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**Aims and Scope**

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Fall-Winter and 2013 Spring-Summer issues are *Equity* and *Leadership*, respectively.

*JMETC* readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

**Manuscript Submission**

*JMETC* seeks conversational manuscripts (2,500-3,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at [www.tc.edu/jmetc](http://www.tc.edu/jmetc).

**Abstract and keywords.** All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120<sup>th</sup> St., New York, NY 10027 or at [JMETS@tc.columbia.edu](mailto:JMETS@tc.columbia.edu).

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## *Journal of Mathematics Education at Teachers College*

### **Call for Papers**

The “theme” of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Equity*. This “call for papers” is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3500 words describing research, experiments, projects, innovations, or practices related to equity in mathematics education. Articles should be submitted to Ms. Krystle Hecker at [JMETC@tc.columbia.edu](mailto:JMETC@tc.columbia.edu) by September 1, 2012. The fall issue’s guest editor, Mr. Nathan N. Alexander, will send contributed articles to editorial panels for “blind review.” Reviews will be completed by October 1, 2012, and final manuscripts of selected papers are to be submitted by October 15, 2012. Publication is expected by November 15, 2012.

### **Call for Volunteers**

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors’ nor reviewers’ names and affiliations will be shared; however, editors’/reviewers’ comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at [hecker@tc.edu](mailto:hecker@tc.edu) or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

### **Looking Ahead**

Anticipated themes for future issues are:

Fall 2012	Equity
Spring 2013	Leadership
Fall 2013	Modeling
Spring 2014	Teaching Aids

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# Journal of Mathematics Education at Teachers College

Spring – Summer 2012

A CENTURY OF LEADERSHIP IN  
MATHEMATICS AND ITS TEACHING

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- Larson, M. R. (2011). *Administrator's Guide: Interpreting the Common Core State Standards to Improve Mathematics Education*. Reston, VA: NCTM.
- Loveless, T. (2012). *How well are American students learning?* Washington, DC: Brookings Institution, Brown Center on Education Policy.
- National Center for Education Statistics [NCES]. (2011). *The nation's report card: Mathematics 2011* (NCES 2012-458). Institute of Education Sciences, US Department of Education. Washington, DC: US Department of Education.
- NCTM. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.
- NCTM. (2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- NCTM. (2006). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*. Reston, VA: NCTM.
- National Mathematics Advisory Panel [NMAP]. (2008). *Foundations for Success: The Final Report of the National Mathematics Advisory Panel*. Washington, DC: US Department of Education.
- Noguera, P. (2004). Transforming high schools. *Educational Leadership*, 61, 26–31.
- Popham, J. W. (2008). *Transformative Assessment*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new US intended curriculum. *Educational Researcher*, 40(3), 103–116.
- Schmoker, M. J. (2006). *Results Now: How We Can Achieve Unprecedented Improvement in Teaching and Learning*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Waters, T., Marzano, R. & McNulty B. (2003). *Balanced Leadership: What 30 years of research tells us about the effect of leadership on student achievement*. Denver, CO: McRel.
- William, D. (2007). *Five "Key Strategies" for Effective Formative Assessment. Research Brief*. Reston, VA: NCTM.
- William, D., & Thompson, M. (2007). Integrating assessment with instruction: What will it take to make it work? In *The Future of Assessment: Shaping Teaching and Learning*, C. A. Dwyer (Ed.), 53–82. Mahwah, NJ: Lawrence Erlbaum.

### Using Item Analysis Data as a Tool to Inform Instruction in the Mathematics Classroom: A Model of Data-Driven Instruction

William Farber  
Mercy College

This article presents a method of using assessment data to help increase student learning and promote a variety of instructional practices in mathematics. The specific model being introduced links the assessment data from the New York State testing program in mathematics to instructional strategies applied in the mathematics classroom. Moreover, this model incorporates the application of an item analysis of the New York State Grades 3–8 Mathematics tests. This item analysis, developed by the New York City Department of Education, provides test data information which will help inform instruction by connecting assessment data to mathematics instructional approaches.

*Keywords:* item analysis, differentiated instruction, assessment, test-taking skills, exemplar, performance indicator, tiered learning.

