Journal of Mathematics Education at Teachers College

Fall – Winter 2011

A Century of Leadership in Mathematics and its Teaching

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

Dr. Robert Taylor was selected by the Teachers College sponsored Teachers for East Africa program to teach mathematics of Uganda's Makerere University. He returned to TC as an instructor in the Department of Mathematics, Statistics, and Computing in Education where he developed an innovative programming language (FPL) intended to introduce educators to the then-new field of computer programming. His seminal work entitled *Computers: Tutor, Tool, Tutee* led to leadership in the new field of computers in education. Dr. Taylor completed 33 years as a member of the Teachers College faculty in 2009.

Dr. Carl N. Shuster completed the doctorate at Teachers College in 1940 under the guidance of William David Reeve. Shuster joined the TC faculty at Reeve's invitation and soon was recognized as the nation's leading advocate of the use of traditional technology, especially measurement technology, in the mathematics classroom. Dr. Shuster served as President of the National Council of Mathematics from 1946 to 1948 and concluded his career as Distinguished Professor of Mathematics at Trenton State University.

Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,000 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select key words from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu

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TABLE OF CONTENTS

TABLE OF CONTENTS, continued

NOTES FROM THE COMPUTER LABORATORY

As technology is incorporated more and more in the mathematics curriculum, teachers will need to integrate it into their lessons. The following are vignettes from beginning teachers describing how they would include technology in their teaching. Topics ranging from using a graphing calculator as a programming tool to re-inventing the textbook in the digital age are discussed in this section.

Insights of Digital Mathematics Textbooks

Hoyun Cho
Mercy College

As ebook readers such as Amazon's Kindle, Barnes & Noble's Nook, or Apple's iPad become popular and widespread in the United States, digital textbooks may replace paper textbooks soon. An educational newspaper reported the story about South Korea's new digital curricular movement (Dwyer, 2011).

South Korea's Education Ministry has announced that it is making a $2.4 billion investment that will enable all of the nation's schools to go digital by 2015. The investment is part of South Korea's new "Smart Education" plan, which will let the students ditch heavy hardback textbooks and instead slip a comparatively light tablet, like the iPad, into their backpacks. The South Korean government plans to build an education-specific cloud computing network where students can store their digital textbooks. The cloud accessibility also means students will be able to access their books through smartphones or laptops. The move will also save schools money since digital textbooks are cheaper than paper textbooks. Since it takes time to buildup the infrastructure to make the switch, traditional texts will still be used during the transition. Elementary schools are scheduled for a full digital adoption by 2014 and middle and high schools by the final 2015 deadline.

In the United State, the Governor of California announced the Digital Textbook Initiative in 2009 (California Learning Resource Network [CLRN], 2011). Following the announcement, CLRN was asked to perform reviews, look closely at the industry, and determine whether these resources met classroom needs. Through three phases, CLRN reviewed open source textbooks in the areas of mathematics, history-social science, and science. In April 2010, 18 middles schools in Indiana have participated in a pilot study using digital curricula in mathematics classes for grades 6-8 mathematics and algebra. Six middle and high schools in Fairfax County in Virginia have begun to adopt online textbooks in their social studies classroom in 2011. Next year Fairfax plans to adopt mathematics digital textbooks. In October 2011, President Obama announced "Digital Promise" center to strengthen technologies for learning and teaching. The education system in the United State is responding to the digital curricula movement.

This new education movement means that traditional textbooks are on a downward trend and are likely to be replaced by digital textbooks with one of three developing models: e-reader, interactive subscription-based online resources, and tablet PC application. Moreover, digital textbooks contain a variety of multimedia content and also provide user interface tools. How should mathematics textbooks be digitized? Digital mathematics textbook should include the following five insights: Living, Interactive, Adaptive, Participative, and Connected.

The first is 'Living.' Students learn and use mathematics not in the learner's perspective but in the perspective of life. Teachers often just provide mathematics problems for students to find the solutions, but the formulation of a problem is so important that students should have the opportunity to invent a problem by themselves and also find its solution. For example, here is a problem. "You have a water tank. How long will it take you to fill the tank?" In the digital textbook, students will not be given any data to solve the problem but be given a video about filling the water tank. They will talk with others to formulate the problem and measure the length and height of the tank to find the solution. Some students might watch the video several times.

