Journal of Mathematics Education at Teachers College

Spring – Summer 2013

A Century of Leadership in Mathematics and its Teaching

TABLE OF CONTENTS

Preface

Articles

TABLE OF CONTENTS

LEADERSHIP NOTES FROM THE FIELD

Other

Journal of Mathematics Education at Teachers College
Spring–Summer 2013, Volume 4

Distributed Leadership: Key to Improving Primary Students' Mathematical Knowledge

Matthew R. Larson
Lincoln Public Schools, Nebraska

Wendy M. Smith
University of Nebraska-Lincoln

The purpose of this article is to present the findings of a quantitative study focused on primary mathematics teachers who participated in an intensive professional development program and then had leadership responsibility for the implementation of a new primary mathematics curriculum in their district. The study examines the effect of the professional development on teachers' and students' mathematical knowledge. Results indicate that effective implementation of a new curriculum can improve students' mathematical knowledge, but that the effect of the curriculum implementation on student mathematical knowledge is enhanced when teachers improve their mathematical knowledge for teaching and are simultaneously involved in the leadership activities of the curriculum implementation.

Keywords: primary mathematics, professional development, curriculum implementation, teacher leadership, mathematical knowledge for teaching

Introduction

NebraskaMATH is a five-year, $9.2 million, Targeted Math Science Partnership funded by the National Science Foundation that began on January 1, 2009. Administered by the University of Nebraska-Lincoln, NebraskaMATH is a P-16 statewide partnership between the University of Nebraska-Lincoln, four of Nebraska's largest school districts, and Nebraska's Educational Service Units. A guiding principle of NebraskaMATH is that quality instruction matters and that teachers of mathematics must be provided opportunities to strengthen their mathematical and pedagogical knowledge in order to increase student achievement and close achievement gaps. Based on research that early math skills have the greatest predictive power on later academic achievement (Duncan, Dowsett, Claessens, et al., 2007), calls to strengthen primary mathematics education (Ginsburg, Lee, & Boyd, 2008; NAEP & NCTM, 2002; NCTM, 2007; NRC, 2009), and a belief that high expectations in secondary mathematics can only be achieved if children's mathematics learning in the primary grades is significantly strengthened, a major component of NebraskaMATH is Primarily Math.

Primarily Math is a program that focuses on providing professional development opportunities for teachers in grades K–3. Primarily Math has two phases: 1. An 18–21 credit hour graduate program that emphasizes mathematics, mathematics specific pedagogy, and working with children in the early grades; 2. Support for teachers after they complete the graduate program and work to strengthen mathematics teaching and learning in the classrooms and schools through on-going study groups co-led by university and school district personnel. The graduate course work emphasizes mathematics content from number and operations, geometry and measurement, and algebraic thinking. The pedagogical course work focuses on working with young children to support their development as mathematical thinkers by emphasizing the NCTM process standards (NCTM, 2000), communities of practice, and an optional leadership course for teachers who were interested in becoming math specialists, math instructional coaches, or assuming other instructional leadership roles in their school districts. The research component of Primarily Math sought answers to two general questions: 1. What is the effect of a professional development program that emphasizes mathematics content and mathematics content-specific pedagogy on teachers' mathematical knowledge for teaching? 2. What is the effect of a professional development program that emphasizes mathematics content and mathematics content-specific pedagogy on primary students' mathematics achievement?

The Study

This study reports on the results of participants in Primarily Math from one of the partner school districts who participated in Primarily Math teacher Cohorts 1 and 3, who began their participation in the summer of 2009 and 2011, respectively. The participants from this single school district were selected for inclusion in this study because the school district had the largest number of participants in Primarily Math and because the district planned to implement a new primary math curriculum in grades K–2 during the 2011–2012 school year, and conducted a year-long K–2 curriculum study and nine-week K–2 curriculum pilot during the 2010–2011 school year. The district curriculum study committee had twelve K–2 teacher members, all of whom had been selected for Primarily Math. The study committee received significant professional development (18 hours), including reading and discussing several research articles and books that were consistent with the instructional philosophy of Primarily Math, and in some cases the same materials that were used during Primarily Math. One-half of the pilot teachers of the program selected for implementation were Primarily Math participants. The pilot teachers received an additional 7.5 hours of professional development consistent with the goals of Primarily Math.

