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Fall – Winter 2011

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

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Aims and Scope
The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

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Current Challenges in Integrating Educational Technology into Elementary and Middle School Mathematics Education

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Developing curriculum and instruction for mathematics education and designing technologically enhanced learning environments are often pursued separately, but may need to be addressed together to effectively link the strengths of technology to performance in mathematics and conceptual understanding. This paper addresses current challenges with educational technology in elementary and middle school mathematics education, as well as technology's influence on curriculum, instruction, and student performance. Three properties of computer-based technology are described; these properties determine technology's unique value for elementary and middle school mathematics: engagement and motivation, informative feedback, and visualization. Three computer-based math applications, Doodle Math, Puzzle Me, and GeoShape, are introduced as examples that combine effective learning resources and technology.

Keywords: Educational technology, pedagogical agents, computer-based math game.

Introduction

When used well, technology can assist instruction, promote students' interest, and improve performance in mathematics. The challenges in integrating digital technology into elementary and middle school mathematics education involves bringing together effective learning resources and an appropriate choice of technology. This is a delicate balance that needs careful consideration. However, school officials seem more concerned about equipping classrooms with hardware and software than inquiring about the skills and concepts that must be considered when integrating technology (Kebritchi, Hirumi, & Bai, 2010). Therefore, teachers are forced to carry the responsibility, develop technological fluency, deal with the underdeveloped and relatively untested nature of these devices, use tools rarely tested in a school setting, and use applications that have little or no link to their existing curriculum (Privateer, 1999). With good reason, teachers are bewildered at the so-called benefits of technology when little gain is seen in academic performance (Kolyda & Bouki, 2005; Edelson, Gordon, & Pea, 1999). As mathematics learning requires a combination of content knowledge and conceptual understanding that continues to evolve as a student's mathematics level progresses, technologies that overlook these features in favor of speed, efficiency, and automation can result in lower levels of student performance and understanding (Bransford, Brown, & Cocking, 2000).

In this paper, we suggest that technology should be one more component within the learning domain, rather than a central player in the learning environment. Achieving technology fluency takes time. Technological and visual literacy is only a steppingstone toward fluency, not the goal. To acquire technological fluency means to become aware of the strengths and weaknesses of educational technology, how it works, the purpose it can serve, and how it can be used efficiently and effectively to achieve specific teaching, learning, and behavioral goals (Privateer, 1999). In other words, successful learning does not depend on a single technology or learning mechanism, but on a confluence of effective learning sources and choice of partnership with technology. This paper contains suggestions for several means of supporting this partnership: (a) help teachers identify the benefits and shortcomings of technology, (b) provide specific examples of how technological properties can be linked to effective methods of instruction, and (c) identify technology-rich learning conditions and activities that can build new pathways for students to self-reflect, reason, and understand mathematical concepts.

The paper begins with an overview of areas in which educational and communication technologies are often applied in mathematics education and identifies key properties in technology that may be useful in instruction. The paper examines how the key properties can further engage the learner, provide immediate feedback, and display representations that prompt innovation and learning. This paper presents three example computer-based mathematics applications, Doodle Math, Puzzle Me, and GeoShape, designed for elementary and middle school mathematics education. The example applications combine effective learning resources and the strengths of educational technology to create computer-based learning environments.

Related Work

Recently, children have found it difficult to identify with the mathematics and science subject area or imagine a career in the Science, Technology, Engineering, and Mathematics (STEM) field. Studies have found that children de-select mathematics and science-related careers as early as elementary school (Eccles, 2007; Steele, 2003). Studies have shown that prior competence and performance do affect math-science identity development. Marsh, Hau, and Kong (2002) say there may be a reciprocal effect between academic self-concept and performance. In other words, earlier performance affects academic self-concept, which in turn affects later performance. Then again, computer-based math games and technology-inspired manipulatives often have an inviting nature that can motivate students to engage in learning complex math concepts, even if they feel that mathematics is not their best subject (Bers & Urrea, 2000).