#### The Need to Examine the Connections Between Instruction and Assessment

Recent declines in scores on tests of achievement in mathematics have become a focus of steadily increasing concern to mathematics educators. In fact, according to The University of the State of New York Office of Communications, “Nationally, about a third of fourth and eighth graders met the cutoff to be assessed as ‘proficient’ in math this year. New York’s students performed a little lower than that” (NYSESED, 2011). The achievement tests in

mathematics are deemed as “high-stakes” tests and generally consist of traditional paper-and-pencil assessments (McMillan, 2011, p.172). These types of assessments have been used for decades for measuring student achievement. In addition, school districts across the United States that use a “high-stakes” testing program have recently changed focus from monitoring student achievement to an accountability system, i.e., monitoring accountability of students, teachers, and school and district supervisors. In the past twenty years, a multitude of reform efforts connecting assessment data to instruction have taken place, which includes professional development for

## ASSESSMENT NOTES FROM THE FIELD

educators and the implementation of process and content standards (e.g., NCTM, CCSSM). These efforts intend to increase student learning and promote a variety of instructional practices using test data to help inform instruction (NCLB).

### The Importance of Connecting Instruction to Assessment

It is essential to consider assessment as a process that aids and develops student learning, and not solely as a document of a student’s knowledge, understanding, and abilities (NAEP, 2011). According to the National Council of Teachers of Mathematics (NCTM), “Assessment should support the learning of important mathematics and furnish useful information to both teachers and students. Assessment should be more than merely a test at the end of instruction to gauge learning, it should be an integral part of instruction that guides teachers and enhances students’ learning” (2000).

### Making Connections: Instructional Strategies in Mathematics Based on Assessment Data

This article focuses on strategies to connect mathematics instructional approaches to assessment data. With regards to the use of assessment to inform instruction, Popham (2003) states: “If each item on an assessment is linked to a specific grade-level expectation in mathematics, teachers can see which concepts individual students have mastered.” The specific model being presented here links assessment data from the New York State test in mathematics to instructional strategies applied in the mathematics classroom. This model incorporates the application of an item analysis. This item analysis was developed by the New York City Department of Education, and serves as a tool providing test data information from the New York State Grades 3–8 Mathematics Tests.

### A Description of the New York State Grades 3–8 Mathematics Testing Program

Before describing the model, it is important first to discuss the features of the New York State grades 3–8 mathematics testing program. All New York State public school students in grades 3–8 take yearly tests in mathematics to assess their mastery of the state’s learning standards. There are

three formats included on the tests. The multiple-choice questions measure basic skills, concepts, and procedures. The short response and extended response questions require students to support their answers by showing or explaining their work. Calculators are used in Grades 7 and 8 for short-response and extended-response questions only (NYSESED, 2011).

### Purpose of the Instructional Model Linking the Item Analysis Tool to Classroom Instruction

The purpose of this instructional model is twofold. First, it provides mathematics educators a sample of test data that can serve as an essential indicator of students’ understanding of the mathematics content. This helps inform teachers of what instructional modifications to make. According to Ashlock (2006), “Assessment is the process of gathering information about student learning and the use of that information to plan instruction” (p. 13). Second, it enhances students’ test-taking skills by examining, discussing, and re-assessing selected test items

