The *Journal of Mathematics Education at Teachers College* is a publication of the Program in Mathematics and Education at Teachers College Columbia University in the City of New York.

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Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Fall-Winter and 2013 Spring-Summer issues are *Equity* and *Leadership*, respectively.

JMETC readers are educators from pre-K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the JMETC include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

Manuscript Submission

JMETC seeks conversational manuscripts (2,500-3,500 words in length) that are insightful and helpful to mathematics educators. Articles should contain fresh information, possibly research-based, that gives practical guidance readers can use to improve practice. Examples from classroom experience are encouraged. Articles must not have been accepted for publication elsewhere. To keep the submission and review process as efficient as possible, all manuscripts may be submitted electronically at www.tc.edu/jmetc.

Abstract and keywords. All manuscripts must include an abstract with keywords. Abstracts describing the essence of the manuscript should not exceed 150 words. Authors should select keywords from the menu on the manuscript submission system so that readers can search for the article after it is published. All inquiries and materials should be submitted to Ms. Krystle Hecker at P.O. Box 210, Teachers College Columbia University, 525 W. 120th St., New York, NY 10027 or at JMETC@tc.columbia.edu.

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Journal of Mathematics Education at Teachers College

Call for Papers

The "theme" of the fall issue of the *Journal of Mathematics Education at Teachers College* will be *Equity*. This "call for papers" is an invitation to mathematics education professionals, especially Teachers College students, alumni and friends, to submit articles of approximately 2500-3500 words describing research, experiments, projects, innovations, or practices related to equity in mathematics education. Articles should be submitted to Ms. Krystle Hecker at JMETC@tc.columbia.edu by September 1, 2012. The fall issue's guest editor, Mr. Nathan N. Alexander, will send contributed articles to editorial panels for "blind review." Reviews will be completed by October 1, 2012, and final manuscripts of selected papers are to be submitted by October 15, 2012. Publication is expected by November 15, 2012.

Call for Volunteers

This *Call for Volunteers* is an invitation to mathematics educators with experience in reading/writing professional papers to join the editorial/review panels for the fall 2012 and subsequent issues of *JMETC*. Reviewers are expected to complete assigned reviews no later than 3 weeks from receipt of the manuscripts in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions seem appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared; however, editors'/reviewers' comments may be sent to contributors of manuscripts to guide further submissions without identifying the editor/reviewer.

If you wish to be considered for review assignments, please request a *Reviewer Information Form*. Return the completed form to Ms. Krystle Hecker at hecker@tc.edu or Teachers College Columbia University, 525 W 120th St., Box 210, New York, NY 10027.

Looking Ahead

Anticipated themes for future issues are:

Fall 2012 Equity
Spring 2013 Leadership
Fall 2013 Modeling
Spring 2014 Teaching Aids

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Journal of Mathematics Education at Teachers College

Spring – Summer 2012

A CENTURY OF LEADERSHIP IN MATHEMATICS AND ITS TEACHING

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The Mathematics Portfolio: An Alternative Tool to Evaluate Students' Progress

Marla A. Sole

Eugene Lang College of the New School for Liberal Arts

This article describes the need for more thorough and varied forms of assessment to evaluate students' level of understanding in mathematics. Portfolios are one type of assessment tool that, when added to a teacher's repertoire can improve students' comprehension and retention and enable students to monitor their own progress and to take more responsibility for their own learning. Portfolio assignments can also help students and teachers to detect and remedy weaknesses and misunderstandings and can increase students' self-confidence in mathematics. This article discusses what a portfolio is, gives an example of a unit portfolio used in an undergraduate Finite Mathematics course to assess students understanding of Venn diagrams, describes the benefits and challenges of using unit portfolios, and discusses how teachers can evaluate portfolios with a rubric. By providing practical advice, this article can be used as a framework for teachers who have the desire to incorporate portfolio assignments, in addition to more traditional tools of assessment, at the undergraduate or K–12 grade levels.

Keywords: assessment, finite mathematics, portfolio, Venn diagrams, rubric.