Manipulatives and computer-based tools are quite successful in their traditional form (non-technology); they help children understand abstract math concepts by making them more concrete through touch and exploration (Case & Okamoto, 1996; Alibali & DiRusso, 1999). Researchers have found that manipulatives improve learning in educational settings (Chao, Stigler, & Woodward, 2000) as hands-on activities prepare students to understand, discover, and interpret. Manipulatives seem to work well with technology as virtual and technology-enhanced manipulatives (e.g., computer-based learning applications, Lego Mindstorms) have been successful in helping students understand elementary and middle school math concepts (Turkle & Papert, 1992). Virtual manipulatives, like physical manipulatives, attempt to represent abstract concepts in visible ways but through digital instead of physical means.

Under certain circumstances, manipulatives can interfere with learning unless the engagement is meaningful or motivating. The engagement can be meaningful in terms of content (e.g., manipulative represents a concept) or self-identity (e.g., student identifies with the task or content area). Uttal, Scudder, and DeLoache (1997) argue that the use of manipulatives is incomplete without an understanding that the manipulative itself represents a concept or written symbol. Unless students understand the relationship between the manipulative and the concept that it represents, not much learning seems to take place. The difficulty in understanding the relationship between concept and representation makes it crucial that the task itself helps the student understand the relationship. Often, this may be the reason for high levels of engagement, but low academic performance.

Computer-based tools only provide a convenient working environment. Just handling the computer-based tool with no guidance can let the user (teacher or student) lapse into meaningless activities. According to Skinner (1986), allowing students to blindly use but not generate can result in challenges similar to those seen in reading mathematics and speaking mathematics; that is, students may easily follow the author of a text solving a sample problem, but find themselves stumbling when they try to solve a similar problem on their own. A positive impact of interaction with these computer-based tools would be lasting cognitive changes that equip students with thinking skills, depth of understanding, and strategies for solving mathematics problems (e.g., similar to internalizing the abacus). Harel (1988) conducted experiments in which students used Logo as an exploration material, rather than a topic to master. In one study, Harel looked at Logo and arithmetic with fractions in a didactic versus a constructivist integrated manner; he found that the constructivist integrated manner led not only to a higher mastery of Logo, but also a deeper understanding of fractions. Thus, goal setting and integration with subject matter seem to affect cognition. Unfortunately, these benefits are not likely to occur automatically. Salomon (1991) mentioned that cognitive effects gained through technology greatly depend on the meaningful engagement of learners in the tasks afforded by these computer-based tools.

Not all learning depends on feedback. Students can learn to model behaviors (Bandura, 1977), though without feedback the learning may not go beyond procedural imitation (Chi & Bjork, 1991). The literature on feedback has emphasized two types of external feedback, performance and informative. Performance feedback is whether a behavior is right or wrong and gives no information on how to achieve the correct answer. Performance feedback usually involves immediate or delayed feedback. Immediate feedback makes it easier to associate the feedback with the relevant behavior (Skinner, 1986), while delayed feedback allows the learner to explore the structure of the problem space (Vollmeyer, Burns, & Holyoak; 1996). Moyer, Bolyard, and Spikell (2002) found that students who were allowed to dynamically control the computer-based manipulative and receive immediate feedback, performed well compared to students who had static manipulatives with no control. Having dynamic control over the manipulative and receiving immediate feedback seems to be an important feature in modeling behavior.

Informative feedback provides information on how to correct a situation. One type of informative feedback comes from control theory (Powers, 1973). People act on their surroundings and try to maintain reference value. Any deviations from the reference value will indicate a change in behavior. Control theory may be useful in gaming environments where the players are constantly monitoring themselves in relation to the game environment. Informative feedback for learning can involve probing a physical environment and students receiving feedback about the way the environment responds (e.g., climate change simulation game). In social contexts, feedback can also include procedural correction ("place the book here, not on the chair") and elaborative feedback that explains why (e.g., Chi, Roy, & Hausmann, 2008).

Levine, Vasilyeva, Lourenco, Newcombe, and Huttenlocher (2005), found that children from low socio-economic status (SES) backgrounds, demonstrate statistically lower spatial abilities. The study suggested that the cause was due to limited amount of parent-child interaction, and less exposure to visual games and toys when growing up. Studies showed that the mental rotation ability of children from low SES groups at third grade was equivalent to the mental rotation ability of a new second grader from upper middle class SES students. Cohen and Hegarty (2007) found that the ability to manipulate and understand computer-based visualization was correlated with their spatial skill level. The results implied that students with poor spatial skills growing up would have a difficult time understanding the visualizations created by technology. Studies from Barke (1993) found that well developed spatial skills are essential for mathematics, and understanding basic and structural sciences.