The second is 'Interactive.' Students will solve a problem without the necessary data. They will explore a problem and use different measurement tools to interact with a problem. From the above example, digital textbook provides the size of the water tank and amount of water needed to fill the tank, zoom in and out functions in the video, a stopwatch, and other measurement tools.

The third is 'Adaptive.' Digital textbooks often update the content along with the students' levels and lessons. If students need more exercise problems, the digital textbook provides similar problems and problems of different levels. The digital textbook company keeps monitoring the students' use of the textbook and adds or removes the content accordingly.

The fourth is ‘Participative.’ Digital textbooks provide a chatting window for the students to discuss with their peers and teachers. This leads to the last component, ‘Connected.’ Digital textbooks provide an internet service that enables students to send emails to their peers and teachers.

Many digital mathematics textbooks are available for use in class, but many of them are simply paper texts converted to digital text. This type of digital textbook could not be distinguished from a regular mathematics textbook. Digital mathematics textbooks should include the above five insights to support student learning properly.

References

California Learning Resource Network. (2011). Digital Textbook Initiative. Retrieved August 10, 2011, from http://www.clrn.org/FDTI/index.cfm

Dwyer, L. (2011, July 6). South Korea’s Making the Switch to Digital Textbook. *Good Education*. Retrieved from http://www.good.is/post/south-korea-s-making-the-switch-to-digital-textbooks

Basic Triangle Properties Through Geometer’s Sketchpad

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Teachers College, MA Candidate

In previous years my lessons on triangle congruence and other basic triangles properties required students to induce individual postulates by exploring with drawings or physical manipulatives (such as pins and rubberbands on foam boards). Having become more familiar with Geometer’s Sketchpad (GSP) through my first student-teaching placement, I can now conceive of more accurate, time-efficient ways of having students deduce these same properties. Ideally, such lessons would be implemented in classrooms in which each student, or at least every student pair, has access to a computer equipped with Geometer’s Sketchpad. The teacher would guide students through a series of constructions and measurements within the program (providing both verbal and written instructions), debriefing each completed task with a pair-share or whole-class discussion.

One such triangle property to elicit would be the formula for triangle area. By using the point and line segment construction tools, students may construct triangles of “arbitrary” dimensions. From here they would use the line tool to construct a line running through the top vertex of the triangle that is parallel to the triangle’s base; this could be accomplished by constructing a line running through the two endpoints of the base, then copying, pasting, and dragging the line so that it runs through the top vertex. Students should then construct a segment from the top vertex that is perpendicular to and terminates at the base of the triangle. They then use the “Measure” tool to calculate the length of the line segment, recognizing that this gives them the “height” of the triangle. Similarly, they should measure the length of the base (see Figure 1). (To save time, the instructor could also have students open a pre-created image as described, then alter the dimensions as desired to create an “arbitrary” triangle.)

Once they have recorded these measurements in a table, they should use the toolbar to calculate the area of the triangle. In dragging the top vertex of the triangle along the top parallel line (to create new triangles while keeping the height and base lengths constant), students will be able to note that while the other two side lengths of the triangles change, the area of the triangles remains constant. Upon completing this process students should be able to conclude that the area of a triangle depends on its base and height. By repeating the process for new triangles (perhaps by partnering with another classmate and comparing their tables of measurements), they should work to find the exact relationship between a triangle’s area and these dimensions.

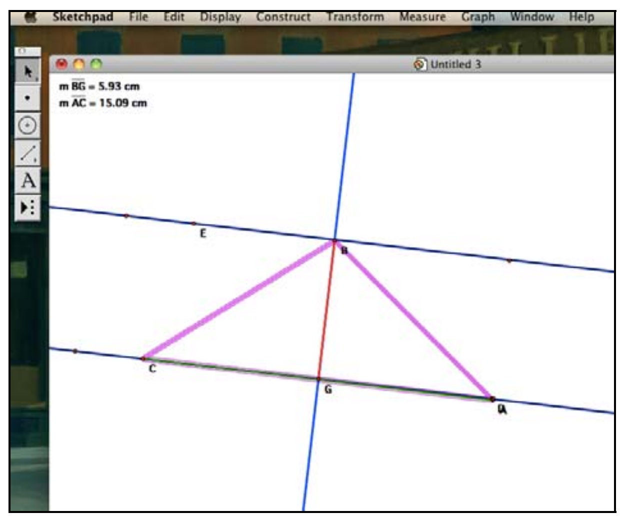


Figure 1.