The K–2 math curriculum the school district selected to implement after a year of study and nine weeks of piloting was *Math Expressions* (Fuson, 2011). The development of *Math Expressions* was partially supported with funding from the National Science Foundation and is considered supportive of and coherent with the instructional goals of the Primarily Math program. Because the partner school district implemented this curriculum during the 2011–2012 school year, it became possible to analyze the effect of a curriculum implementation on student achievement when the curriculum implementation professional development activities are intellectually designed and led by a group of teacher leaders who had the opportunity to participate in an intensive professional development program that emphasized both mathematics content and content-specific pedagogical strategies that were congruent with the selected curriculum and the goals of the district.

School districts have found that purchasing new textbooks is not enough to improve student outcomes (e.g., Bay, 2000). A district needs both extensive buy-in from teachers (Stein, Grover, & Henningsen, 1996) and significant supports in place for implementation in order for teachers to effectively utilize curricular materials to improve classroom practices and student outcomes (Bay, B. Reys, & R. Reys, 1999). A distributed leadership model where the teacher leaders are "on the same page" with philosophies of teaching and learning mathematics, aligned to district goals and new curricula, has the potential to achieve meaningful change (Hargreaves & Fink, 2005; Spillane, 2006; Spillane & Diamond, 2007). Recent research indicates that mathematics teacher leaders can serve as critical brokers of advice and information about mathematics and mathematics instruction within and between schools, and provide coherence between district curriculum and classroom teachers, thereby supporting implementation of district-mandated curricular initiatives (Hopkins, Spillane, Millerd, & Heaton, in press). This study builds on this research by examining the effect of teacher leadership of a curriculum implementation on student mathematical understanding.

The district's implementation of *Math Expressions* was supported by an initial 3.5-hour professional development session and a monthly one-hour session (eight total), led by a district math curriculum team that both designed and delivered the implementation professional development, consisting of the following:

- K–12 District Curriculum Specialist for Mathematics
- K–2 District Math Coordinator
- Three, grade level (K, 1, & 2) district-wide teacher leaders
- Thirty-three teacher leaders, 11 at each grade K, 1, & 2
- Thirty-seven elementary building K–2 math liaisons

The District K–2 Math Coordinator and the three district-wide grade-level leaders were all Primarily Math participants, as were 13 of the 33 teacher leaders, i.e. 46% of the curriculum implementation teacher leaders were Primarily Math participants, versus 7% of the district's grades K–2 teachers. The district curriculum specialist, K–2 district math coordinator, and the three district-wide grade level teacher leaders, planned the monthly curriculum professional development sessions for all of the district's K–2 math teachers. This team then trained the thirty-three teacher leaders, who delivered the monthly professional development sessions to small groups of 12–15 teachers. Primarily Math participants provided the intellectual guidance to the design of the implementation professional development and in many cases directly delivered the professional development.

In addition, over one-third of the building math liaisons were Primarily Math participants. Building math liaisons met monthly with the district curriculum specialist and the K–2 district math coordinator to problem solve implementation issues in the buildings and provide a two-way communication structure between the district and teachers in school buildings. Therefore, for the school district, through the intentional use of Primarily Math participants in the intellectual design and leadership of the implementation of *Math Expressions,* the curriculum implementation in effect became an extension of Primarily Math for all of the district's K–2 teachers, albeit significantly less intense compared to an eighteen-hour graduate program. Nonetheless, the on-going monthly professional development sessions were "mini best-of" Primarily Math sessions directly linked to the *Math Expressions* curriculum, and all teachers in the district benefited from Primarily Math even if they had no teachers who were directly involved with Primarily Math. In this way the school district took advantage of its Primarily Math participants to provide distributed leadership across the entire school district to support the curriculum implementation.

Design

The partner district in the study is a Midwestern public school district of approximately 37,000 students in grades K–12, with 38 elementary schools (37 at the time of the study), 11 middle schools, and six comprehensive high schools. In grades K–3, 69.7% of the students are white and 45.9% of the students participate in the free or reduced lunch program. Approximately 150 teachers teach mathematics at each grade K–3. Primarily Math participant teachers were selected on the basis of a competitive application process that was skewed in favor of teachers from low-achieving and/or economically and ethnically diverse schools. Sixteen grades K–3 teachers were selected from the partner district to participate in Cohort 1 of Primarily Math and 34 grades K–3 teachers from the partner district were selected to participate in Cohort 3. A group of twelve grades K–3 non Primarily Math participants from the partner district were selected to serve as a control group. The control group teachers were selected at random from buildings with a socio-economic status similar to those of the Primarily Math teachers.