Visualization is an excellent way to (a) help students remember, (b) strategize when problem solving, and (c) use visualization as a way to help learners see how they structure their thoughts. In the simplest form, visuals can be an excellent way to help students remember. Standing (1973) found that people have far more expansive memory for pictures than words or sentences (Standing, 1973). Paivio (1986) observed that when people saw a visual scene and made a perceptual code, they often explained the content to themselves, which also yielded a verbal code in memory. Paivio found that this "dual coding" increased chances of retrieval. A similar effect was seen with pictures and captions (Bower, Karlin and Dueck, 1975).

Schoenfeld (1992) conducted a study where students were blindly solving math word problems where the situation was impossible, but the students never realized that the problem was insolvable because they did not try to model the problem. Although imagery can provide determinate structures, people need to learn that they should construct images. In solving math word problems, children need to learn how to combine linguistic, mathematical, and spatial processes. It is particularly important to encourage early readers to imagine narratives so they can better understand the content.

Imagery does not always arise spontaneously, as children have difficulties with imagery compared to adults (Reiser, Garing, & Young, 1994). Often, children may need special support or training to construct images. Bamberger (1991) describes how children, given encouragement and prompting, will invent increasingly precise visual representations on the pitch and duration of musical notes, and visual representation on motion (diSessa, Hammer, Sherin, & Kolpakowski, 1991). Glenberg, Gutierrez, Levin, Japuntich, and Kaschak (2004) found that encouraging imagery through the initial use of physical modeling (e.g., figurines that portrayed the actions in the passage) helped improve children's reading comprehension.

Schwartz (1995) found that very few adolescents spontaneously construct visualizations to solve problems. However, once they were encouraged to invent their own representations, and experienced the benefits of visualization for problem solving, the students started to invent their own forms when solving new problems. One of the benefits of visual representations is that students can bring to bear their spatial abilities when working with structure. Larkin and Simon (1987) found that how a spatial representation is structured can easily determine the complexity level of the search. For example, a matrix that uses rows and columns permits easy detection of linear and curvilinear patterns, thus helping learners see how they structure their thoughts. The research on Teachable Agents use dynamic visual representations to show knowledge structures and reasoning mechanisms in the form of a concept map. Using a pedagogical agent called Betty, the students use Betty's directed graph structure to introduce and organize those concepts. The students' visualization through Betty's graph structure allowed students' ideas and agent reasoning to come together as a shared representation. Betty animates the reasoning to make critical relations visually explicit for the child. This helps students organize and reason with their own concepts (Biswas, Schwartz, Bransford, & TAG-V, 2001).

The next section provides a discussion of three key properties in educational and communications technology that may assist teachers and students in instruction.

Educational and Communication Technologies with Unique Value for Elementary and Middle School Mathematics Education

Three key properties in educational and communications technology determine its unique value for mathematics education. Engagement and motivation, informative feedback and assessment, and visualization will be highlighted separately in this section.

Engagement and Motivation

Computer-based learning applications and manipulatives invite high levels of engagement but often result in little gain in academic achievement. One way to make engagement meaningful is to design tasks that are integrated into a relevant narrative (or storyline). Too often we see commercial math games that use a familiar narrative from a favorite cartoon character and add arithmetic problem sets (e.g., Star Wars Jedi Math video game). When relevant narratives are found to be quite effective in increasing motivation, evaluating students' learning process, and helping students bridge the gap between task and math concepts. Cordova and Lepper (1996) engaged students in a computer math game on arithmetical order-of-operation rules. They compared a task with no narrative to three tasks that integrated relevant contextualization, personalization feature, or control over choices (e.g., Space Quest, where strategic use of the order of operation determines how you navigate through the galaxy maze). Compared to the condition with no narrative, the conditions with contextualization, personalization, or choice all produced dramatic increases in student motivation, learning, aspiration, and perceived confidence levels.