**2009 Mathematics Tests Standard and Performance Indicator Map with Answer Key  
Grade 8**

Question	Type	Points	Strand	Content Performance Indicator	Answer Key
<b>Book 1</b>					
1	Multiple Choice	1	Algebra	7.A02 Add and subtract monomials with exponents of one	C
2	Multiple Choice	1	Geometry	8.G05 Calculate the missing angle measurements when given two parallel lines cut by a transversal	B
3	Multiple Choice	1	Geometry	7.G05 Identify the right angle, hypotenuse, and legs of a right triangle	B
4	Multiple Choice	1	Number Sense and Operations	8.N01 Develop and apply the laws of exponents for multiplication and division	C
5	Multiple Choice	1	Geometry	8.G05 Calculate the missing angle measurements when given two parallel lines cut by a transversal	B
6	Multiple Choice	1	Algebra	7.A04 Solve multi-step equations by combining like terms, using the distributive property, or moving variables to one side of the equation	C
7	Multiple Choice	1	Algebra	8.A03 Describe a situation involving relationships that matches a given graph	D
8	Multiple Choice	1	Geometry	7.G08 Use the Pythagorean Theorem to determine the unknown length of a side of a right triangle	D
9	Multiple Choice	1	Geometry	8.G04 Determine angle pair relationships when given two parallel lines cut by a transversal	C
10	Multiple Choice	1	Algebra	8.A08 Multiply a binomial by a monomial or binomial (integer coefficients)	D
11	Multiple Choice	1	Algebra	8.A03 Describe a situation involving relationships that matches a given graph	C
12	Multiple Choice	1	Number Sense and Operations	8.N05 Estimate a percent of quantity, given an application	B
13	Multiple Choice	1	Geometry	8.G04 Determine angle pair relationships when given two parallel lines cut by a transversal	C
14	Multiple Choice	1	Algebra	8.A02 Write verbal expressions that match given mathematical expressions	B
15	Multiple Choice	1	Geometry	8.G01 Identify pairs of vertical angles as congruent	A

Figure 1. 2009 New York State Grade 8 Mathematics Test Items, Standard, and Performance Indicator Map (Test Items 1–15)



## ASSESSMENT NOTES FROM THE FIELD

in a collaborative classroom setting. In recent years there has been increasing interest in teaching test-taking skills as part of a comprehensive test preparation program (Nolting, 1989; Fueyo, 1979). Claims have been made that such training leads to increased test scores.

### Description of an Instructional Model Linking the Item Analysis Tool to Classroom Instruction

The following example will demonstrate the features of the instructional model linking the item analysis tool to classroom instruction. This item analysis tool is posted on the New York City Department of Education website and contains comprehensive item analyses for the tests for grades three through eight. The data files contain school by school item analyses for Grades 3–8 for the 2008 and 2009 tests. The 2009 data include summaries by network and district. The data tool is organized as follows:

1. Each test item can be viewed, cross-referenced with performance indicators, and analyzed for varied levels of difficulty;
2. Student achievement data are reported for whole populations, or as aggregate data;
3. Data are also disaggregated, i.e., test scores may be analyzed by specific subgroups of students;
4. Graphical representation of data is displayed in several formats;
5. The test items are displayed showing
  - the top ten test items having the percentage of students obtaining a correct answer,
  - the bottom ten test items having the percentage of students obtaining an incorrect answer, and
  - the ‘middle test items’ are displayed in ranked order based on the percentage of students who answered the test item correctly; and
6. Hyperlinks are provided to each test item to display the actual question from the test with associated strand and standard.

### Using the Item Analysis Tool to Help Inform Instruction

The first step is to access the relevant grade-level and district data from the website, for example, New York City Schools (citywide). The data are presented in a chart format; let us call this chart the “Preliminary Information Chart.” This chart includes five columns with basic information:

**Column One:** The highest ten (the top ten test items having the percentage of students obtaining a correct answer) and the bottom ten test items (the bottom ten having the percentage of students obtaining an incorrect answer);

**Column Two:** Test item number;

**Column Three:** Percent of students who obtained the correct answer to the given test item;

**Column Four:** Type of question given, i.e., multiple choice, short response, or extended response item; and

**Column Five:** Mathematical strand correlated to each test item.

It is also possible to access summary data. This summary includes the total number of students tested on the particular test, the mean scale score (scale scores determine the student’s performance level), the percentage of students at each scoring level (the performance level that shows whether or not the student meets or exceeds New York State Learning Standards for mathematics), the percentage of each type of test item, and the percentage of test items that are categorized in one of the five mathematical content strands. Each question type has a specific weighted value of points (1, 2, and 3 points for multiple choice, short answer, and extended response, respectively).

The website <http://schools.nyc.gov/Academics/Mathematics/EducatorResources/Item+Analyses.htm> displays the grades 3–8 test items so that it is possible to match the item number with the actual question that was posed.

Figure 1 is part of the 2009 New York State Grade 8 Mathematics Test Items, Standard, and Performance Indicator Map.

The chart displays the following information:

- The question item number;
- The type of test question, i.e., multiple choice, short answer, or extended response;
- The point values allotted for each test question;
- The mathematical strand; and
- The performance indicator for each test question. The performance indicators are the specific topics taken from the New York State Learning Standards for Mathematics (2005) (<http://schools.nyc.gov/Academics/Mathematics/EducatorResources/SS.htm>).

