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A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

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Strengthening a Country by Building a Strong Public School Teaching Profession¹

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What would be one of the most sensible ways for a country to invest to achieve maximal economic growth? A recent study (Chetty, Friedman, & Rockoff, 2011) by economists at Harvard and Columbia Universities shows that better teacher quality results in significantly higher students' lifetime earnings. And investing in public school teachers results in an expanding skilled work force, the foundation for maximal economic growth.

Although Japan's economic growth has declined in recent decades, Japan has been and still is one of the several countries successfully educating students to participate in a large, highly skilled work force that generates a high per capita GDP.

In this paper, characteristics of Japanese public schools that have contributed to attracting and retaining superior teachers will be described from the perspective of a high school mathematics teacher with 20 years teaching experience in Japanese public high schools and more than 13 years teaching in the United States; and more recently as department chairperson, hiring mathematics teachers for a private, residential high school.

The paper will explore how better to select teachers with reference to actual practice in Japan and in the U.S. and will include an analysis of actual qualifying/employment examinations and their outcomes. Prospects to foster and maintain a top tier, public school teaching profession that will expand the high skilled population and counteract "brain drain" for a developing country will be discussed.

Keywords: mathematics teacher, teacher education, teacher employment examination, public education, economic growth.

Introduction

A recent report from the Organization for Economic Cooperation and Development, OECD (2011) states that with technology progressing rapidly, the failure of a country's educational system to meet the demand for skilled workers results in widening income inequality—a factor which may destabilize the country—and that a country's educational goal should be not merely to increase school attendance but to improve the competence and employability of its people. Simply put, a country's education system forms the basis of its future success.

What is, then, a major factor that contributes to students' future success? In the United States, Chetty, Friedman and Rockoff (2011) showed that public school teacher quality in teaching English and Mathematics, assessed by a "value-

added"² measure, had a statistically significant positive correlation with students' future success in many aspects of life. The researchers tracked one million U.S. children from 4th grade to adulthood and found that as their lives progressed, those who had had better teachers were more likely to attend college, command a higher salary, live in a better neighborhood, save more for retirement, and have a lower likelihood of having a child before adulthood. The study stated, "Replacing a teacher [for one year] whose value-added is in the bottom 5% with a teacher of average quality would generate lifetime earnings gains worth more than \$250,000 for the average classroom [of 28.3 students]" and "Given that many teachers have long careers, the cumulative gains from deselecting a low value-added teacher can be quite large." In short, "Good teachers create substantial economic value." This has a very important implication for a country building its education system. And it is interesting that even in the famous "exam schools" of the New York and Boston public school systems, Duke University and MIT Researchers Abdulkadiroglu, Angrist, and Pathak (2011)

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² "Value-added" is defined as the average test-score gain for his or her students, adjusted for differences across classrooms in student characteristics such as prior scores. (Chetty et al., 2011)

found no increase in performance that could be attributed to the exam school in the absence of superior teachers—even with a rich offering of advance placement courses and a highly capable, highly motivated peer environment. For teachers, exam school staffing decisions are prescribed by the union contract in force. In their table of exam school resources, Abdulkadiroglu, Angrist, and Pathak list more senior teaching staff (though not necessarily the best teachers). Evidently, for exam schools, even with superior students, superior course resources, and presumably superior family and peer support, without superior teachers, there is no superior outcome.

To build a high quality teaching force, the researchers in the above Chetty et al.'s study (2011) suggest replacing teachers at the bottom 5% of accumulated “value-added” measure for three years. There will, of course, be always the bottom 5% however many times we repeat the replacement, and the study does not indicate the target number of “value-added” for a country to achieve. Also, a loss imposed on students assigned to low quality teachers before they are replaced will be likely to remain as a loss. It will be much better if we can select individuals who will become high “value-added” teachers before they enter the teaching profession.

Is there any good predictor for a person to be a high “value-added” teacher? Since “value-added” is defined “as the average test-score gain for his or her students, adjusted for differences across classrooms in student characteristics such as prior scores” (Chetty et al., 2011), high “value-added” teachers can be described as the ones who can educate their students to the extent that their education outcomes can be measured as high score gains on well-constructed tests. Therefore, the above question can be stated as whether or not there is any good predictor for a person to be a teacher whose students yield a high score gains on well-constructed tests.

Programme for International Student Assessment (PISA) is one of the most highly regarded international tests examining 15-year-old students’ mathematical literacy. The COACTIV project conducted by researchers at Max Planck Institute, the University of Kassel and the University of Oldenburg in Germany “correlated the achievement gains of the students (on class level), measured by the gain in PISA scores from 9th to 10th grade (in 2003 and 2004), to the degree of professional knowledge of the class teacher” (Neubrand, 2008). In particular, teachers’ pedagogical content knowledge (PCK)³ and content knowledge (CK)⁴ were assessed by the tests developed by the researchers. Baumert, Kunter and Blum (2010) reported the outcome of the study stating that the levels of CK and PCK demonstrated by teachers were

strong predictors of students’ score gains. Neubrand (2008) went on to say that “It is not simply PCK which makes an effect on students’ achievement. PCK seems to be positively influenced by a sound CK.” According the study, strong PCK with strong CK seems to be a good predictor for a person to become a teacher whose students yield high score gains in well-constructed tests, in other words, a high “value-added” teacher, who will presumably strengthen economic growth of the country.