Similarly, students can use GSP to uncover slightly more advanced triangle properties such as the various congruence postulates. For instance, by using the fourth option for the arrow button in the toolbox, students can drag individual triangle sides, keeping the others constant, to create similar (incongruent) triangles with equivalent angle measures—thus eliciting that two triangles can have identical angle measures without being congruent (i.e. that no AAA Postulate exists).

By allowing students to experiment with triangles of any dimensions, these various GSP activities enable students to elicit general properties through a broad range of examples. One should note, though, that these activities do not inherently require students to rationalize these various properties or theorems, but merely to acknowledge their existence (a useful activity for intuiting postulates, for instance). While the comparatively minimal opportunity for human error makes GSP an effective means of having students recognize these trends initially, I would choose to pair these activities with others that require more deductive reasoning.

Discovering Blackbeard's Treasure by SMART Board™

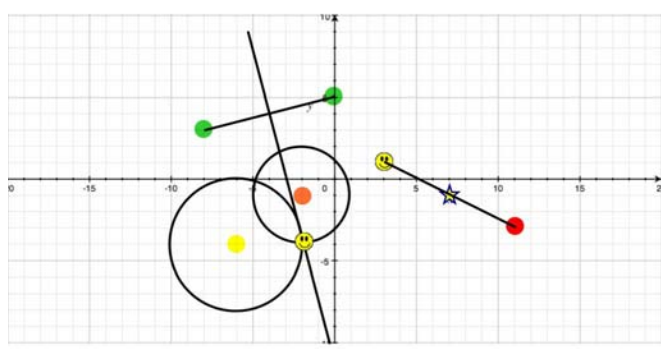
Emily Ying Liao
Uncommon Charter High School

Students living in this day and age are really lucky to have countless forms of technology available to cater to their every need. Mathematics has also become more accessible to the general public through different facets of technology. Video games that require problem solving and analytic thinking are becoming increasing popularly. My favorite game series is the Professor Layton games, which showcase many logical problems as well as puzzles that are closely modeled after graph theory. With the abundance of technology out there, it is important for the teacher to bring some of these new developments into the classroom. One of my uses of technology is the SMART Board, which I incorporated into a lesson to teach and review compound loci with 10th grade geometry students.

My lesson is a collaborative learning activity that combines both group work and whole class instruction. The students have previously learned to apply compound loci to different simple situations and construct compound loci using a compass, straightedge, and graph paper. They are also able to write equations of lines and circles using coordinate geometry, and the general equations for lines and circles. For the activity, students will receive a description of the task detailing Blackbeard's treasure located on Ocracoke Island in North Carolina. In their groups, students need to combine their knowledge of geometry to discover the final resting place of Blackbeard's famed treasure.

For the first part of the activity, students need to work together to recognize the different types of loci from Blackbeard's letter. They need to find a point equidistant from two points by finding the midpoint and to find circles centered on certain points. Once they determine some of these important equations of circles and lines, we will come back together as a class to construct the compound loci on the SMART Board. A student will provide each piece of the construction, and the constructions will be displayed to the whole class. As a class, we will determine whether each step and/or construction is correct. Having the class complete this portion of the activity together will make it more exciting for the students as well as give them a review of how to perform certain constructions. They will discuss the different types of loci from the problem as a review of previous lessons.

Using the SMART Board allows the students to visualize the problem better and problem solve as a group. The SMART Board is easily erasable, so mistakes are encouraged and expected. Also, it is a very hands-on activity as students are building and generating the information from the first step through the final step. Also, they will be able to see how different loci are visually represented. For example, when creating the circle, the image will go from a small circle to the circle with the desired diameter. Having a treasure hunt also adds a creative dimension to the lesson to keep students involved. This topic will be more accessible to students; they will see the equations of circles and lines on paper and different types of loci and immediately see the images on the SMART Board as well. This will build the connection between what the written formula looks like and what the formula actually refers to. The best part of this activity is the beautiful constructions possible on the SMART Board. The following is one way to complete the activity with the gold star marking the final resting place of Blackbeard's treasure. Through this interactive activity and use of the SMART Board, students will better understand compound loci and coordinate geometry.