### Item Analysis Data Provide a Basis for the General Improvement of Classroom Instruction

Before delving into the benefits of the item analysis tool, it is important first to provide a brief definition and description of an item analysis. An item analysis is a statistical tool used for collecting assessment data and observing student responses to test items. For example, an item analysis may include information involving the percentage of students who obtained either a correct or incorrect answer to a given test item. Item analysis data provide valuable information about student misconceptions and topics that may need additional work (Miller, Linn, & Gronlund, 2009). According to Miller et al. (2009), “Item-analysis data provide a basis for the general improvement of classroom instruction. Item- analysis data can assist in evaluating learning outcomes and course content for the particular students being taught.” For example, topics that

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**6** Solve the equation below for  $x$ .

$$2(6 + 2x) = 8x$$

**A**  $x = 1$

**B**  $x = 2$

**C**  $x = 3$

**D**  $x = 6$

Figure 2.

are generally too easy or too challenging may suggest instructional modifications or a change in teaching focus. Similarly, misconceptions in student thinking that consistently appear in item analysis data may point attention to the need for more effective teaching techniques. “In these and similar ways, item- analysis data can reveal instructional weaknesses and clues for improvement” (Miller et al., 2009).

### Using the Item Analysis Tool to Inform Mathematics Instruction in the Classroom

One method of using the item analysis tool is first to examine the test items in the Preliminary Information Chart for the desired district and grade. Test items can be selected for a variety of reasons based on either the educational objectives of the lesson or based on the curriculum strand being addressed. A specific test item contains one or more attributes and may be selected according to any of the options provided (item number, percent of students who answered correctly or incorrectly, type of question, etc.). The remaining portions of the item analysis process are given through two exemplar models.

### Classroom Vignettes: A Practical Application of the Item Analysis Tool

According to Stecher et al. (2006), “vignettes measure important aspects of reform-oriented instruction that are not captured by other measurement methods.” Therefore, the major purpose of the following two vignettes is to serve as models, which demonstrate the power of the item analysis tool. “Responding to vignettes individually or in small groups can be an effective way to experiment with ideas, build on the ideas of others, and work toward consensus in a nonthreatening manner” (Campbell, 1996).

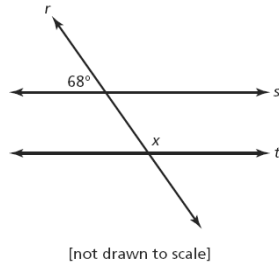
### *Vignette One—Examining a Single Test Item Using the Item Analysis Tool*

Ms. Lopez is a New York City middle school mathematics teacher with twelve years of teaching experience. She presently teaches four eighth grade mathematics classes of students with varying abilities. After carefully reviewing and analyzing the results of her class assessments, Ms. Lopez observed that many of her students were having difficulty understanding how to solve multi-step linear equations. Subsequently, Ms. Lopez’ goal was to access the data information to help reinforce the various methods, skills, and concepts for solving multi-step linear equations, and at the same time teach her students test-taking tips and strategies (Bangert-Drowns, Kulik, & Kulik, 1983; Fueyo, 1977; Ligon & Jones, 1981). First, Ms. Lopez referenced the chart entitled “2009 New York State Grade 8 Mathematics Test Items, Standard, and Performance Indicator Map” (Figure 1 gives the first 15 items), then selected a question that correlates to the desired performance indicator that she intends to explore with her classes, in this case, 7.A.4, “Solving multi-step equations by combining like terms, using the distributive property, or moving variables to one side of the equation” (NYSED, 2005). Ms. Lopez then decided to select test item 6 (Figure 2), which involves a linear equation requiring a multi-step approach and is aligned with performance indicator 7.A.4.