The Mathematical Knowledge for Teaching (MKT) questionnaire (Hill, Schilling, & Ball, 2004) measures K–6 teachers' mathematical content knowledge for teaching, concentrated in the areas of numbers and operations, geometry, and patterns, functions and algebra. A sample item is shown in Figure 1; items are designed to measure the specific knowledge teachers need in order to teach mathematics effectively. Teachers participating in the Primarily Math research project took the MKT annually 2009–2013. MKT scores are reported as standardized scores (mean 0, standard deviation 1); it is expected that a group of K–3 teachers would score somewhat below the mean set by a national sample of K–6 teachers.

The student subjects consisted of a subset of the students from twenty-five Primarily Math teachers in Cohorts 1 and 3 (selected at random), and the twelve control group teachers. Student math achievement was assessed using The Test of Early Mathematics Ability, 3rd Edition [TEMA-3] (Ginsburg & Baroody, 2003). The TEMA-3 calculates a standardized score called the Math Ability Score, which is an indicator of a child's overall mathematical ability. The TEMA-3 has a mean of 100 and a standard deviation of 15. The score is calculated through a combination of raw scores and age. Each fall, 15 students were randomly selected from teachers' classrooms to be assessed using TEMA-3; each spring, as many of the 15 students who remained in the selected classrooms were reassessed. On the TEMA-3, because age

Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

I) $\begin{array}{r} {}^{1} \\ 38 \\ 49 \\ +\,65 \\ \hline 142 \end{array}$ II) $\begin{array}{r} {}^{1} \\ 45 \\ 37 \\ +\,29 \\ \hline 101 \end{array}$ III) $\begin{array}{r} {}^{1} \\ 32 \\ 14 \\ +\,19 \\ \hline 64 \end{array}$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

Source: Hill, Schilling, & Ball, 2004.

Figure 1. Sample MKT Item

is included in the math ability score, a child making normal/average growth would have the same score in the fall and the spring. Therefore, increases in scores show students who are making growth larger than the national norm group for their age; growth in Math Ability Scores of at least 8 points is considered statistically significant. Table 1 provides guidelines for interpreting the Math Ability Score on the TEMA-3.

Table 1. Guidelines for Interpreting Math Ability Scores

MAS	Description
≥ 131	Very Superior
121–130	Superior
111–120	Above Average
90–110	Average
80–89	Below Average
70–79	Poor
≤ 69	Very Poor

Results

Student Results: TEMA-3

Table 2 shows student scores on the TEMA-3 for Cohort 1, Cohort 3, and the control group, for the 2009–2010, 2010–2011, and 2011–2012 school years. For each school year, students are grouped by fall score into above average or higher (scores greater than 110), average (scores 90–110), and below-average or lower (scores less than 90) groups, and the table shows each group's median score and how the median score changed from fall to spring. Table 2 shows that student TEMA scores increased every year, for all groups of students. Larger gains were seen in students who were below-average in the fall, but this is to be expected since these students had more room for growth. When considering only the above average and average groups, growth rates were the greatest between fall and spring for students in classrooms whose teachers were participants in Primarily Math. Although TEMA score improvement began prior to the district's implementation of a new curriculum, growth was greatest after the district implemented the new curriculum as evidenced by the growth of achievement from students in the control group during the 2011–2012 school year.

Teacher Results: MKT

Table 3 shows Mathematical Knowledge for Teaching scores for Cohort 1 teachers (16). There was a significant change in the MKT subscale scores ($p<0.05$ using a least-squares mean difference test) for numbers and operations, algebra, and geometry between the years of 2009 to 2010 (during their time in Primarily Math). This increase was retained during the 2010–2011 and 2011–2012 school years.

Table 4 shows Mathematical Knowledge for Teaching Scores for Cohort 3 teachers (34) who began Primarily Math in 2011. Teachers in this cohort experienced significant increases in their algebra and geometry subscale scores from 2010 to 2011, prior to participation in Primarily Math, but after participation in the district's math curriculum study and curriculum pilot. This group did not experience any significant gains in scores during Primarily Math; their post-Primarily Math assessment will occur in spring 2013. Table 5 shows Mathematical Knowledge for Teaching scores for the 12 control teachers. Control teachers' scores showed some fluctuations across time, but did not have any sustained significant increases.