In 1995, the National Council of Teachers of Mathematics (NCTM) published the Assessment Standards for School Mathematics which urged teachers to revaluate the mathematics students need to know, how students would use mathematics outside of the classroom, and how students' level of understanding would be evaluated. According to the NCTM, students should be able to apply the material they have learned in a mathematics class to other academic disciplines and to solve real-world problems, should develop mathematical reasoning skills and be able to communicate mathematically, and, as students become more comfortable and proficient with mathematics, they should gain confidence in their mathematical abilities and come to value mathematics (NCTM, 1989, 1995). Although mathematics educators share these goals, the propensity of some teachers to rely almost exclusively on tests, quizzes, and homework problems from the assigned textbook to assess students' learning and to evaluate their performance is not wellaligned with the aforementioned goals.

More traditional tools of assessment, such as tests and homework, provide valuable feedback that is relatively easy to evaluate. However, this feedback alone is not a sufficient means of thoroughly evaluating a student's level of understanding. Moreover, it is unclear whether students who perform poorly on tests, quizzes, and homework are able to monitor their own comprehension effectively and detect any difficulties or misunderstandings before their work was submitted and graded. Portfolios, on the other hand, are completed over a period of time and may undergo several revisions. This can help students clarify any misunderstandings they might have and help increase students' level of mathematical self-confidence.

Portfolios have been identified as a valuable assessment tool (Birgin, 2011; Burks, 2010; Crowley,

1993; Fukawa-Connelly & Buck, 2010; Jones, 2010; NCTM, 1989; Robinson, 1998) that can be used to supplement or replace more traditional mechanisms of assessment. However, despite the fact that portfolios have been reported to increase the level of student performance (Burks, 2010), they have been underutilized (NCTM, 1989). For teachers who are considering expanding the repertoire of tools they use to evaluate students' comprehension to include a portfolio assignment, this paper provides practical advice. The article begins by describing what a portfolio is and then an example is provided of a unit portfolio assignment about Venn diagrams that was used in an undergraduate Finite Mathematics course. Next, after the benefits and challenges of using portfolios are described, how to evaluate portfolios using a rubric is described. This article provides practical suggestions for integrating a portfolio assignment into a mathematics course curriculum and should interest mathematics teachers who teach undergraduate courses or at the K-12 grade level, as the sample portfolio assignment could be modified.

What is a Portfolio?

Mathematics portfolios consist of a collection of work that a student has done, which can include the following: homework, tests, quizzes, class notes, group projects, in-class activities, written assignments, a mathematics autobiography, a research project, mathematical vocabulary, a mathematical proof, a book review, photographs or drawings of three dimensional objects, multiple solutions to challenging exercises, and student generated problems. Among mathematics educators there is no consensus as to what types of work should go into a portfolio (Birgin, 2011; Kuhs, 1994; Tillema & Smith, 2000). According to

Haladyna (1997), there are five distinct types of portfolios. The types of portfolios are 1) an "ideal portfolio," which contains all of the work a student completed; 2) a "showcase portfolio," which consists of a selected collection of a student's finest work; 3) a "documentation portfolio," which contains work that the student has done over a period of time and therefore provides evidence of skills that have been improved upon or general growth in mathematical thinking; 4) an "evaluation portfolio," which includes some work that had not previously been submitted, which could be graded; and 5) a "class portfolio," which contains a student's grade and an evaluative assessment of the student by the teacher.

When deciding which type of portfolio will be used, teachers need to take into account the level of the course. age of the student, and how the portfolio will be used and evaluated. If the portfolio will be graded, teachers need to decide whether work that had been previously submitted and assigned a grade should be included. According to Crowley (1993), although some new work should be included if the portfolio is to be graded, the portfolio should not consist of entirely new work because this would result in the portfolio no longer having a "selected" feeling, which Crowley argued was an essential part of a portfolio. By contrast, Fukawa-Connelly and Buck, (2010) used portfolio assignments for specific units that consisted entirely of new material created for the portfolio and reported that this alternative model worked well. In my course, I also used a portfolio that consisted of entirely new material that had not previously been evaluated, which I found worked best.

Whether a portfolio consists of new or previously evaluated work, teachers need to decide whether a student alone would select the work to be included or whether the student would discuss with the teacher which work would be included. In either case, according to Asturias (1994), the primary responsibility for assembling the portfolio rests with the student.

Designing a Unit Portfolio Assignment

As part of an undergraduate Finite Mathematics course, students learn the vocabulary used to describe Venn diagrams and sets, how to illustrate relationships with Venn diagrams, and how to interpret all of the regions of a Venn diagram. Even when students mastered the homework problems and were able to answer test questions about Venn diagrams correctly, I was not sure that they would think to construct a Venn diagram if they were given a data set and asked to provide a visual representation.