Ms. Lopez can now introduce techniques to her classes utilizing a variety of methods to differentiate the instruction for solving the multi-step equation in test item 6 (Tomlinson, 2008). For example, Ms. Lopez can discuss the traditional method of isolating the variable using the distributive property and inverse operations, or she can introduce methods that incorporate the use of concrete materials, i.e., the use of a balance scale and weights to represent the constants, coefficients, and variable of the equation, moreover, she can have students incorporate the use of technology by accessing websites such as NCTM’s Illuminations and the National Library of Virtual Manipulatives. Also, because test item 6 is a multiple choice question, Ms. Lopez suggested a strategy that may enhance a student’s test-taking skills (Bangert-Drowns, Kulik, & Kulik, 1983; Fueyo, 1977; Ligon, & Jones, 1981), which entails substituting each of the four choices (one at a time) into the equation in order to determine which choice yields the correct answer.

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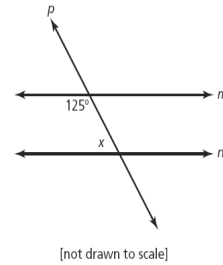
- 2** In the diagram below, line  $s$  is parallel to line  $t$ , and line  $r$  is



What is the measure of  $\angle x$ ?

- A  $158^\circ$
- B  $112^\circ$
- C  $68^\circ$
- D  $22^\circ$

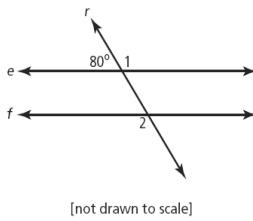
- 5** In the diagram below, line  $m$  and line  $n$  are parallel, and line  $p$  is a transversal.



What is the measure of  $\angle x$ ?

- A  $35^\circ$
- B  $55^\circ$
- C  $125^\circ$
- D  $215^\circ$

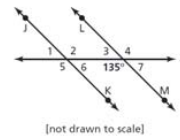
- 23** In the diagram below, line  $e$  and line  $f$  are parallel, and line  $r$  is a transversal.



What is the sum of the measures of  $\angle 1$  and  $\angle 2$ ?

- A  $100^\circ$
- B  $160^\circ$
- C  $180^\circ$
- D  $200^\circ$

- 29** In the diagram below,  $\overleftrightarrow{JK}$  and  $\overleftrightarrow{LM}$  are parallel, and line  $n$  is a transversal.



What is the measure of  $\angle 1$ ?

**Answer** \_\_\_\_\_ degrees

On the lines below, explain how you determined your answer.

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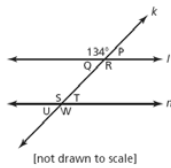


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- 34** In the diagram below, line  $l$  and line  $m$  are parallel, and line  $k$  is a transversal.



What is the measure of  $\angle U$ ?

**Answer** \_\_\_\_\_ degrees

On the lines below, explain how you determined your answer.

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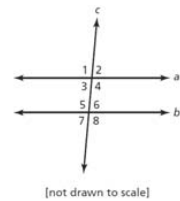


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- 39** In the diagram below, line  $a$  and line  $b$  are parallel, line  $c$  is a transversal, and the measure of  $\angle 1$  is  $100^\circ$ .



Is  $\angle 3$  congruent to  $\angle 1$ ? On the lines below, explain how you determined your answer. If it is not congruent, give the correct measure of  $\angle 3$ .

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Figure 3. Test Item Exemplars Aligned to Performance Indicator 8.G.5

## ASSESSMENT NOTES FROM THE FIELD

Test Item No.	Type of Question	# of Students with Correct Answer	Percent of Students Getting the Correct Answer
2	Multiple-Choice	62,355	86%
5	Multiple-Choice	60,905	84%
23	Multiple-Choice	26,102	36%
29	Short Answer	52,929	73%
34	Short Answer	55,104	76%
39	Short Answer	41,328	57%

Figure 4. Statistical information for each test question correlated to performance indicator 8.G.5

### *Vignette Two—Examining Several Test Items Using the Item Analysis Tool*

Ms. Lopez observed that her students were having difficulty with identifying and justifying geometric relationships pertaining to calculating missing angle measurements when two parallel lines are cut by a transversal. According to the New York State Mathematics Standards, this performance indicator is 8.G.5, “Calculate the missing angle measurements when given two parallel lines cut by a transversal” (NYSED, 2005). Subsequently, Ms. Lopez then accessed the item analysis chart (Figure 1) with the intention of selecting all the test items that correlated to performance indicator, 8.G.5. Ms. Lopez identified the following six test items, which will serve as exemplars: 2, 5, 23, 29, 34, and 39 (Figure 3).

