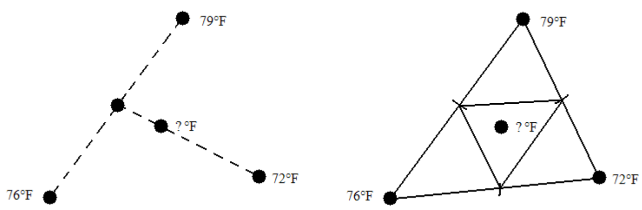

Figure 2

Mathematical modeling should be used in a classroom to teach students that mathematics can be found in many real-world situations; the mathematics necessary to solve a problem within that situation already may be known or new methods may need to be invented. Mathematical modeling activities like the one described here will help a student discover how mathematics is developed and how innovations occur.

Packing Oranges

Kai Chung Tam
Macau, PRC

We know how storekeepers stack oranges in a nice way (Figure 1), but Kepler asked if this is indeed the most efficient way and if there are no other competitive methods[1]? Being inspired by Kepler's problem of sphere packing, we will compare the efficiency of different ways of packing by looking at how many oranges are contained in a box of a certain size. How do we count the number of oranges? Theoretically, someone counts one-by-one until there are no more oranges. But when the amount is large, other techniques are needed to pursue the answer. Here is an important aspect of mathematical modeling: real-world situation comes first, the mathematics follows naturally. When Archimedes wanted to figure out the number of grains of sand that would fit in the universe, he came up with the idea of using repeated multiplication as a simpler way to represent very large numbers[2].

Figure 1. Stack of oranges leads to sphere packing problem

In the design of *Packing Oranges*, I use several scenarios in which there is potential mathematical content about packing. Here are the scenarios:

Scenario 1: Sam shows to the guests a full box of randomly arranged oranges, stating that anyone who is able to figure out the right number of oranges can take away as many oranges as s/he would like. Well, is it just a gimmick? Or by what means can we estimate quickly?

Scenario 2: Wait. How does Sam know the exact number of oranges? If not, can he be cheating? Sam is glad to tell us how he determined the number. He basically knows the total weight of the box of oranges (although the guests don't know) and so he can divide the total by the weight of one orange.

Scenario 3: Suppose that we have successfully won the prize. How can we take away as many oranges as possible? Sam offers some (other, not huge) boxes that you can use, and the problem becomes not only counting, but also what kind of arrangement allows the most oranges in a certain container.

A variety of mathematical tasks go along with these scenarios. For example, using Scenario 1, students can begin by thinking about a simpler problem. Two-dimensional experiments can be used to verify a method of efficient counting, based on a division of the total area by the area of one circle. This method has an obvious flaw in that it does not count the spaces in between the circles; students are meant to discover this flaw and suggest an improvement based on the experimental results and

[1] George Szpiro (2003) wrote on this topic a monograph for the general audience. *Kepler's Conjecture*. John Wiley & Sons, Inc., Hoboken.

[2] Heather Hasan (2006), *Archimedes: The Father of Mathematics*. Rosen, New York.

conceptual thinking. Another experiment based on Scenario 2 can be done by weighing real objects, which is not hard to achieve if a digital scale is available.

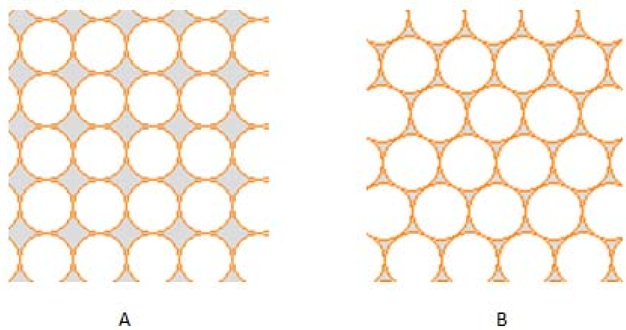

Figure 2. Two regular arrangements in 2-d

In Scenario 3, a winner can take as many oranges as possible, so it is natural to ask: what is the maximum number? Such a question provokes the thinking about efficient ways of packing oranges, and also the definition of packing density. They can be randomly or regularly arranged in the box. This lesson focuses mainly on the ways of regular arrangements (Figure 2), for which the counting becomes a geometry exercise suitable for high school students[3]. However, an interesting relationship can be explored between if carefully packed oranges are significantly more efficient than randomly packed oranges. In particular, do the "benefits" for packing outweigh the "costs" in time? Regular arrangements can be defined as those that repeat periodically in every direction, causing the density of the arrangement to be well-defined also. This is where important mathematization takes place. In reality, boundaries of containers prevent any perfectly regular arrangements, but students might realize that the differences are small. One observation for this lesson may be that students notice that the difference is even more negligible for larger containers, which can serve as an introduction or reinforcement to the concept of limit. The difficulties involved in packing oranges can produce interesting discussions of significant mathematics based on the various mathematical models used.

Note: This lesson is designed to match four CCSS modeling standards in Geometry (G-GMD.3, and G-MG.1,2,3), although the concept of modeling could be broadened so that Geometry itself becomes a set of modeling activities. Indeed, as will be exemplified in the lesson, Geometry can be seen as a model containing objects and their spatial relationships idealized from the physical world.

[3] If we neglect the mathematical difficulties of probability theory, we can use simulation programs to demonstrate the packing density. Just as there is not a unique way to arrange oranges regularly, there are also different ways to randomize packing, and the resulting density depends quite strongly on the randomization (cf. *Science Magazine* **303**, page 968). This idea could be developed as a "follow-up" modeling lesson.