Limitations

Limitations of this study include ceiling effects associated with the TEMA-3 scores. Across three years of TEMA-3 data, approximately 50% of 3rd graders and 10% of 2nd graders scored high enough each fall to preclude statistically significant improvements in spring scores. Additionally, the norm group for the TEMA-3 test has a maximum age of 8 years 11 months; most 3rd graders turn 9 during the school year, with a majority being 9 years old by the time of spring testing. Thus, the ceiling effects of TEMA-3 scores, both in 3rd graders' ages and in high fall scores for 2nd and 3rd graders combine to potentially mask some of the student growth. Additionally, during the first year of the study, research permissions were delayed and so "fall" TEMA-3 data were collected in November and December; in subsequent years fall data were collected in September.

Another limitation to this study is that initial correlational statistical models have revealed no significant correlations between teacher MKT number & operation scores and student TEMA-3 scores. We note that others using MKT data and attempting to find correlations with elementary student achievement scores report similar non-significant findings (Hill, 2013). More advanced statistical models, including structural equation models, are being analyzed to better understand potential relationships between teacher MKT and student TEMA-3 outcomes. Since Primarily Math teachers took the MKT annually, resulting in gains between most consecutive administrations, and students took the TEMA-3 each fall and spring, one initial complex decision is which teacher score (or gain between consecutive scores) to compare to which student scores (or gains between consecutive scores).

Finally, in this exploratory study our findings are limited to the context examined here and we make no attempt to generalize beyond the district in this study. However, we believe the findings are useful to similar districts that

Table 2. Median Student TEMA-3 Score

	Above Average			Average			Below Average		
Cohort 1	Fall	Spring	Growth	Fall	Spring	Growth	Fall	Spring	Growth
2009–2010	117	117	0	100	104	4	82	84	2
Number	26	25		58	49		23	19	
2010–2011	115	122	7	100	107	7	83	90	7
Number	24	24		117	112		56	51	
2011–2012	115	128	13	98	114	16	84	98	14
Number	14	14		62	60		25	25	
Cohort 3	Fall	Spring	Growth	Fall	Spring	Growth	Fall	Spring	Growth
2009–2010	117	119	2	100	107	7	85	97	12
Number	64	55		88	84		36	26	
2010–2011	117	118	1	100	109	9	83	93	10
Number	38	29		115	102		51	44	
2011–2012	117	124	7	100	113	13	84	97	13
Number	33	31		98	88		52	49	
Control	Fall	Spring	Growth	Fall	Spring	Growth	Fall	Spring	Growth
2009–2010	117	118	1	100	109	9	82	92	10
Number	38	33		69	64		27	19	
2010–2011	119	118	−1	99	110	11	85	99	14
Number	24	24		66	57		32	27	
2011–2012	115	124	9	102	110	8	84	100	16
Number	16	14		68	61		35	30	

are considering the implementation of a reform-oriented curriculum. Although both teacher and student mathematical knowledge were considered, teachers' beliefs and practices were not examined. Additional research is needed to understand the effect of the professional development and curriculum implementation on teachers' beliefs and practices.

Discussion

For teachers in Cohort 1 of Primarily Math, their participation in year-one of the program had an immediate and positive impact on their mathematical knowledge for teaching. In addition, their mathematical knowledge for teaching gains continued to grow after year-two in Primarily Math and were maintained, and in the case of their overall MKT scores, actually increased after their participation in Primarily Math ended. Because there were significant gains in their MKT scores prior to the district's curriculum study and pilot, it can reasonably be concluded that these initial gains are attributable to their participation in the Primarily Math program and that their gains continued to improve and were maintained through their participation in the district's curriculum initiatives, including the math study, curriculum pilot, curriculum implementation, their participation in the

Table 3. Cohort 1 (Primarily Math Participants) Mathematical Knowledge for Teaching Scores

Assessment Year	Numbers & Operations	Algebra	Geometry	MKT	N
2009	0.1234	−0.0538	−0.2394	−0.0566	13
2010[1]	0.8775*	0.6770*	0.6162*	0.7236*	12
2011[2]	0.9992	0.5860	0.7080	0.7644	10
2012[3]	0.9863	0.6451	0.7524	0.7946	11

*Significantly higher than previous year's score ($p<0.05$)
[1]Assessment following first-year of Primarily Math participation.
[2]Assessment following second-year of Primarily Math participation, and district math study and curriculum pilot.
[3]Assessment following first-year of curriculum implementation.