Each of the six selected test items are aligned with performance indicator 8.G.5, however, each item requires a varying degree of cognitive demand (Smith & Stein, 1998). Ms. Lopez referenced the statistical information appearing in the Preliminary Information Chart and created another chart (Figure 4) listing the observed statistical information for each test question correlated to performance indicator 8.G.5. There were 72,506 students who took the test.

Ms. Lopez noticed the disparity of test results of the six selected test items. Although each of the six test items is within the same performance indicator (8.G.5), the level of difficulty and cognitive demand for each test item varies. This becomes a teaching opportunity for Ms. Lopez by utilizing the six test items as test item exemplars, which, in turn will help her students better understand performance indicator 8.G.5. Also, because of the varying degree of cognitive demand of these six test item exemplars, Ms. Lopez is now in the position to differentiate the instruction for content, processes, and product (Earl, 2003). According to Earl, “Differentiation also counsels that—armed with assessment information and other knowledge about a student—the teacher should adapt teaching plans to attend to learner readiness, interest, and preferred modes of learning. Once we understand what a student knows (and doesn't know), what motivates that student to learn, and how the student learns best,

differentiation is simply what comes next” (Earl, 2003). Using the statistical information above, Ms. Lopez can discern the approximate difficulty of each item, and therefore, Ms. Lopez can strategically use this information to plan a differentiated instructional approach. Her approach may include one or more of the following differentiated instructional strategies:

- Tiered learning, which is having students work on essentially the same performance indicator (8.G.5), but at different levels of difficulty (Pierce & Adams, 2005);
- Technologies, i.e., websites, videos, computers and calculators (NCTM, 2003);
- Cooperative problem solving, which offers students multiple modes of exploration (NCTM, 1989);
- The use of learning centers, where each learning center can feature one or two of the selected test item exemplars; and
- Present multiple perspectives on performance indicator 8.G.5.

### Implications for the Future

#### *Limitations and Possibilities of the Item Analysis Tool*

Each question type of New York State tests has a specific weighted value of points. Although the item analysis tool indicates the percentage of students getting a correct answer for any of the above mentioned items, the tool does not indicate “partial credit” for short answer or extended response test items, which is a limitation. However, a modification of the tool can be developed to indicate partial credit of short answer and extended responses. In fact, Appleseed Analytics produces customized item analyses and is able to produce a tool to include data pertaining to short or extended answer responses and partial credit which allows the teacher to identify specific strengths and weaknesses.

## ASSESSMENT NOTES FROM THE FIELD

### *New York State's Testing Policies: Suggestions for Change*

The policies of New York State Testing program have been modified for tests administered in 2011–2012. “As in 2011, the 2012 Grades 3–8 Mathematics Tests must be kept secure following their administration in April 2012 and cannot be released or posted on any web site; no part of the test may be used for instructional or staff development purposes. However, the tests administered from 2006 through 2010 will continue to be available on the NYSED website” (NYSED, 2011). Although this limits access to future New York State test questions, the format of the item analysis tool discussed can still serve as a valuable tool. For example, the item analysis tool may be used for classroom tests, school wide assessments, interim assessments, or district wide tests. Also, the performance indicators to be assessed have not changed and are still based upon the New York State Learning Standards which are available on the Department of Education’s web site.

### *The Common Core State Standards in Mathematics and Future Assessments*

There are assessment instruments being developed that are aligned to the Common Core State Standards in Mathematics (CCSSM). “The new state assessments, expected to be operational in 2014–2015, will focus on measuring deeper levels of thinking, and will therefore be more predictive of college and career readiness” (PARCC). The test data generated by these assessments can be analyzed using a customized version of the item analysis tool, which will continue to help inform instruction of the mathematics content embedded in the CCSSM.

### Summary

This paper addresses how educators can use item analysis test data as an invaluable tool to help inform mathematics instruction. The item analysis tool described and suggested in this article serves as a prototype or model that can provide powerful information to the classroom teacher, and may develop into the basis for instructional design and/or lesson planning and lesson modification. The two vignettes presented serve as illustrations that demonstrate and promote data-driven instructional strategies through careful observation of the data presented in the item analysis tool. The item analysis data tool reveals areas of strengths and weaknesses that may require needed changes in classroom instruction. It also provides data to help in assessing learning outcomes and course content for students. More importantly, misconceptions in student thinking that constantly emerge in item analysis data may focus attention to the need for more effective teaching procedures. Therefore, item analysis data can uncover instructional weaknesses and clues for improvement.