From which group of its people can a country most effectively recruit prospective mathematics teachers with strong PCK and strong CK? The researchers in the above study unexpectedly found that mathematics major students at the end stage of their university education demonstrated not just higher level of content knowledge, CK, but also higher level of pedagogical content knowledge, PCK, than students at the end stage of their university education in the most rigorous teacher education track qualifying them as mathematics teachers at gymnasium (upper level high school for academic, college bound students). However, the latter appears to continue acquiring PCK eventually surpassing that of the former over the years as they gain experience and progress in the course of their gymnasium teaching careers. Through this process, they also acquire CK close to the level of students majoring in mathematics as shown in Table 1, Teacher Mathematical Knowledge Comparison, made with the data provided by the study (Krauss, Neubrand, Blum, 2008, p. 11).

According to the data, it seems to be reasonable to say that (1) setting a separate teacher education track is not likely to produce the highest quality teaching force and (2) recruiting future teachers from among the full range of mathematics majoring students is most likely to yield the best high school mathematics teachers.

But, will just being a mathematics major be enough to be a high quality mathematics teacher? In fact, the researchers of the above study address this in a part of their study entitled “The unexpectedly high PCK scores of students majoring in mathematics” and argue that the higher level of pedagogical content knowledge demonstrated by the mathematics major students in comparison to the students in the gymnasium teacher education track are due to the following factors:

1. The mathematics major students have studied only one subject, which means that they have had more than twice the time to engage in mathematics compared to the students in the teacher education track that requires them to study two subjects plus general pedagogy and psychology in Germany.
2. The mathematics major students were most probably not a representative sample because only volunteers took the PCK test, in contrast to the teacher education track student sample.

³ “knowledge about ‘the ways of representing and formulating the subject that make it comprehensible to others’ (Shulman, 1986, p. 9)” (Neubrand, 2008, p. 2).

⁴ “a deeper understanding of the mathematics taught at school” (Neubrand, 2008, p. 2).

Table 1. Teacher Mathematical Knowledge Comparison

	PCK	CK
Gymnasium Mathematics Teacher (Average age: 47.2 years)	21.0	8.5
University Student [Mathematics Major]	19.7	8.6
University Student [Gymnasium Mathematics Teacher Program]	18.2	6.6
Non-Gymnasium Mathematics Teacher	16.8	4.0
High School Students in Advanced Mathematics Course	9.7	2.6
Gymnasium Chemistry/Biology Teacher	7.6	0.4

Note: From “The Professional Knowledge of German Secondary Mathematics Teachers: Investigations in the Context of the COACTIV Project,” by S. Krauss, M. Neubrand, W. Blum, & J. Baumert, (2008). Presented at the 11th International Congress of Mathematical Education (ICME-11).

3. The test developed by the researchers focused more on algebra where the mathematics major students are typically better; in fact, the teacher education track students outperformed significantly the mathematics major students in geometry problems.

Suppose that the above explanations are valid. Explanation (1) indicates that to be worthwhile, a separate education track for mathematics teachers requiring them to study two subjects plus general pedagogy and psychology at the expense of CK and PCK would have to significantly increase PK (pedagogical knowledge), which includes skills such as classroom management, but which was not measured in this study. In addition, gains in PK as the result of the teacher training track program would have to match or significantly exceed that gained by a mathematics major being mentored on the job in his/her first year teaching high school. Explanation (2) indicates that the best future high school mathematics teachers can be recruited from mathematics majors who volunteer to take the PCK test such as those who volunteered to participate in the study. Explanation (3) indicates that having mathematics major students study more geometry will create even better mathematics teachers. Regardless of how the test data are interpreted, the researchers do note that “these findings may indicate that *very strong* subject matter competence can indeed be one route to pedagogical content knowledge.” (Krauss, Baumert, & Blum, 2008, p. 888)

Thus, the next question would be ‘how can we select the best teacher candidates?’ According to the above study, majoring in mathematics seems to be a good predictor for a person to become a good mathematics teacher, but as stated above, we need to select among the mathematics majors, those

with a high level of pedagogical content knowledge. Also, mathematics majors may not be the only possible candidates for good mathematics teachers. Although, as indicated above, requiring students to major in two subjects and take courses in psychology and pedagogy may be less than ideal, this does not imply pedagogical knowledge is unnecessary. Indeed for a novice teacher, pedagogical knowledge such as classroom management is essential, but in practice, this may be developed through apprenticeships, mentoring and on-the-job training as well if not better than in formal education courses. What about students majoring in a subject with an intimate connection to mathematics, such as physics, who studied in the same depth that mathematics majors study mathematics? Can physics or engineering majors become good mathematics teachers? Finally, can those who are actually involved in hiring individuals for teaching positions rely on degrees from highly selective universities, teaching certificates or interviews alone? Or is some kind of further testing also necessary?

In an attempt to answer these questions the following topics are presented below:

- A brief historical overview of public high school teacher education, training and recruitment in Japan.
- A specific focus, with examples, on examination questions that prospective mathematics teacher candidates are likely to encounter in their college applications and that college graduates are likely to encounter in their teacher employment applications in Japan.
- Japanese public school teacher recruitment with emphasis on salary policy; professional self-regulation of teachers with emphasis on the equal distribution of teacher quality throughout a system of district high schools independent of academic and/or socio-economic status; student/teacher relations and the three year student/teacher “teams;” teacher/teacher relations and the “teachers’ room.”
- An introduction to the controversy and critical analysis of current K–12 public school education in the United States.
- A case study conducted by the author of the present paper illustrating development and outcome of a procedure designed to select candidates for mathematic teaching positions at a highly selective, accredited, private, residential high school in the United States nearly all of whose students enroll in selective/highly-selective universities after graduation.