Other features that often contribute to motivation and engagement exist in highly customizable and personalized environments with some form of social interaction with humans or virtual characters (Okita, Bailenson, & Schwartz, 2006). These features can range from superficial (e.g., background color of the screen, what the character is wearing) to analytical (e.g., creating an environment similar to your classroom or an application that keeps track of your progress), but must be carefully integrated so as not to distract the learner. Technology-enhanced manipulatives and computer-based learning games can be easily customized (e.g., choice of narrative, a comfortable and familiar layout) or personalized (e.g., math game remembering the problems students have trouble with at school), which allows students to make the learning environment their own (Bailenson, Yee, Blascovich, Beall, Lundblad, & Jin, 2008).

Computer-based math learning applications can be ideal for skills that require a lot of practice. However, trial-and-error learning is usually fragmented and may need a narrative that guides students in linking the pieces together. Cordova and Lepper (1996) found that including a narrative (e.g., Space Quest, Treasure Hunt) increased not only interest, but also confidence level, learning, and positive attitude toward mathematics. Choosing a favorite topic for a narrative may make it easier for students to identify with math concepts. Social interaction can be designed and integrated into the narrative to elicit motivation through competition (e.g., competing and winning points with correct answers) and collaboration (e.g., checking your teammate's answer so your team doesn't lose a point) as the virtual character can act as a more capable peer who prompts within the student's zone of proximal development (ZPD).

Informative Feedback and Assessment

Computer-based learning environments allow rapid interactive cycles with instant feedback that keeps students occupied, thinking, and alert. However, keeping students engaged requires a careful balance of skill and challenge level within reach for each student. This is where feedback and assessment become important contributors to designing personalized or customized learning environments. Through the use of well-designed computer-based learning environments, teachers will be able to identify feedback sources that can regulate student learning in desirable ways. Often, simple strategies like trial and error can inform strategic decisions on the level of difficulty in the game that will represent the optimal challenge for the student. Providing clear indicators of progress through immediate feedback is important to make students think that improved skills are within reach (e.g., as in ZPD). Such feedback often motivates students to feel that they can, for example, solve the puzzle or determine the algorithm behind the learning game application. Technology-rich learning environments can also be ideal for developing metacognitive skills (e.g., monitoring, self-correct) that require a lot of practice (Schwartz, Chase, Chin, Oppezzo, Kwong, Okita, Biswas, Roscoe, Jeong, & Wagster, 2009). Features to record and monitor student activities can be embedded in the environment to provide self-reflecting feedback and assessment for both teachers and students.

Studies have found that teachers who give informative feedback, confront learners with cases of conflict, and provide students with the opportunity to contribute are likely to trigger mental experimentation, encouraging students to interrelate different concepts and understand their generic attributes (Gelman & Brown, 1986). Salomon, Perkins, and Globerson (1991) considered the ability to self-regulate and guide as a cognitive impact that would serve the individual in numerous instances, even when away from computer-based tools. Obtaining informative feedback can be difficult if teachers are not sure what to look for in technology-rich learning environments. According to Salomon, Perkins, and Globerson (1991), student performance can be assessed two ways. One is the performance that students display while equipped with technology/computer-based tools. Usually, this means that technology plays a significant part in the cognitive process that students would usually have to manage manually on their own. However, the students' potential can be evaluated while they work with a computer-based tool. Such a partnership with technology is similar to having a more capable peer; it allows learners to engage in cognitive processes that are a bit higher than the level they would engage in if they were problem solving alone. An intelligent computer-based tool can operate within a student's zone of proximal development (ZPD), where individual performance is assessed, as well as in conditions under which the student challenges his or her cognitive ability (e.g., use of pedagogical agents). In addition to seeing how students perform and work effectively with computer-based tools (e.g., learning game applications, math devices, graphics software), one could explore how partnerships with computer-based tools can be designed to achieve cognitive impact.