Programming Probability

Paul Morrill
New Design Middle School

Probability is simultaneously one of the simplest and most complicated areas of high school mathematics. The nuances of conditional probability, expected value, and the inherent "word problem" nature of probability make the subject very complicated even for advanced mathematics students. One topic many students often have problems with is the idea of randomness. A classic example is when a teacher forms two groups of students as follows: the first group flips 100 coins and records the results, while the second group makes up the results for 100 flips. The teacher is almost always able to distinguish between the two groups, as the students who make up the results often interpret "random" to mean "non-repeating," seldom having more than 2 or 3 of the same result in a row.

One way to get students acquainted with this phenomenon is to enable them to use a probability simulator. While there are many excellent simulators available online, getting the students to code their own simulator can convince them that the results are accurate, as well as empower them in understanding how randomness can work. Given any TI-83 (or higher) calculator, the students can create a simple code using the following lines:

```
i = 0
For i < 100
        x = Rand
        if x < .5, print H
        else print T
i = i + 1
Loop
```

While the specifics of the code can be changed, this simple code can result in a very straightforward and easy to implement program. By running the program multiple times, each run simulating 100 flips, the student can verify that 50 or more heads (or tails) is common, that there can be a large discrepancy in the number of heads or tails, but that in general the results will tend to be centered around 50-50.

Another benefit is that the data can be stored and compared. This type of exercise naturally lends itself to looking at probability distributions, including applying the binomial theorem to determine the likelihood of outcomes. Such a task can be incredibly time consuming using more traditional methods, such as coin flipping.

A third advantage is that the students are empowered by the mathematics. The idea of coding such a simple program that provides such a wide range of data highlights some of the intricacies of randomness, while simultaneously driving the students to better appreciate the logic behind the mathematics.

A final benefit is that this activity provides a good introduction to basic programming. Computer science is in some regards a field of applied mathematics, specifically logic and reasoning. While there is a learning curve associated with any programming language, much of the coding knowledge required by the TI calculators is rather straightforward. A lesson like this can be tiered, as the instruction can range from giving the students the code to having the students generate it individually or in groups.

Lessons such as this use technology as a tool, rather than as a crutch, giving students an opportunity to experience a side of mathematics they would not have access to in a traditional classroom.

GeoGebra in the Geometry Classroom

Christina Constantinou
High School East, Half Hollows Hills Central School District

What began in 2001 as a master's thesis and continued as a Ph.D. thesis in Mathematics Education at the University of Salzburg (Austria) has emerged as an essential tool in a teacher's repertoire to teach some of the more challenging topics in geometry clearly and effectively. We are indebted to creator Markus Hohenwarter and his team of researchers who developed and continue to enhance this amazing software that is free to anyone who wants to explore the relationships among algebra, geometry and beyond.

The program is downloadable and installable on any computer or you can use the software directly on the web without installation. You will notice that Geogebra uses a coordinate based system, but that does not mean you have to rely on the coordinates to solve problems. This program can do everything that the popular Geometer's Sketchpad can do, and in some cases better than what can be done on Sketchpad. GeoGebra has won many awards since the launching of the program in Europe and the United States.

Geogebra provides three different views of mathematical objects: a *Graphics View*, a numeric *Algebra View,* and a *Spreadsheet View.* They allow you to display mathematical objects in three different representations: graphically, algebraically, and in spreadsheet cells. The three views are linked and adapt automatically to any changes made to any of the representations regardless of how they were initially created.

Diagrams with proper labels can be created in GeoGebra that can be used for worksheets and demonstrations in the classroom. In addition, I have used this program in my classroom to introduce many theorems that are used throughout the course to effectively explain concepts and motivate students. Some of these theorems include triangle geometry proofs and circle geometry proofs.