Table 4. Cohort 3 (Primarily Math Participants) Mathematical Knowledge for Teaching Scores

Assessment Year	Numbers & Operations	Algebra	Geometry	MKT	N
2009	−0.3906	−0.5297	−0.2641	−0.3948	22
2010	−0.2511	−0.4830	−0.1028	−0.2790	22
2011[4]	0.1370*	0.1183**	0.1245**	0.1266	31
2012[5]	0.0794	0.1423	0.1967	0.1395	31

*Significantly higher than previous year's score ($p<0.10$)
**Significantly higher than previous year's score ($p<0.05$)
[4]Assessment following first-year of Primarily Math participation and district math study and curriculum pilot.
[5]Assessment following second-year of Primarily Math participation and district curriculum implementation.

Table 5. Control Teachers (non-Primarily Math Participants) Mathematical Knowledge for Teaching Scores

Assessment Year	Numbers & Operations	Algebra	Geometry	MKT	N
2009	−0.7560	−0.8783	−0.1840	−0.6061	12
2010	−0.1301	−0.734	−0.2141	−0.3594	10
2011	−0.7342	−0.6078	−0.0962	−0.4794	10
2012[6]	−0.2560	−0.5750	−0.4569	−0.4293	9

[6]Assessment following district curriculum implementation.

leadership of these activities within the district, and the on-going study groups. Cohort 1 MKT scores were significantly stronger than those of Cohort 3.

Teachers in Cohort 3 saw some improvement in their mathematical knowledge for teaching scores between the 2009 and 2010 assessments, prior to their participation in the Primarily Math program. This gain can be attributed to district professional development activities. Cohort 3 MKT score gains were greater between the 2010 and 2011 assessments, after their participation in the first-year of Primarily Math. The mathematical knowledge for teaching scores of the control group teachers fluctuated from year to year, even after their implementation of the district's new curriculum, and never reached a positive standardized score.

Consideration of Cohort 1, 3, and control teacher MKT scores as well as student TEMA scores as a whole leads to the following conclusions:

- The Primarily Math program did have a positive effect on teachers' mathematical knowledge for teaching.
- The effect of Primarily Math on teachers' mathematical knowledge for teaching was greater on Cohort 1 than Cohort 3. This may have been due to the fact that Cohort 1 teachers had one year of Primarily Math participation prior to engaging in the district's curriculum study, pilot, and leadership activities. It

is possible that participation in year-one of Primarily Math allowed the Cohort 1 teachers to take advantage of the associated district leadership and curriculum activities in ways that Cohort 3 teachers did not have, providing them an opportunity to integrate their knowledge and build cognitive coherence that Cohort 3 teachers did not have.

- Because the control group teachers did not experience a gain in their mathematical knowledge for teaching, it appears that a curriculum implementation alone is insufficient to improve teachers' mathematical knowledge for teaching, no matter how innovative or how much "professional development" is embedded within the curriculum for teachers.
- Implementation of a new curriculum alone can have a positive effect on student achievement, but achievement gains are greater when combined with coherent and on-going professional development that emphasizes mathematics content and pedagogy.

Implications

Over the next two years school districts in 45 states and the District of Columbia will make significant changes to their mathematics programs to align them with the Common Core State Standards for Mathematics (CCSSI, 2010). In many cases this will result in the adoption and implementation of a new curriculum. New curricula, if truly aligned with the Common Core, will require that teachers make significant changes in their instructional practices (Kanold & Larson, 2012) and teach content they have previously not taught due to the increased rigor of the standards (Porter, McMaken, Hwang, & Yang, 2011). Currently in many districts the focus of elementary teachers' professional development is reading, and when mathematics is addressed it is typically only addressed at the time of a curriculum implementation and narrowly emphasizes the use of the adopted curriculum materials (Larson et al., 2012).