Public High School Teaching, Training and Recruitment in Japan

Although Japan's students' performance has declined somewhat on recent international tests—which is likely to be a result of a recently reversed national curriculum revision that reduced subject content and class hours—Japan has been and still is one of the countries successfully educating students who on average enter a highly skilled work force and generate a high per capita GDP.

Scholars seem to agree that Japan succeeded in the second half of the twentieth century in building a strong public school teaching profession by selecting and recruiting capable, intelligent people with a demonstrated expertise in subjects they teach, and mentoring them in their first years of teaching.

Education System of Post-World War II Japan

After World War II, with the strong leadership of the American occupation officials, a pre-war tracking system, in which students had to decide by the time they would finish the compulsory education whether they would have just two more years of education at upper elementary school or would pursue further education at middle school, was consolidated into the single 6-3-3-4 system. Following optional three years of preschool/kindergarten education, nine years of compulsory education is provided mostly at municipal schools and at small number of private and national schools. The nine years are divided into elementary education from age of six to twelve and middle school education from age of twelve to fifteen. Children are assigned to elementary school and middle school according to their residence unless they wish to attend a private or national school.

After their compulsory education, students who wish to enter high school must pass an entrance examination. There are two types of high schools: academic high school and vocational training high school. Currently 97% of middle school graduates attend high school and more than 70% of these attend academic high school (Ministry of Education, Culture, Sports, Science and Technology of Japan, 2003). Students are given a choice to attend schools in the high school district of their residence or other schools that accept non-resident students. Within a high school district, high schools are typically ranked “by the number of graduates they send to universities and the level of the prestige of the universities to which their graduates are accepted” (Trelfa, 1998, p. 54).

College Entrance Examinations in Japan

After high school education, higher education is provided by public and private universities, most of which also have

graduate programs. To attend, students must pass a highly competitive entrance examination open to all graduates of high schools including vocational training high schools. Currently 35% of high school graduates attend four-year colleges and 10% attend two-year junior colleges. Regarding the selection of students, the statement in “The Educational System in Japan” (Trelfa, 1998, p. 65) that “Japanese universities generally admit students entirely on the basis of the Center Entrance Examination,” which is a national examination and “is made up of multiple choice questions [that] has been likened to the SAT in the United States,” is not entirely accurate in that universities select students based on both scores of the Center Entrance Examination and their own entrance examinations.

Teacher Education and Entrance Examinations at Teacher Colleges in Japan

Teacher education, which had been on a separate track from the university system in the pre-war teacher training system, was consolidated into the Japanese university system after World War II. For training elementary and middle school teachers, special colleges were newly established in the university system. These colleges also give their own entrance examination to select students. There is no difference in levels of difficulty between teacher college entrance examination problems and university entrance examination problems in Japan in contrast to the United States where, in general, getting into a major university requires significantly higher SAT or ACT scores than getting into a teachers college (Berliner & Biddle, 1995; Eide, Goldhaber, & Brewer, 2004; Gross, 1999). Figure 1, Mathematics College Entrance Examination Problems in Japan, shows examples of mathematics problems from teacher college and university faculty of education entrance examinations.

Currently, all university students—not just students in faculties of education or teacher colleges—can obtain a teacher certification upon earning sufficient credits to graduate, credits in the subjects relevant to the certificate, and credits in specialized subjects relevant to the teaching profession that are designed by each university. At both universities and teacher colleges, the emphasis is mainly on the subject matter content (“Teacher Training Course,” Waseda University, 2003).

Teacher Employment Examination in Japan

To become a teacher at a public elementary, middle, or high school certified graduates have to pass an employment examination, which is highly competitive. The examination for prefectural high school teaching positions usually consists of two parts: written tests and an interview. The written tests usually consists of three kinds of examinations: (1) general

knowledge of social and natural science, mathematics, Japanese literature and English; (2) knowledge of teaching as a profession, such as history of education, education system, laws regarding education, psychology, education theories, school and class management and issues in current education; and (3) academic competency on the subject an applicant wishes to teach. Figure 2, Teaching Qualification Exam, is a translation of the written mathematics examination given by Kyoto prefecture in 2002 (Naigai Kyouiku Kenkyuukai, 2002a, 2002b).

Teacher Recruitment in Japan

To secure teachers of high quality, the salary schedule for public school teachers has been revised several times since the end of World War II. In 1954, the teacher salary schedule was differentiated from other government employees to allow a generally higher pay scale, and divided into three increasing categories: elementary and middle school teachers, high school teachers, and university teachers. In

1972, a special adjustment was added to the basic salary of teachers to compensate for overtime work, which was not required of teachers by administrators but was found to be unavoidable for teachers wishing to fulfill what they felt was required by their profession. In 1974, “Human Resource Recruitment Law” was enacted, and teachers’ salaries have been increased significantly three times since then. To secure the necessary numbers of teachers in elementary and middle schools, most of which are funded by municipal government, the national government has been providing half of the salary for the teachers (Ministry of Education, Sports, Science and Technology, 2003). In 1984, according to the U.S. Department of Education, “the ratios of the average teacher’s salary to the average wage in manufacturing, to average salary in all nonagricultural activities, and to salaries in various other occupations, are all higher in Japan than in the United States” (U.S. Department of Education, 1987). In 2001, Japanese national average of high school teachers’ yearly wage was approximately \$45,730 (@ 120 yen/dollar) (Roudou Hourei Kyoukai, 2001) while Japanese national average yearly wage was approximately \$28,590 (@ 120 yen/dollar) (Ministry of Health, Labour and Welfare of Japan, 2001). This amount does not include the bonuses that teachers and most other Japanese employees receive in June and December and which can add up to the equivalent of two or three months’ salary. It should be noted that teachers in Japan are full-year employees. The numbers of days for summer and winter vacations available to Japanese public high school teachers are five and six, respectively. Though teachers do not have classes during students’ vacations—forty days in the summer, two weeks in the spring and two weeks in the winter—they are required to come to school 5 days a week, or to submit a report of school-related study they have done outside.