Visualization

Technology has continued to develop at an unprecedented rate, increasing computer processing speeds and making high-performance video graphics cards more affordable for the educational setting. Computer-based tools and virtual manipulatives can now provide more spatial methods for interacting with information. Children today are quite familiar with visualization as seen on their smart phone applications, interactive maps, and game interfaces, where most of the information must fit on a 2-inch by 3-inch screen. Children are no longer impartial or passive receivers of visual messages and are capable of making critical selections between the necessary and unnecessary; they can also identify soliciting messages from real and valuable information. Students may be up-to-date, but teachers also need to ensure that they develop visual literacy to understand and create visual representations for learning (Averinou & Ericson, 1997).

Technology has enabled pictures and diagrams to give feedback to the learner, and can be useful in helping students draw inferences and develop spatial skills. Visual representations can range from symbols, maps, pictures, and graphics to simulations and animations. Computer visuals and simulation tools provide different ways to visualize data, procedures, relations, dynamics, and movement. Visualizations and simulations are valuable tools that help students form interpretations from multiple representations, develop spatial skills and reasoning skills, and communicate information through visuals. Virtual manipulatives that combine the abilities to move and draw on pieces seem to add more flexibility and variety in representation than traditional manipulatives (Ainsworth, Bibby, & Wood, 2002; Clements, 2002). Studies have found that manipulating computer graphics tools can help children develop concepts of two-dimensional space (Martin & Schwartz, 2005; Sarama, Clements, Swaminathan, McMillen, & Gonzalez Gomez, 2003).

Often, visual perception is tightly coupled with people taking action. Visual perception can guide motor action (e.g., moving your head to see the screen in the movie theater), and motor action can guide visual perception (e.g., touch helps people figure out the shape of an object) (Gibson, 1962). According to Parsons (1987), for successful imagery, people usually had to first imagine the consequence of an action, and then animate their image, that were often the actions they can actually take (e.g., walking down the street, change perspectives, using a tool). Computer-based tools, visual perception and action in relation to imagery can be useful when the task is difficult for children to precipitate (e.g., simulating molecular rotation, or folding and unfolding origami paper). Computer-based tools can easily coordinate visual perception and action, as children can easily learn how to operate game interfaces, and manipulate virtual objects through touch (e.g., tablet computers, game consoles).

Mathematics problem solving is often based on linguistic representations in which logical and sequential reasoning is preferred over visual representations. However, recently, the use of pictures and diagrammatic explanations is emerging as a promising tool for assisting in comprehension of elementary and middle school mathematics. Rieber (1995) advocated the use of instructional materials that can generate multiple representations and enable visually oriented problem solving. Studies have found that applying visual literacy skills to learning strategies like mind maps and concept maps improves teaching and learning (Buzan, 1996).

In situations where children need to combine linguistic, mathematical, and spatial processes, it is particularly important for early readers to imagine narratives to better understand the problem. Virtual reality technology (e.g., Second Life) may make imagining narratives easier for children. Computer graphic characters can be created and easily controlled in a virtual reality environment, the teacher avatar can be represented differently to communicate with the learner in the most optimal way (e.g., gender, age, similar cultural appearance), and students can experience content from different point of views (e.g., cross-cultural atmosphere, first person, third person, birds-eye view). The seating in virtual classrooms can even be positioned based on the learner's attention level (Bailenson, Yee, Blascovich, Beall, Lundblad, & Jin, 2008).

The next section contains an analysis of three separate computer-based learning applications that attempt to integrate these unique properties of technology.

Integrating Properties of Technology into Computer-Based Math Learning Applications

Three computer-based learning environments for math, Doodle Math, Puzzle Me, and GeoShape, were created as a testing environment and are used to highlight the three useful properties of technology. To highlight the interactive and communicative properties of technology, we emphasized them separately, but all three are at play in each example.

Engagement and Motivation: Computer-Based Math Learning Application Puzzle Me

The learning environment of Puzzle Me includes the properties of engagement and motivation, as well as informative feedback (see Figure 1). Puzzle Me attempts to develop metacognitive skills (e.g., self-correction of calculation mistakes) by having students monitor the reasoning of a pedagogical computer agent solving math problems. The motivation behind the math application is that children often find it difficult to be attentive to their own mistakes when they are concentrating on a problem or