Arithmetic and Algebra to Solve Fairness Problems

Joseph Malkevitch
York College

One big concern all of us have is being treated fairly. Not only do fairness issues arise in our daily lives, but they are a big aspect of the news—ranging from fair tax systems to whether disaster relief money is distributed fairly after an emergency such as the current earthquake/nuclear crisis in Japan.

Here is an applied problem which leads to a system of linear equations and provides students with the opportunity to connect mathematics with something that may arise in their daily lives or which they may see in the news. The problem may help motivate students in wanting to master the algebra skills needed to solve systems of equations.

Suppose that there are disaster relief funds available amounting to $140. (I will use relatively small numbers but the problem can be scaled so that one talks about $140,000 or $1,400,000 instead.) For simplicity assume there are exactly two claimants, A and B, whose claims have been verified to be $150 and $60, respectively. We will assume that there is a "judge" who must distribute the $140 whose work is paid for with money other than the $140, that all the money is going to be distributed to the claimants, and that these are the honest amounts that are due them. How much money should be given to A and B?

To answer a question of this kind one has to apply some principles of what is "fair." One notion of being fair is that everyone be treated equally. It is this reasoning that is behind "one person, one vote." Rich people, middle income people, poorer people, highly educated and uneducated people are all treated equally. However, in this example if we merely give half of the amount available to each claimant, $70, we give B more than he requested (was entitled to). In light of this, back in the Middle Ages, the scholar Maimonides suggested the idea that "gains" be distributed to claimants as close to equal as possible but not exceeding the claim. This would mean giving each claimant $60, but using the remaining $20 ($140 – $120) to add to what is given to A. The final settlement would be $80 to A and $60 to B.

Another notion is that claimants owed larger amounts should be compensated more than claimants with smaller claims, which will not typically happen when all claimants are treated equally. This leads to the idea of settling claims

SEARCH EXTENDED

Teachers College Columbia University
Department of Mathematics, Science, and Technology

MATHEMATICS EDUCATION FACULTY VACANCY

Teachers College invites applicants for a faculty position in mathematics education. A successful candidate will have an earned doctorate in mathematics or mathematics education at the time of appointment and a demonstrated ability to pursue an active research agenda in a significant area of mathematics education. Applicants should be qualified to teach graduate-level courses in at least two areas of mathematics and in two or more areas of mathematics education. Preference will be given to candidates with prior experience teaching school mathematics. All candidates are expected to demonstrate an ability to establish a research program and a potential to obtain external funding. Candidates also are expected to establish a record of continuous, scholarly productivity and a record of leadership. Minority candidates are strongly encouraged to apply.

Rank: Open Rank, Tenure Track

Send CV, a cover letter explaining your interest in the position, representative publications, and names of three references to Professor Bruce Vogeli, Search Committee Chair, Teachers College Columbia University, 525 West 120th Street, Box 195, New York, NY 10027.

Review of applications will begin by November 15, 2011 and continue until the search is completed. Appointment begins September 2012.

Teachers College as an institution is committed to a policy of equal opportunity in employment. In offering education, psychology, and health studies, the College is committed to providing expanding employment opportunities to persons of color, women, and persons with disabilities in its own activities and in society.

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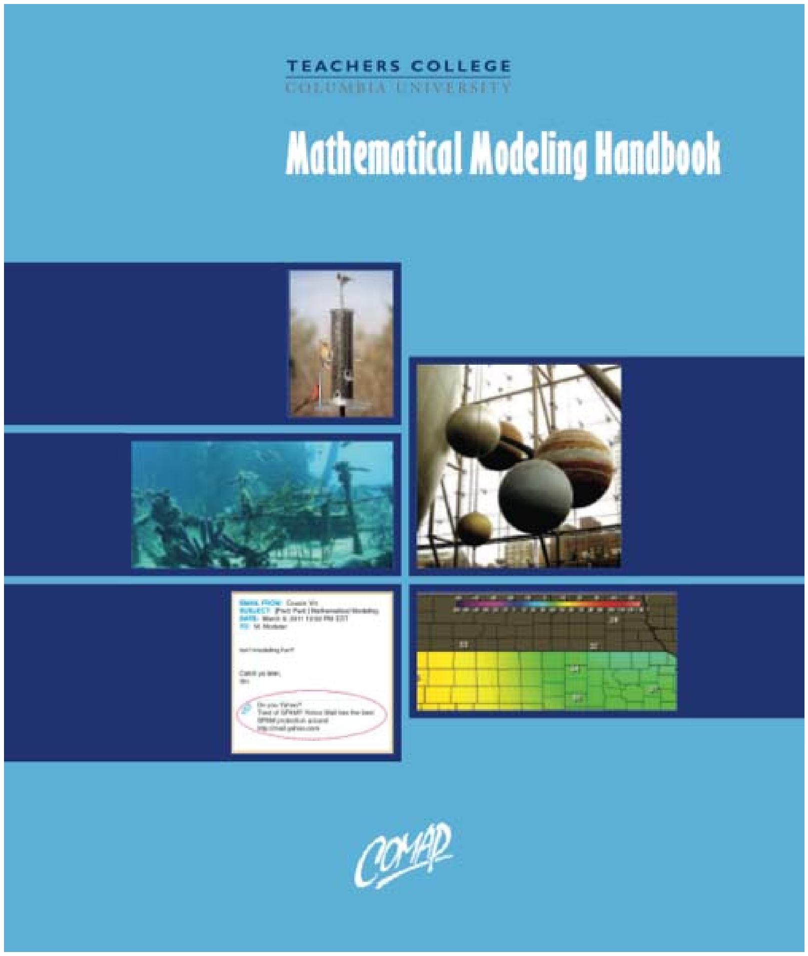

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