It should be also noted that in the Japanese public school system, the teacher quality of a school is not determined by the academic level of the school. As described in “The Educational System in Japan” (Kinney, 1998), public school teachers in Japan are employed by a local government, not by an individual school, and they are subject to transfers to any school in the district. The school a teacher will be transferred to is determined not only by seniority, but also by a job rotation system designed to give every teacher working experience in a wide range of schools and academic levels. Most teachers want to work in a school with less distractive behaviors in class, and the higher academic level a school has, the less a teacher encounters problematic behaviors. This creates a situation in which most teachers try to move to, and stay in upper level schools. To break this cycle in Kanagawa Prefecture, a professional association of teachers initiated a dialogue

<p>[Osaka Teacher College, 1995]</p> <p>A (1, 0), B (2, 0) and the line L: $y = mx$ ($m \neq 0$) are given in the xy-plane.</p> <p>(1) Express x and y of P(x, y) on L in terms of m that minimizes AP + BP</p> <p>(2) Find the equation of the locus of the point P with all values of m.</p> <p>[Gifu University, Faculty of Education, 1996]</p> <p>For any real number x, $[x]$ is the largest integer that does not exceed x.</p> <p>(1) Prove that any real number ‘a’ that is equal to or greater than 1 satisfies the following condition (C).</p> <p>Condition (C): For any real number x and y that are equal to or greater than 0, $[x + y] + a > 2(xy)^{\frac{1}{2}}$</p> <p>(2) Prove that minimum value of ‘a’ that satisfies the condition (C) is 1.</p> <p>[Gifu University, Faculty of Education, 1988]</p> <p>P (t, t^2) is the point on the parabola $y = x^2$. Consider a circle with the radius $(1 + 4t^2)^{\frac{1}{2}}$ and a tangent at P that is also a tangent of the parabola at P. Find the equation of the locus of the center of the circle with all positive real values of t. Also graph the equation.</p>

Figure 1. Mathematics College Entrance Examination Problems in Japan

- [1] Assume that $f(x) > 0$ for any x , and $f(x_1 + x_2) = f(x_1)f(x_2)$.
- (1) Express $f(-x)$ in terms of $f(x)$.
 - (2) Evaluate $f(x)$ if $f(1) = a$.
- [2] Roll m dice at the same time, and repeat the trial n times. Find the following probabilities.
- (1) Getting at least one 1 in every trial.
 - (2) Getting m 1s in at least one trial.
- [3] In triangle ABC , D is the intersection of the bisector of the angle A and the side BC , and I is the intersection of the bisector of the angle B and AD . Assume that $AB : BC : CA = 2 : 3 : 4$. Express vector AD and vector AI in terms of vector AB and vector AC .
- [4] $y = x + e^{(x-1)}$, $x = 1$ and $x = 0$ are given.
- (1) Find the area surrounded by above given graphs and x -axis.
 - (2) Find the volume of the solid made by rotating the figure described in (1) around the x -axis.
- [5] Show how to solve the sequence $a_{n+1} = 5a_n + 1$ for a_n with three different methods. Discuss advantages and disadvantages of each method.
- [6] Write Cauchy's Theorem of which part is given as follows, and show a proof:
 "For functions $f(x)$ and $g(x)$ where $g(x)$ is not equal to 0, and for a and b there exists c such that . . ."
- [7] Describe concretely how you will introduce matrices for the first time with real life examples that can grab the attention of students.
- [8] Describe what you will write on the blackboard to explain and prove the Law of Sines to tenth grade students who are studying the theorem for the first time.

Figure 2. Teaching Qualification Exam

with the local government and created a job rotation system, which worked in such a way that a teacher in a top academic level school was most likely to be transferred to a bottom or close to a bottom academic level school at some point in his/her career.

Characteristics of Japanese Public High Schools—Teachers' Room and Longitudinal Team of Teachers

In the United States, teachers stay and teach in their classroom almost all day separated from their fellow teachers, and the teachers' room is usually a small lounge where teachers can retreat to for a break between classes. In

Japan, the teachers' room is a large, office style room where teachers spend a major portion of their day. Except for the time they spend in class—which is three class periods a day on average—teachers work at their own desks in the teachers' room, beside other teachers.

The teachers' room facilitates free and spontaneous conversations among teachers. This is where teachers share mathematical knowledge with other teachers. This is where they come back from their classes and share their classroom experience and information about students. In the teachers' room, teachers constantly exchange their daily teaching experiences with their fellow teachers, particularly the mistakes that students make and stumbling blocks they encounter.