Results of this study indicate that this all too common approach to professional development and curriculum implementation is insufficient to maximize the impact on student achievement. While the results of this study indicate that the effective implementation of a new curriculum can improve students' mathematical knowledge, the effect of a curriculum implementation on student mathematical knowledge is improved when teachers have an opportunity to increase their mathematical knowledge for teaching and are simultaneously actively involved in the leadership, including the intellectual design and delivery, of the curriculum implementation. Active engagement in the planning and distributed leadership of a curriculum implementation appears to allow teachers to make sense of the curriculum and translate it both for their students and colleagues in ways they would not otherwise have, ultimately improving its effectiveness with students. Additionally, it appears that the most positive effects on student learning are achieved when teachers have the opportunity to begin their professional development experiences well in advance of the curriculum implementation. The implications for leaders planning to implement new curricula in the Common Core era are significant.

Authors' Note

Support for this research comes from the National Science Foundation (DUE-0831835). All opinions and findings are those of the authors and do not necessarily reflect the views of the funding agency.

References

Bay, J. M. (April, 2000). *The dynamics of implementing and sustaining standards-based mathematics curricula in middle schools*. Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Bay, J. M., Reys, B. J., & Reys, R. E. (1999). The top 10 elements that must be in place to implement standards-based mathematics curricula. *Phi Delta Kappan, 80,* 503–506.

Common Core State Standards Initiative [CCSSI] (2010). *Common Core State Standards for Mathematics*. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math). Washington, D.C.: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org.

Duncan, G. J., Dowsett, C. J., Claessens, A., Magnuson, K., Huston, A. C., Klebanov, P., Pagani, L.S., Feinstein, L., Engel, M., Brooks-Gunn, J., Sexton, H., Duckworth, K., & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology, 43*(6), 1428–1446.

Fuson, K. C. (2011). *Math Expressions* (K–6 curriculum). Boston: Houghton Mifflin Harcourt.

Ginsgburg, H. P., & Baroody, A. J. (2003). *Test of Early Mathematics Ability, 3rd Edition.* Torrance, CA: Western Psychological Services.

Ginsburg, H. P., Lee, J. S., & Boyd, J. S. (2008). *Mathematics education for young children: What it is and how to promote it.* Social Policy Report, Society for Research in Child Development.

Hargreaves, A. & Fink, D. (2005). *Sustainable leadership.* San Francisco: Jossey-Bass.

Hill, H. C. (Jan., 2013). The multi-dimensional nature of mathematics instructional quality: A first look at teacher and teaching elements that contribute to student outcomes. Presentation to the MSPnet Academy, http://hub.mspnet.org/index.cfm/nature_of_mathematics/

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal, 105*(1), 11–30.

Hopkins, M., Spillane, J. P., Millerd, P., & Heaton, R. M. (in press). Infrastructure redesign and instructional reform in mathematics: Formal structure and teacher leadership. *Elementary School Journal.*

Kanold, T. D., & Larson, M. R. (2012). *Common core mathematics in a PLC at work™: Leader's Guide.* Bloomington, IN: Solution Tree Press; Reston, VA: NCTM.

Larson, M. R., Fennell, F., Adams, T. L., Dixon, J. K., Kobett, B. M., & Wray, J. A. (2012). *Common core mathematics in a PLC at work™: Grades K–2.* Bloomington, IN: Solution Tree Press; Reston, VA: NCTM.

National Association for the Education of Young Children [NAEYC] and National Council of Teachers of Mathematics [NCTM]. (2002). Position Statement. *Early childhood mathematics: Promoting good beginnings.* From http://www.naeyc.org/about/positions/psmath.asp.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics [NCTM]. (2007). Position Statement. *What is important in early childhood mathematics?* From www.nctm.org/about/content.aspx?id=12590.

National Research Council [NRC]. (2009). *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity.* Committee on Early Childhood Mathematics, Christopher T. Cross, Taniesha A. Woods, and Heidi Schweingruber, Editors. Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academies Press.

Porter, A., McMaken, J., Hwang, J., & Yang, R. (2011). Common core standards: The new U.S. intended curriculum. *Educational Researcher, 40*(3), 103–116.

Spillane, J. & Diamond, J. (Eds). (2007). *Distributed leadership in practice.* New York: Teachers College Press.

Spillane, J. P. (2006). *Distributed leadership*: San Francisco, CA: Jossey-Bass.

Stein M. K., Grover, B. W. & Henningsen, M. A. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Education Research Journal, 33,* 455–488.