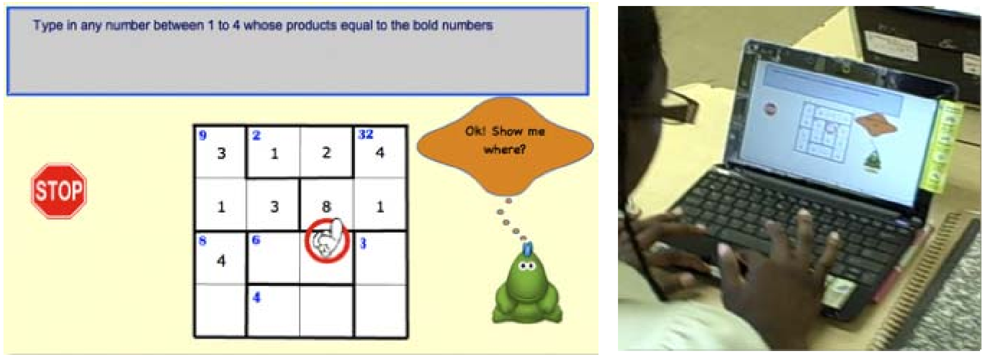

Figure 1. Computer-Based Puzzle Me Application.

task. However, they may find it relatively easy and are often motivated to catch other people's mistakes. This math application attempts to use this checking/monitoring behavior to students' advantage; by monitoring the agent for calculation mistakes, students may learn to self-correct and solve math problems with better accuracy. The student and agent both solve math problems using specific math strategies (e.g., like grouping numbers, divisibility rules), so the reasoning of the agent is more visible and easier to monitor/check for potential mistakes. The storyline is that the student and agent (e.g., dinosaur character agent) are a collaborative team, and the student is responsible for catching any mistakes the agent makes to avoid penalty for the team. The agent can be correct or incorrect. The student takes turns with the computer agent to solve addition or multiplication problems.

A series of informative feedback comments is provided to the student while he or she is monitoring the agent. The student is not aware of it, but the agent includes different calculation mistakes, general calculation mistakes common at the grade level, and calculation mistakes the student has made in previous sessions. If the student is consistent and uses only the same divisibility value (e.g., always uses 2 x 6 for 12), the agent will use a different divisibility rule to solve the problem in an attempt to familiarize the student with alternative ways to solve the problem (e.g., 3 x 4 for 12). The teacher also receives informative feedback that helps in assessing the student's learning process. In addition to the student's accuracy level in calculation, the application records when the student recognizes the mistake and whether monitoring the agent has helped the student self-correct when solving problems on his or her own. The log feature in the application keeps track of what the student types and indicates whether students catch their own mistakes and change their answers in the puzzle.

Informative Feedback: Computer-Based Math Learning Application Doodle Math

The next application, Doodle Math, has features similar to Puzzle Me, but includes additional features for informative feedback. When studying elementary and middle school mathematics, we observed several behaviors that led to mistakes that had little to do with the inability to calculate correctly. Some had to do with mixing operations (e.g., students adding when they should be multiplying), forgetting to add/subtract a number (e.g., when grouping numbers, forgetting a number or adding the same number twice), or loosing track because of sloppy handwriting and bad organization (e.g., in calculating bits and pieces of the problem on paper, the seven looks like a nine). In an attempt to address this problem, in Doodle Math, students solve math problems by writing out the calculations using an electronic pen and using an electronic tablet instead of paper (see Figure 2). As in Puzzle Me, the agent can play back the recording of the student as if it is the agent solving the problem with bad handwriting. This feature was created with the hope that self-reflection and self-monitoring may help change students' behavior. More specifically, it was designed so that students can realize as they monitor the agent how difficult it is to follow reasoning when the calculations are scattered, operations mixed, and handwriting sloppy (see Figure 3). The recording of the handwritten calculation provides informative feedback for the teacher because the recording can be played back in real time (e.g., at the student's actual calculation speed). In addition to seeing how the student solved the problem, the teacher can play back the recording and see where in the problem the student paused (or hesitated). This is the type of data that are difficult to obtain without technology, but easy to record with the processing capacity in computer-based applications. When technology is viewed as only a tool to assist with existing curriculum and instruction (rather than a central player in

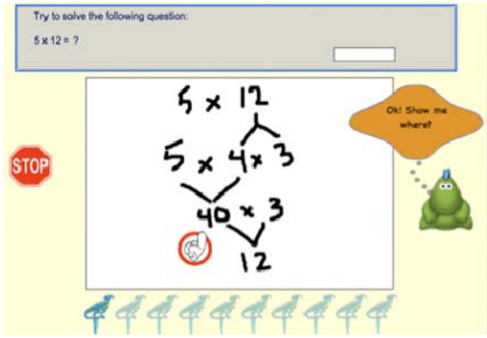

Figure 2. Doodle Math Application.