For this assignment, students were given the data shown in Table 1, which gives the number of people living in poverty by gender for the years 2005–2010 compiled by the United States Census Bureau (http://www.census.gov/hhes/www/poverty/data/historical/people.html). All of the numbers are in thousands of people. Students were asked to describe the relationships using the correct mathematical vocabulary, to construct several Venn diagrams, and to write a few sentences to accompany each diagram. Students were also asked to read two articles discussing poverty in the United States during the years 2005 to 2010 that focused on gender and to include a few findings from the articles.

I chose to use these data to connect the concepts learned in class to a real-world issue, which is supported by the NCTM (2000) and to have students become better informed about an important problem that unfortunately is growing. Although the students spent only a few days learning about Venn diagrams in class, they were given several weeks to complete this assignment. This led to a more thorough analysis and increased the quality of the work submitted, a finding other researchers have reported (Fukawa-Connelly & Buck, 2010; Rohrer & Taylor, 2006). Although students quickly realized that the men living below poverty was a subset of all of the men, it took students some time to realize that for each of the years they could subtract the number of men and women living below poverty from the respective totals, and then using the two non-overlapping categories men/women and below

Men			Women	
Year	Total	Below Poverty	Total	Below Poverty
2010	150,413	21,012	155,275	25,167
2009	149,237	19,475	154,582	24,094
2008	147,862	17,698	153,179	22,131
2007	146,655	16,302	152,044	20,973
2006	145,486	16,000	150,964	20,460
2005	143,803	15,950	149,331	21,000

poverty/not below poverty could construct four different Venn diagrams.

The design was purposely open-ended because several different types of Venn diagrams could be constructed and I wanted students to realize that there was not just one correct solution that I was looking for them to produce. I have found that open-ended questions yield more thoughtful explorations and more varied problem solving strategies, a finding supported by Hancock (1995). Robinson (1998) reported that portfolio assignments that do not have a good deal of formal structure enhance students' creativity.

This assignment was designed to be completed outside of class time. However, in order to help students organize their work and clarify potential misunderstandings, students had the opportunity to receive feedback in class and during office hours. Students also had the opportunity to receive feedback from their peers. The class was instructed to construct as many different visual representations as possible. As a challenging extension, students could try to calculate the number of different Venn diagrams that could be constructed. At the end of the semester, students presented their portfolios to the class.

Benefits of Incorporating Portfolios

One benefit of incorporating a portfolio was that this assignment appealed to students who had different learning styles. In particular, students who had some apprehension about studying mathematics or who became anxious taking tests appreciated having the chance to complete a portfolio assignment. Many students who have either mathematics or test anxiety stated that they were eager to work on an assignment that could reflect their strengths and work ethic.

Another benefit was that while working on this assignment, students needed to manage their time and monitor their progress. As a result, students took more responsibility for their own learning and worked autonomously. Additionally, because students could receive feedback, they were more likely to be able to identify their strengths and weaknesses, and, more importantly, had the chance to fix errors in their work and clarify any points of confusion. This led to a deeper level of understanding and, as a result, the quality of the work they submitted was of a high caliber. Other researchers (Asturias, 1994; Birgin, 2011; Burks, 2010; Crowley, 1993; Fukawa-Connelly & Buck, 2010; Jones, 2010; Rohrer & Taylor, 2006) have reported similar benefits.

The written component of the portfolio assignment improved students' communication skills, as they were more likely to be able to recall and use the correct mathematical terminology when describing sets.

I thoroughly enjoyed incorporating portfolios as an evaluative tool, because portfolios gave students a feel for how mathematicians work and for how discoveries in the

field of mathematics are made. Mathematics is not about memorizing formulas and facts. In fact, mathematics is quite creative (Kogelman & Warren, 1979). Mathematicians look to establish some yet unproven concept is, in fact, true (Tao, 2007). This assignment models the way mathematicians work as students thought about the portfolio assignment for several weeks, received feedback or assessed their own work, and then made revisions. Students also seemed to enjoy creating an original portfolio.