Without the teachers' room, teachers have to call or to visit one another to talk. In this situation, communication among teachers—especially spontaneous communication—is certainly less likely to occur than when there is a large, common teachers' room. In fact, when the author of the present paper was teaching at an American girls' private, residential high school, she was in her classroom almost all day, usually surrounded by students. The only time to get together with other teachers was at lunchtime, but the presence of students in the same cafeteria made it hard for teachers to talk to each other about their classroom experiences. Mathematics teachers had department meetings, but the meeting was rarely the time to talk about "small" experiences in the classroom. In

addition, the meetings were not held frequently enough for a teacher to keep momentum or initial excitement to talk about what happened in class several days earlier.

In Japanese high schools, teachers are usually assigned to a team of teachers that follows a given class from year to year. Mathematics teachers teach algebra when their class is in 10th grade, precalculus when it is in 11th grade and calculus when it is in 12th grade. By the time their students reach the final grade of high school, teachers have gotten to know them particularly well and can confidently guide them to choices of colleges or other careers that best fit their individual needs and talents.

BUILDING A STRONG TEACHING PROFESSION

United States Teacher Education and Development Study in Mathematics (TEDS-M)

In the United States Teacher Education and Development Study in Mathematics (U.S. TEDS-M⁵), mathematics content knowledge of future elementary school teachers and future middle school mathematics teachers finishing their teacher preparing programs at colleges, universities or normal schools in 16 countries⁶ were examined with a test. A report entitled “Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation” (Michigan State University the Center for Research in Math and Science Education, 2010) revealed that the future U.S. teachers’ mathematics content knowledge was not as high as the country would like it to be. Comparing the results with elementary and middle school students’ test scores in the Trends in International Mathematics and Science Study (TIMSS), the report stated, “The parallelism of the results between the knowledge base of the future teachers and the corresponding knowledge level of those whom they will teach is most likely more than just a coincidence” (p. 31).

In particular, “Perhaps, not surprisingly, the performance of the U.S. elementary future teachers internationally is quite consistent with the performance of third and fourth graders in the Trend of International Mathematics and Science Study (TIMSS)—mired near the international mean.” And middle school teachers’ mathematics content knowledge level was also “in the middle of the international distribution” and “it is rather disconcerting [to the U.S. authors] that the future Taiwanese teachers scored over one and a half standard deviations higher on the mathematics content knowledge test. This is a substantially large difference in performance between the United States and Taiwan. For the Russian Federation and Singapore their future teachers outperformed those of the United States by a half a standard deviation or more.”

The report also revealed a great variation among future teachers within the U.S. in mathematical knowledge. It says, regarding the future elementary teachers, “There are some institutions in the U.S. whose performance places them in the lower end of the distribution for Taiwanese

primary future teachers and at the upper end of those from the Russian Federation, Poland, Switzerland and Singapore (where there is only one institution). On the other hand, there are universities or colleges in the United States where the average performance of the future teachers in that institution is like that of some institutions in the poorest performing countries.” Regarding the future middle school teachers, the report says, “There are U.S. teacher preparation universities or colleges whose students perform at the highest levels in terms of mathematics content knowledge as is found in the top achieving countries. On the other hand, there are certain U.S. institutions where the average level of knowledge of mathematics of its future teachers places them in the middle among the Georgia institutions and at the upper end of the Chile and Botswana distributions.”

One of the most important findings is that teachers’ mathematical content knowledge and their students’ international test scores are closely related to each other. The report states, “It is quite striking that the performance of the future teachers in terms of their mathematics content knowledge at both levels parallels so closely that of the students they teach.”

High School Teacher Candidates’ Mathematical Competence in the United States—A Case Study by the Author of the Present Paper

The following is a case study by the author involving the testing of individuals applying for mathematics teaching positions at a private, residential high school in the United States. The mathematical competence of the applicants was assessed with a test as an essential part of the selection process, and the results are presented along with other material from applicant resumes. The test was originally designed as a 90-minute placement test to assess 11th grade students’ mathematical competence to determine their qualification for a 12th grade Advanced Placement type calculus class. Basically, the test is a straightforward 11th grade level mathematics content knowledge test.

Mathematics Department Profile and Mathematics Employment Examination in the Case Study

The school is a private, college preparatory school located in a suburb of New York City and serving a primarily foreign student population, all of whom attend a selective/highly selective college after graduation. Currently, with the inclusion of the two recent hires described below, the mathematics department consists of five teachers, one with elementary through college education in a European country,

⁵ “TEDS-M is a collaborative effort of worldwide institutions to study the mathematics preparation of future primary and secondary teachers....The lead center is the International Study Center at Michigan State University. . . . TEDS-M builds on other cross-national studies such as IEA’s Trends in International Mathematics and Science Study (TIMSS).” (U.S. TEDS-M, 2010) (<https://arc.uchicago.edu/reese/projects/teacher-education-and-development-study-mathematics-teds-m>.)

⁶ The countries are U.S., Germany, Norway, Poland, the Russian Federation, Spain, Switzerland, Taiwan, Singapore, Thailand, Malaysia, Botswana, the Philippines, Chile, Georgia, and Oman.

and four with elementary and middle school education in Asian countries, one of whom attended undergraduate college in the US. Two teachers majored in mathematics in college, one teacher majored in physics, one teacher majored in engineering, and one majored in geoscience. All teachers have a master's degree either in mathematics, in physics, in engineering, or in mathematical statistics and computer science. Two teachers also have PhD's, one in physics and one in mathematics education. The department requires all 9th graders to take algebra and geometry, all 10th graders algebra and trigonometry, all 11th graders pre-calculus and all 12th graders calculus, and it expects all teachers to be able to teach any course, even on short notice, with mathematical rigor. No teacher has American teaching certificates of any kind. First year teachers are mentored by the department chairperson and receive on-the-job training on how to teach.