### References

- Ashlock, R.B. (2006). *Error patterns in computation: Using error patterns to improve instruction*, 9th ed., Upper Saddle River, NJ: Pearson Prentice Hall.
- Bangert-Drowns, R.L., Kulik, J.A., & Kulik, C.-L. (1983). Effects of coaching programs on achievement test performance. *Review of Educational Research*, 53, 571–585.
- Bartlett, J. E., II, Kotrlik, J. W., & Higgins, C. (2001). Organizational research: Determining appropriate sample size for survey research. *Information technology, learning, and performance journal*, 19(1) 43-50.
- Boudett, K.P., City, E.A., & Murnane, R.J. (Eds.) (2005). *Data wise: A step-by-step guide to using assessment results to improve teaching and learning*. Cambridge, MA: Harvard Education Press.
- Campbell, P.B. (1996). How would I handle that? Using vignettes to promote good math and science education. Washington, DC: American Association for the Advancement of Science, Collaboration for Equity.
- Data Driven Decision Making. (2005). *No Child Left Behind: A tool kit for teachers*.
- Earl, L. (2003). *Assessment as learning: Using classroom assessment to maximize student learning*. Thousand Oaks, CA: Corwin.
- Fairleigh Dickinson University. (2005). Action plan to promote data driven instruction to improve test performance.
- Fueyo, V. (1979). Training test-taking skills: A critical analysis. *Psychology in the Schools*, 14, 180–184.
- Goertz, M.E., Olah, L.N., & Riggan, M. (2009). *Can interim assessments be used for instructional change? CPRE Policy Brief*. University of Pennsylvania, Philadelphia, PA: Consortium for Policy Research in Education. Retrieved from [http://www.cpre.org/images/stories/cpre\\_pdfs/rb\\_51\\_role%20policy%20brief\\_final%20web.pdf](http://www.cpre.org/images/stories/cpre_pdfs/rb_51_role%20policy%20brief_final%20web.pdf)
- Ligon, G.D. & Jones, P. (1981). Preparing students for standardized testing: One district’s perspective. ERIC Document 218319.
- McMillan, J.H. (2011). *Classroom assessment: Principles and practice for effective standards-based instruction*, (5th ed.). MA: Pearson Publishing Company.
- Miller, D., Linn, R.L., & Gronlund, N.E. (2009). *Measurement and assessment in teaching* (10th ed.). Upper Saddle River, NJ: Pearson Prentice Hall.
- Murnane, R. J., Sharkey, N. S., & Boudett, K. P. (2005). Using student-assessment results to improve instruction: Lessons from a workshop. *Journal of Education for Students Placed at Risk*, 10(3), 269–80.
- National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