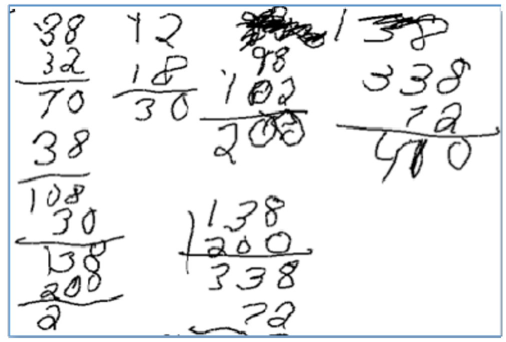

Figure 3. Disorganized Layout That Leads to a Calculation Mistake.

the learning environment), teachers may see more clearly where and how to integrate technology into their work.

Visualization: Computer-Based Math Learning Environment GeoShape

The application GeoShape has features similar to Puzzle Me, but includes additional features in visualization to develop geometric concepts, imagery, and spatial skills. Geometry is an important part of the mathematics curriculum, where children begin to develop spatial skills. It is through understanding the different characteristics seen in geometric shapes that students begin to analyze the relationships between structures. Cohen and Hegarty (2007) found that the ability to manipulate and understand computer-based visualization is correlated with students' spatial skill level, implying that those with poor spatial skills growing up will have difficulty understanding the visualizations created by technology. Studies from Barke (1993) found that well-developed spatial skills are essential for mathematics and the understanding of basic and structural sciences.

The GeoShape application allows students to build and manipulate visual representations of two- and three-dimensional objects through origami paper-folding tasks (see Figures 4 through 6). The origami puzzle is used to introduce students to the nature of basic geometric forms and help them understand spatial reasoning and the concepts of symmetry, congruence, angles, and patterns. The GeoShape application consists of a puzzle game that attempts to help students with visualization and imagery through mental manipulation of geometric figures. This computer-based application attempts to develop the visual and spatial skills needed to perceive an object from different perspectives with the hope of preparing students for future learning (e.g., calculating area and volume).

The GeoShape application first introduces students to the basic concepts of paper folding through representation of how a flat piece of paper can be turned into a three-dimensional object. Starting with simple folding steps, the visualized origami paper shows how a square can be folded to make two triangles or two rectangles, and prompting questions that appear on the screen ask whether the triangle and rectangle are the same size. Such questions are included to help students see visual representations and understand the relationship of squares, rectangles, and triangles. The GeoShape application consists of three puzzle game environments where students can perceive the visual origami models from different perspectives, which is known to help children interact and learn geometric math concepts that may otherwise be somewhat abstract.

In the first puzzle (folding explorations), students are presented with a visualization of one completed master origami model and four flat origami papers. Each of the four papers will show, step by step, how the model develops as the paper progresses through the folding sequence. Each paper will be developed into a different origami model (e.g., box, bird, boat, animal). In the puzzle, the students follow the four origami papers as they progress through the folding sequence. Students carefully observe each folding sequence and identify the one that matches the master origami model (see Figure 4). The earlier the student identifies the correct folding sequence, the more reward points he or she receives (i.e., for all puzzles, gain 8 points if right, lose 5 points if wrong to avoid random guessing). The second puzzle (relevant step match) shows several pieces of paper at different stages in the folding process, and students examine the shapes and patterns and guess which fold (in progress) will become the displayed origami model (see Figure 5). The third part (crease puzzle) provides an analysis of origami objects presented in the previous two parts. The three-dimensional origami objects are unfolded so that the crease pattern can be examined. Students look at several different crease patterns and determine which crease pattern belongs to the displayed origami model. Students must observe the details of the three-dimensional origami model and map them onto the flat paper (see Figure 6).