Challenges of Incorporating Portfolios

Portfolio assignments present unique challenges to students and teachers. For students who may have previously only been asked to complete shorter assignments in mathematics courses, time management can be an issue. Birgin (2011) noted that this can be a source of stress for students. Some of the students in my class expressed feeling a bit unsure how to go about answering an open-ended question. Despite some students' initial discomfort with these types of questions, because openended exercises mimic the way problems arise outside of the classroom, I believe this approach ultimately facilitated learning.

For teachers, the biggest challenges of using portfolios as compared to tests, quizzes, and homework assignments are that portfolios take more time to design, implement, and evaluate. After years of incorporating portfolios into the course curriculum, I have found that the time required to plan unit portfolio assignments and to address students' questions decreased significantly, as I was able to modify existing assignments and anticipate and address many of the questions and concerns students had when the assignment was handed out. However, grading portfolios has remained an extremely time-consuming and demanding task when compared with the ease with which tests are assigned numerical grades. Fukawa-Connelly and Buck (2010) have reported similar challenges grading.

Grading with a Rubric

Other researchers (Asturias, 1994; Thompson & Senk, 1998) have suggested that open-ended questions and portfolios could be evaluated with a rubric. Therefore, I used a rubric to ensure that all students who had met the same expectations received the same grade.

A single grade was assigned to the entire portfolio. I wanted students to be thorough and for their portfolios to contain multiple Venn diagrams that were similar. Some teachers may prefer to have students select only their best work. Teachers who use open-ended questions with portfolio assignments need to inform the class when the assignment is handed out whether they expect students to submit a sample of the work completed or a large number

of similar exercises, so that students who include only their best work are not penalized for having a thin portfolio.

Stutzman and Race (2004) provided a model of a fourlevel rubric that I used as a rough guide to design a rubric for this type of assignment, modifying or expanding the descriptions of work that I classified as excellent, good, needing revision, or poor. Work that was classified as excellent met or exceeded expectations, had multiple visual representations, and demonstrated a high level of critical thinking and understanding. Excellent work was also creative, and had clear, complete sentences accompanying the diagrams using the appropriate mathematical vocabulary. Work that was classified as good met expectations and demonstrated a competent level of understanding. However, this work was less thorough. This work had clear but less thoughtful or eloquently written comments, and did not always use the correct mathematical terminology or had some minor grammatical or spelling mistakes. Work that was classified as needing to be revised did not demonstrate a clear level of understanding. Although some of the visual representations were correct, there was little variety in the types of visual representations produced. Additionally, some of the written comments used the mathematical vocabulary incorrectly or there may have been some grammatical or spelling mistakes. Work that was classified as poor had some work that was mathematically incorrect. There was also little variety in the work produced. Assignments classified as poor either had no written comments to accompany the visual representations or the mathematical vocabulary was used incorrectly or the sentences, although mathematically correct, had several grammatical or spelling mistakes.

Conclusion

Traditional forms of assessment are easy to design, implement, and evaluate, which may be one reason why they have been heavily relied upon. However, although it is more challenging and time-consuming to design, employ, and assess unit portfolio assignments, making portfolio assignments an integral part of a mathematics course can be beneficial. I have found that these assignments increase students' level of understanding and provide teachers with a more dynamic and comprehensive assessment of students' understanding.

Portfolio assignments provide a more complete picture of students' learning than a single assessment or a single grade. Whereas traditional measures of assessment provide a snapshot of students' performance on the course objectives, portfolios can show growth over time. More importantly, because students have the opportunity to revise their work and receive feedback from their teacher and their peers, portfolios shed light on misunderstandings, which can then be clarified. The opportunity to work on an assignment over several weeks and to make revisions led

to increased comprehension and retention of key concepts. Incorporating a written component helped students use and recall the correct mathematical terminology.

Portfolio assignments appealed to students with a variety of learning styles or to those students who may suffer from mathematics anxiety or test anxiety. As a result, portfolios appeared to increase students' level of self-confidence with mathematics. Because students monitored their time and progress, they were more apt to work independently.

Portfolio assignments gave students the opportunity to understand how mathematicians work. These assignments reinforced the idea that there is not always one correct way to approach a problem, as no two portfolios were identical. As a result, students had the opportunity to see the creative side of mathematics.

Despite the difficulties that may arise, incorporating well-designed portfolio assignments to supplement more traditional modes of assessment can be beneficial, as portfolios promote learning, provide a more comprehensive mode of assessment, and reinforce the idea that mathematics is a field of study that values originality.

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