## ASSESSMENT NOTES FROM THE FIELD

- National Council of Teachers of Mathematics [NCTM]. (1995). *Assessment standards for school mathematics*. Reston, VA: NCTM.
- National Council of Teachers of Mathematics [NCTM]. (1989). *New directions for elementary school mathematics*. 1989 Yearbook of the NCTM, P. R. Trafton (Ed.). Reston, VA: NCTM.
- National Council of Teachers of Mathematics [NCTM]. (2003). NCTM position statement, The use of technology in the learning and teaching of mathematics.
- The National Assessment of Educational Progress [NAEP], (2011).
- The Nation's Report Card: Trial Urban District Assessment Mathematics 2011. Retrieved from <http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2012452>
- New York City Department of Education. (2012). NYC DOE and the NY State P-12 Common Core learning standards. Retrieved from <http://schools.nyc.gov/Academics/CommonCoreLibrary/Why/NYSStandards/default.htm>
- New York State Board of Regents. (2011). The State Education Department, The University of the State of New York, Office of Communications, Albany, NY. Retrieved from <http://www.nysed.gov>
- New York State Education Department [NYSED]. (2005). Learning standards for mathematics. Albany, NY: NYSED.
- New York State Testing Program, Developed and published under contract with the New York State Education Department by CTB/McGraw-Hill LLC, a subsidiary of The McGraw-Hill Companies, Inc., Monterey, CA.
- Noiting, P.D. (1989). *Winning at math*. Academic Success Press, Inc.
- Paek, P. L. (2008, January). Connecting mathematics assessment to instruction: Anchorage School District. Profile from practices worthy of attention: Local innovations in strengthening secondary mathematics. Austin, TX: Charles A. Dana Center at The University of Texas at Austin.
- Pellegrino, J.W. & Goldman, S. R. (2008). Beyond rhetoric: Realities and complexities of integrating assessment into classroom teaching and learning. In C. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 7–52). New York: Lawrence Erlbaum Associates.
- Pierce, R. & Adams, C. (2005). Using tiered lessons in mathematics. *Mathematics teaching in the middle school*, 11(3), 144.
- Popham, W.J. (2003). *Test better, teach better: The instructional role of assessment*. Alexandria, VA: Association for Supervision and Curriculum Development.
- Smith, M.S., & Stein, M.K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics teaching in the middle school*, 3, pp. 344–350.
- Stecher, B., Le, V.-N., Hamilton, L, Ryan, G., Robyn, A., & Lockwood, J. R. (2006). Using structured classroom vignettes to measure instructional practices in mathematics. *Educational evaluation and policy analysis*, 28(2), pp. 101–130.
- Tomlinson, C.A. (2008). The goals of differentiation. *Educational leadership*, 66(3), pp. 26–30.
- William, D., & Thompson, M. (2008). Integrating assessment with learning: What will it take to make it work? In C. Dwyer (Ed.), *The future of assessment: Shaping teaching and learning* (pp. 53–82). New York: Lawrence Erlbaum Associates.
- US Department of Education, National Center for Education Statistics. (2011). Digest of education statistics, 2010 (NCES 2011-015) (Chapter 2).
- US Department of Education. (2004). No Child Left Behind (NCLB).

### On-Line Resources:

- Appleseed Analytics:  
[http://www.appleseedanalytics.com/home/?\\_r=default](http://www.appleseedanalytics.com/home/?_r=default)
- Common Core State Standards Library of Resources:  
<http://schools.nyc.gov/Academics/CommonCoreLibrary/Why/NYSStandards/default.htm>
- Information on the Use of Calculators:  
<http://www.emsc.nysed.gov/3-8/7-8calculators.htm>
- The National Assessment of Educational Progress (NAEP):  
<http://nces.ed.gov/nationsreportcard/>
- National Library of Virtual Manipulatives:  
<http://nlvm.usu.edu/en/nav/vlibrary.html>
- New York State Department of Education [NYSED]:  
<http://www.nysed.gov>
- New York State Department of Education [NYSED] - General Information on Testing Accommodations:  
<http://www.vesid.nysed.gov/specialed/publications/policy/testaccess/guide.htm>
- New York State Education Department Parent Resources:  
<http://usny.nysed.gov/parents/>
- New York State Grade 8 Mathematics Test Items, Standard, and Performance Indicator Map (2009): <http://www.nysedregents.org/Grade8/Mathematics/20090309scoringkey.pdf>
- New York State Department of Education. (2003). Learning Standards for Mathematics: <http://www.emsc.nysed.gov/ciai/ela/pub/elalearn.pdf>.
- New York State Department of Education – Mathematics Tests, Grades 3–8: <http://www.p12.nysed.gov/apda/samplers/>
- New York City Department of Education, Educator Resources:  
<http://schools.nyc.gov/Academics/Mathematics/EducatorResources/default.htm>
- New York City Department of Education: <http://schools.nyc.gov>
- New York City Department of Education – Item Analysis Tool:  
[http://schools.nyc.gov/Documents/STEM/Math/ItemAnalyses/Gr6-Gr8MItemAnal2009\\_v2.xls](http://schools.nyc.gov/Documents/STEM/Math/ItemAnalyses/Gr6-Gr8MItemAnal2009_v2.xls)
- Partnership for the Assessment of Readiness for College and Careers (PARCC): <http://schools.nyc.gov/Academics/CommonCoreLibrary/Why/PARCC>