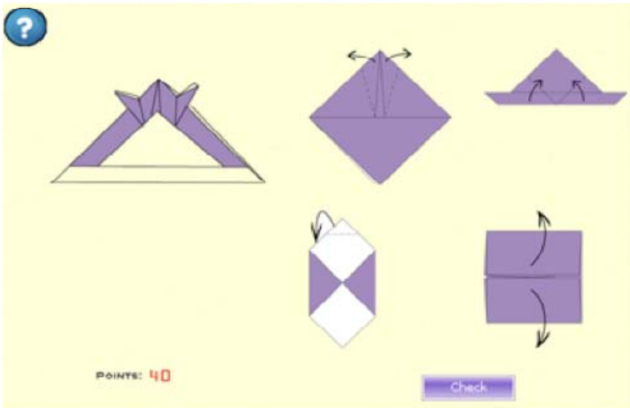

Figure 4. GeoShape Application: Folding Exploration.

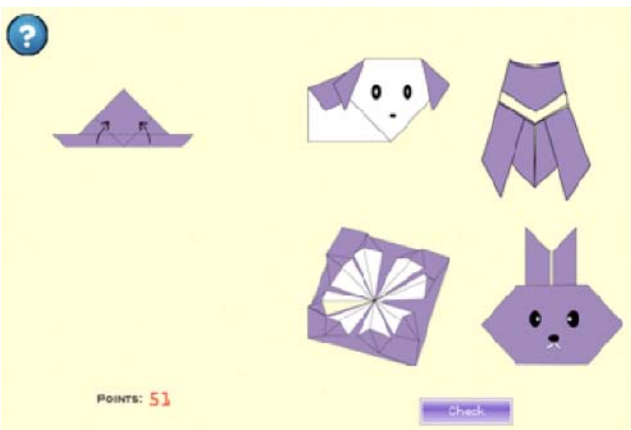

Figure 5. GeoShape Application: Relevant Step Match.

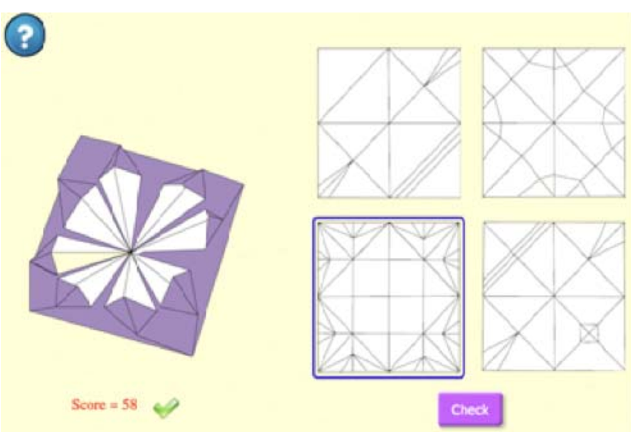

Figure 6. GeoShape Application: Crease Puzzle.

Future Work

As successful learning does not depend on a single device or learning mechanism, technology should be thought of as another factor within the learning domain. As learning often depends on situations that bring together a well-chosen confluence of learning resources and choices of technology, there is still much work to be done in providing teachers with usable concepts and frameworks for curriculum and instruction. Researchers must continue to test theories, develop robust learning and behavioral measures, and identify technology-rich learning conditions that positively influence learning. The next step in our work will be to conduct a full-scale study using our three computer-based math applications.

Summary

This paper addressed current challenges associated with educational technology in elementary and middle school mathematics education and technology's influence on curriculum, instruction, and student performance. The paper made several suggestions for supporting the partnership between mathematics education and technology: (a) help teachers identify the benefits and shortcomings of technology and provide specific examples of how technological properties can be linked to curriculum and instruction, (b) design activities that integrate tasks into relevant narratives so that engagement can lead to academic performance, and (c) create computer-based learning environments that encourage students to contribute content so that teachers can make informed assessments of students' learning process. Three properties of computer-based technology—motivation and engagement, informative feedback, and visualization—were described as having unique value for elementary and middle school mathematics. Three computer-based mathematics learning environments, the Doodle Math application, Puzzle Me application, and GeoShape application, were introduced as examples that bring together effective learning sources and technology.

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