The department has had an opening in 2009 and in 2010. It was announced on the school's web site, in the New York Times and through two teacher recruitment agencies stating, "Bachelor's degree in mathematics/physics/engineering required. Teaching experience of high school level mathematics through AP calculus BC preferred. Master's degree in mathematics or physics is desirable. A 60 minute written examination, which consists of 10 problems on the following topics will be given on the day of an interview: quadratic functions, exponential functions, logarithmic functions, trigonometric functions, matrices, vectors, sequences and series."

Calculators were not allowed and all work had to be shown although the department did not lower the score just because an answer did not show every step. Each of the ten questions carried 10 points with a total of 100. Partial credits were given rather generously whenever an answer showed at least some understanding of a concept relating to a given problem. When the test was given to students as a placement test, they were given 90 minutes and the average score was 50 in a range from 12 to 95. Teacher candidates were given 60 minutes although they were told that extra time would be given if necessary. None of them, however, requested the extension at the end of the 60 minutes. The questions are shown in Figure 3, Teacher Employment Written Examination.

Teacher Candidates' Profiles and Their Test Performances in 2009

In 2009, nine applicants were invited for an interview and then the test. The resume of one of

the candidates impressed the human resource manager of the school, who then invited him first among the nine candidates for an interview and the test. He was one of the former hedge fund managers. He had decided to become a teacher and was finishing an accelerated certification program. In the interview, he spoke highly of the department policy of regarding a candidate's mathematical competence as most important. The department chairperson and the teacher from Europe, the other interviewer, who had also worked on Wall Street, enjoyed talking with him so much that the meeting lasted more than twice as long as scheduled. After grading his test, however, the European teacher was so disappointed with the candidate's low test score that he suggested that next time they give the test first. They subsequently tried to move on to the test as quickly as possible whenever an interview was scheduled.

1. Simplify (10 points each.)

(a) $\sqrt[3]{54} + \frac{3}{2}\sqrt[4]{4} + \sqrt[3]{-\frac{1}{4}}$

(b) $4\log_2 \sqrt{2} - \frac{1}{2}\log_2 3 + \log_2 \frac{\sqrt{3}}{2}$

2. Solve the following equation/inequality for x ($0 \leq x \leq 2\pi$). (10 points each.)

(a) $3\sin x - 1 = \cos 2x$

(b) $\sin x < \sin 2x$

3. Suppose $\sin \alpha + \sin \beta = a$, $\cos \alpha + \cos \beta = b$. Express $\cos(\alpha - \beta)$ in terms of a and b .

4. Find the minimum of the function $y = 9^x - 2 \cdot 3^x + 5$.

5. Which is greater, $\log_{10} x^2$ or $(\log_{10} x)^2$? Explain.

6. Express $\sum_{m=1}^n \left\{ \sum_{l=1}^m \left(\sum_{k=1}^l 1 \right) \right\}$ in terms of n .

7. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}$. Find the matrix X that satisfies $XA = B$.

8. Two vectors \mathbf{a} and \mathbf{b} are given. Suppose that

$$\|\mathbf{a}\| = \|\mathbf{b}\| \text{ and } (5\mathbf{a} - 4\mathbf{b}) \perp (\mathbf{a} + 2\mathbf{b}), \text{ where } \mathbf{a} \neq \mathbf{0} \text{ and } \mathbf{b} \neq \mathbf{0}.$$

Find the angle θ between the vector \mathbf{a} and the vector \mathbf{b} .

Figure 3. Teacher Employment Written Examination

The highest score was 100 and the lowest 6. Possession of a state teaching certificate was found to be negatively associated with mathematic content test performance. In particular, none of the candidates whose test score was in the upper half of all the test scores of candidates who took the test possessed teaching certificates of any kind. On the other hand, all but one whose test scores were in the lower half possessed a state high school teaching certificate. Ultimately, the candidate with the score of 100 was hired. But the initial resume review process was filled with a lot of issues and conflicting views. First of all, the human resource manager did not like his resume listing assistant sushi chef and tutor as his only job experience, and agreed to invite him for an interview only after he came to know that a visa could not be secured for the leading contender who had already been interviewed and tested with the score of 85. The candidate's phone conversation and a preliminary meeting with the manager did not go well either. He gave the manager an impression that he might not be able to handle teenagers trying to take advantage of a new young teacher who did not look very strict. The department chairperson got the same impression in the interview while only the European teacher was favorably impressed because of his somewhat reserved demeanor. After reviewing his test results, however, the department chairperson and the European teacher were both very positive and decided to recommend him. They eventually overcame the human resource manager's reservation and he was hired.

During the first year of the new teacher, the department chairperson sat in his classes several times, reviewed all tests and talked with him every day. Although he had trouble managing the class at first, by the middle of the school year, the strength of his mathematical knowledge and his sincere effort had won over the class. His students tested well, and overall, his performance exceeded the department chairperson's expectation.