Proving the Relationship of Mid-Segment of a Triangle

As a demonstration, we will prove the theorem that if the midpoints of two sides of a triangle are connected, then that segment is parallel to the third side and its measure is equal to one half the length of the third side. Currently, throughout the Geometry curriculum, the students are exposed to long tedious proofs in which many high school students often get lost and do not grasp the reasoning behind the proof or why the proof is important. The Euclidean proof for this theorem takes more than twenty steps and the analytic proof involves a lot of algebra. After working with this program for a while, I was motivated to use a method other than the usual Euclidean or analytic proof.

First, I hide the axes and construct the triangle by plotting three random points. Notice that the program automatically stores the coordinates and measures the lengths of the line segments in the algebra window. Next, I construct the midpoints of two of the sides and connect them (Figure 1).

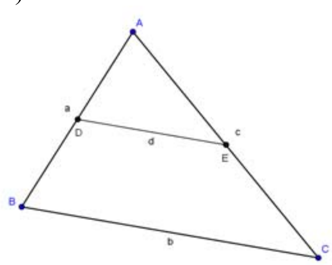


Figure 1.

In order to prove the line segments are parallel, I could measure the angles to show that if the measures of corresponding angles are equal, then the lines are parallel; or I could measure the slopes to see if they are the same, which is the more simple approach. See Figure 2.

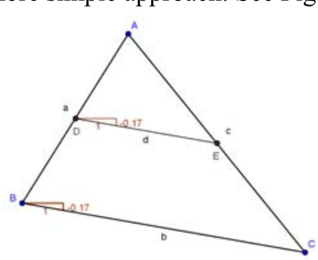


Figure 2.

Even though the measurements look the same, I can confirm that they are the same by using the "Relation between two objects" tool located within the second-to-last task icon. When I select the slope measurements in the algebra window, you will notice that the software confirms that the measurements are equal.

In order to prove that line segment "d" is one-half the length of "b," I need to perform the calculation and then determine the relation between the calculation and "b." In the "input" task bar at the bottom of the screen, I created a new variable called "Halfb" and set it equal to b/2, then used the comparison tool again with "Halfb" and "d."

It should not be difficult to convince your students that this holds true for any triangle by selecting one of the vertices and dragging it to another place on the screen and then examining the results. Through this example, we can generalize the process we just used in GeoGebra to construct a Euclidean proof if desired.

Proving the Pythagorean Theorem

We have been exposing our students to the Pythagorean Theorem since Middle School. After enough practice, they are all familiar with the numeric relationship, $a^2 + b^2 = c^2$. Most of our students just repeat this theorem mindlessly and have come to a point where there is no meaning behind it. With the use of GeoGebra, we can finally prove this famous theorem to our students geometrically.

We are going to demonstrate that the area of a square that is formed using the hypotenuse is equal to the sum of the areas of the squares that are formed by using the legs. First, we need to construct a right triangle. Draw a line anywhere on the screen by selecting the third icon from the left and clicking on the screen at two points.

Now select the fourth icon from the left and select the "perpendicular line" tool. Click on the line, and then click on a point on the line to construct the perpendicular line. Select the second icon from the left and click on the perpendicular line to identify a point on that line. See Figure 3.

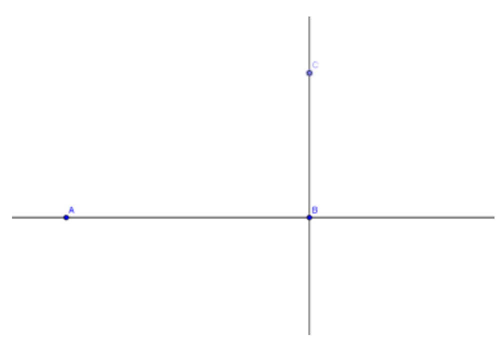


Figure 3.

Hide each of the lines with a "right click" on either the line or the object in the algebra window, and then deselect "show object." Then select the third icon, select "segment between two points", and click on the endpoints of each segment to create the triangle as shown in Figure 4.

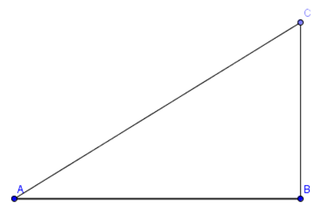


Figure 4.