Teacher Candidates' Profiles and their Test Performances in 2010 in the Case Study

In 2009, as described in a previous section, to let applicants know what to expect on the test, topics on the test were posted on the school web site. However, without sample questions, the candidates educated in the United States may not have expected the level of difficulty of the test questions. A research scientist who took the test in 2009 sent the school the following email after the test:

Besides enjoying my conversation with your faculty, I was very impressed by the test questions. They showed a level of thinking and skill that I am not accustomed to see in U.S. high schools. Even though I don't think I did very well on this test,

some of the questions stayed with me and think I solved them on my way home from the interview. If this is the level at which you are teaching math at [(school name)] you can be very proud. Thank you for the interesting experience and all the best.

In 2010, in an attempt to select out less capable applicants, and also to allow capable applicants who have been out of school for a long time to see what materials they may have to review for the test, the department posted a sample test on the school website. Nine candidates were invited for an interview, but this time before scheduling it, they were encouraged to try out the sample test. Four of them called later to withdraw their applications saying, in one way or another, that it would be a waste of the school's time and their own to come to take the test and implying, at the very least, that they could not manage the sample test questions with anything like the facility necessary to teach students. Their resumes looked great. Three were mathematics or engineering majors, and all but one were from most selective colleges. Also all but one had a state high school teaching certificate.

Five candidates were invited for the test and interview. The highest test score was 90 and the lowest 0. Again, possession of a state teaching certificate was found to be negatively associated with mathematic content test performance. The department decided to hire the candidate with the highest score. Why the candidates who got 14 or 0 decided to continue is a mystery. Perhaps they mistakenly thought that they had done well on the sample test since they did not have access to the correct answers.

Examples of Mistakes Candidates Made in the Case Study

Except for the candidates with scores of 85, 90 or 100, every one of the other candidates gave at least one answer containing mistakes that the department consider egregious. Instruction from teachers who make mistakes such as these can actually harm students. Here are some examples of mistakes candidates made.

Question 1(a) is about simplifying the following radical expression:

$$\sqrt[3]{54} + \frac{3}{2}\sqrt[6]{4} + \sqrt[3]{-\frac{1}{4}}$$

A candidate made the following egregious mistake:

$$54 + \frac{3}{2}\sqrt[3]{4} - \frac{1}{4}$$

What was he thinking? Perhaps he thought that each term could be cubed separately. Still, the second term does not make any sense.

Question 1(b) is about simplifying the following logarithmic expression:

$$4\log_2 \sqrt{2} - \frac{1}{2}\log_2 3 + \log_2 \frac{\sqrt{3}}{2}$$

A candidate made the following egregious mistake:

$$4(2e)\sqrt{2} - \frac{1}{2}(2e)3 + (2e)\frac{\sqrt{3}}{2}$$

What was he thinking? Perhaps he thought that

$$\log_2$$

is the same as

$$2e.$$

Question 2(a) is about solving the following trigonometric equation for x ($0 \leq x \leq 2\pi$):

$$3\sin x - 1 = \cos 2x$$

After correctly factoring

$$(2\sin x - 1)(\sin x + 2) = 0,$$

a candidate made the following egregious mistake:

$$\sin x = -2, x = \sin^{-1}(-2)$$

What was he thinking? Perhaps he forgot the fact that

$$-1 \leq \sin x \leq 1.$$

Question 2(b) is about the following trigonometric inequality x ($0 \leq x \leq 2\pi$):

$$\sin x < \sin 2x$$

After correctly using the double angle formula

$$\sin x < \sin 2x \cos x,$$

a candidate made the following egregious mistake:

$$1 < \frac{2\sin x \cos x}{\sin x}, \quad 1 < 2\cos x$$

What was he thinking? Perhaps he did not think about the possibility of

$$\sin x < 0.$$

Question 3 is about trigonometric identities as follows:

$$\text{Suppose } \sin \alpha + \sin \beta = a, \quad \cos \alpha + \cos \beta = b.$$

$$\text{Express } \cos(\alpha - \beta) \text{ in terms of } a \text{ and } b.$$

A candidate made the following egregious mistake:

$$\sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}, \quad \cos 60^\circ + \cos 30^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$\cos(\alpha - \beta) = \cos(60^\circ - 30^\circ) = \cos 30^\circ = \frac{1}{2}$$

$$a\left(\frac{1+\sqrt{3}}{2}\right) = \frac{1}{2}b, \quad \cos(\alpha - \beta) = \frac{b}{(1+\sqrt{3})a}$$

What was he thinking? Perhaps he thought that just giving an example without solving the general case would be all right. His answer degenerates into nonsense.

Question 4 is about finding the minimum value of the following exponential function:

$$y = 9^x - 2 \cdot 3^x + 5$$

A candidate made the following egregious mistake:

$$2(3)^x = 6^x$$

What was he thinking? Perhaps she completely forgot the law of exponents. Another candidate made the following egregious mistake:

$$\ln y = \ln 9^x - \ln(2 \cdot 3^x) + \ln 5$$

What was he thinking? Perhaps he thought that he could put "ln" in front of each term.