Now we need to construct squares out of each of the three sides of the triangle. Select the third icon and then select "regular polygon." Click on the two points in clockwise order and then type 4 to create the square. Notice in Figure 5 that the areas of each square have already been calculated and displayed.

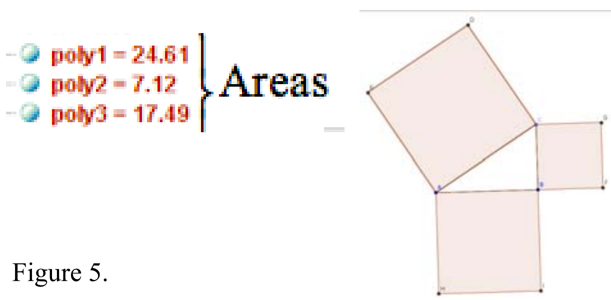


Figure 5.

By "right clicking" each name in the algebra window, and selecting "rename" can change the name of each polygon. The polygons are now called "AreaHyp," "AreaLeg1," and "AreaLeg2." In order to show that the sum of the areas of the squares of the legs is equal to the square of the hypotenuse, I created a new variable called "AreaSum" in the "Input bar" at the bottom of the screen by typing "AreaSum = AreaLeg1 + AreaLeg2." It seems that "AreaSum" and "AreaHyp" are the same.

In order to verify that these quantities are equal, select the second to last icon and then select "relation between two objects." Then select "AreaHyp" and then "AreaSum" to see that they are equal.

Let's see what would happen if we constructed a regular polygon other than a square. I created equilateral triangles by using the "regular polygon" command and selecting 3 sides (Figure 6).

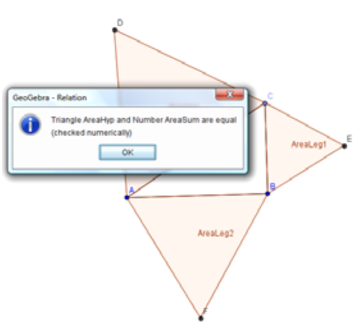


Figure 6.

As you can see in Figure 6, the sum of the areas of the equilateral triangles formed with the legs is equal to the area of the equilateral triangle formed with the hypotenuse. The theorem holds true for any regular polygons that you construct. You can even have your students explore this through the program themselves.

Proving the Measure of an Angle in Relation to its Intercepted Arc

The geometry of the circle is one of the most difficult units for students. Often times, they are just given the relationships between angles and their intercepted arcs. With GeoGebra, we can construct a circle with angles and use the dragging tool for the students to discover the relationships on their own as shown in Figure 7.

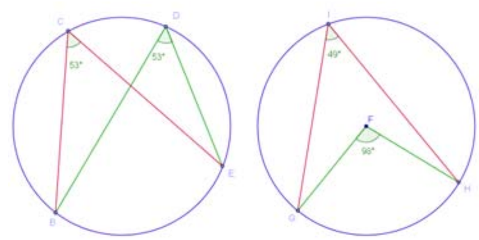


Figure 7.

After the students have constructed the desired diagrams in GeoGebra, they are ready for the discovery portion of the lesson. The students are then instructed to drag the vertex of each of our angles to different locations around the circle as well as the points that affect the measure of the arc. They should also be instructed to make inferences on what kind of relationships they discovered through this activity. After some time, a class discussion can be held to bring out the relationships between the angles in addition to the relationship certain angles have with their intercepted arcs. The diagram on the left will show that angles inscribed in the same arc have equal measure. The diagram on the right shows how an inscribed angle is one half the measure of a central angle inscribed in the same arc.

Visit www.geogebra.org and discover for yourself how easy it is to deliver concepts in an entertaining and informative way.

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Evaluation*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3000 words describing research, experiments, projects, innovations, or practices related to evaluation in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by January 21, 2012. The spring issue's guest editor, Ms. Heather Gould, will send contributed articles to editorial panels for "blind review." Reviews will be completed by February 1, 2012, and final drafts of selected papers are to be submitted by March 1, 2012. Publication is expected by April 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the spring 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Spring 2012	Evaluation
Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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