Question 5 is about comparing the following values of logarithmic functions:

$$\log_{10} x^2 \text{ or } (\log_{10} x)^2$$

A candidate made the following egregious mistake:

$$2\log_{10} x = (\log_{10} x)(\log_{10} x)$$

What was he thinking? Perhaps he thought that

$$2\log_{10} x = (\log_{10} x)^2,$$

confusing with the fact that

$$2\log_{10} x = \log_{10} x^2.$$

Question 6 is about the following summation of series:

$$\sum_{m=1}^n \left\{ \sum_{l=1}^m \left(\sum_{k=1}^l 1 \right) \right\}$$

A candidate made the following egregious mistake:

$$\sum_{k=1}^l 1 = \frac{n(n+1)}{2}$$

What was he thinking? Perhaps he just focused on the right side of the formula

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

neglecting the meaning of

$$\sum_{k=1}^l 1.$$

Question 7 is about solving a matrix equation as follows:

$$\text{Let } A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 0 \\ 1 & 1 \end{pmatrix}. \text{ Find the matrix } X \text{ that}$$

$$\text{satisfies } XA = B.$$

A candidate made the following egregious mistake:

$$XA = \begin{vmatrix} 1-x & 2-x \\ 3-x & 4-x \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 1 & 1 \end{vmatrix}$$

What was he thinking? Perhaps he confused matrices with determinants and multiplication with subtraction. Still, the answer does not make any sense.

Question 8 is about finding an angle between two vectors as follows:

Suppose that $\|a\| = \|b\|$ and $(5a - 4b) \perp (a + 2b)$, where $a \neq 0$ and $b \neq 0$. Find the angle θ between the vector a and the vector b .

A candidate made the following egregious mistake:

$$\perp \text{ so } 5a - 4b = -\frac{1}{a + 2b}$$

What was he thinking? Perhaps he was thinking that vectors were the same as slopes of lines.

Case Study Discussion

New York Times (2010) reported that although manufacturers could expect a large number of qualified people to fill their openings after so many workers were laid off during the worst of the recession, some of them could not fill their openings. As an example, the article cites a Cleveland suburb contract drug maker for pharmaceutical companies that could find only 47 people out of 3600 job applicants to hire because “a significant portion” of applicants failed to “pass a basic skill test showing they can read and understand math at a ninth-grade level.”

The drug maker’s predicament seems to be exactly the same as the one in which the previously described private high school mathematics department found itself during its attempt to hire a teacher with the mathematical understanding and competence to teach 11th grade level mathematics. Neither a bachelor’s degree in mathematics from selective universities, nor a U.S. mathematics teaching certificate, nor a master’s degree in mathematics education seems to guarantee that a candidate would be competent at 11th grade level mathematics. The test demonstrated this beyond any doubt.

Describing the history of U.S. K–12 education in her book entitled “Left Back: A Century of Failed School Reform,” Ravitch (2000) quotes Isaac L. Kandel, an internationally regarded American scholar, defending a national examination developed by the American Council on Education in 1939 and first offered in 1940⁷, stating that “critics of the examination reflected an old American tradition that all a teacher needed to know was how to teach” and that education psychology and philosophy at his time “were aligned with a native tradition of anti-intellectualism. (p. 316)” These traditions still seem to be alive in 21st century America.

⁷ According to Ravitch (2000), the National Teacher Examination was developed “in the depth of the Depression, when teaching jobs were highly prized and there were more applicants than positions.” School officials liked the examination “as long as teachers were abundant.” When a teacher shortage occurred due to World War II, “school districts no longer cared how well their teachers scored on an examination.”

What is interesting in American education is that at the graduate school level, it is quite successful in producing many internationally recognized researchers as Cole (2010) describes in his book entitled “The great American university.” Cole (2010) attributes the success to the uniquely American relationship between professors and students in which they work as a team as intellectual equals.

On the other hand, at the K–12 level, a Michigan State University report entitled “Breaking the cycle: An International Comparison of U.S. Mathematics Teacher Preparation” states, “the weak K–12 mathematics curriculum taught by teachers with an inadequate mathematics background produces high school graduates who are similarly weak. Some of them then become future teachers who are not given a strong preparation in mathematics and then they teach and the cycle continues” (p. 31).

Perhaps, the mathematics department may have been seeing something of the effect of this cycle in its hiring attempts. But, in particular, what the department has learned is that judgments concerning potential mathematics teachers based on degrees, certificates, resumes and interviews alone are insufficient when not corroborated by an in-depth, written, comprehensive, mathematics content knowledge test.

Conclusions

International Association for the Evaluation of Educational Achievement (IEA) and TEDS-M project at Michigan State University conducted a study of twenty countries investigating a relationship between teacher pay—as compared to mathematics-oriented professions, such as engineers and scientists—and student mathematics scores on TIMSS 2003 and PISA 2003. The study stated that “students in countries where teachers are paid more relative to males’ salaries in competing professions do better on mathematics knowledge tests” (Carnoy et al., 2009) with the proviso that this relationship may not necessarily be causal.

Another study by OECD (2010) reported that the average teacher salary in Korea and in Finland, the countries whose students’ performances on international tests were at the top level, was slightly above 80% and 90% of the average salaries of all college graduate workers in the respective country. By contrast, the average teacher salary in the U.S. where the average student score on international tests is significantly lower than those of Korea and Finland is slightly below 60% of the average salary of college graduate workers.

Recently, it has been reported that in the United States, local governments and/or schools often invest in education technology even though the effectiveness of the technology has not been proven (New York Times, 2011). Investing in technology at the expense of teachers’ pay might well be a mistake.

John Friedman (2012) wrote, “In the long run, the best way to improve teaching will likely require making teaching a highly prestigious and well rewarded profession that attracts top talent.” And for a developing country it may be crucial in preventing “brain drain,” that is, the loss of a country’s educated elite to jobs abroad.

A developing country typically does not have the capital, the technology, or the research and academic institutional infrastructure to support a high rate of economic growth. It does have a population of young people that it can educate—a critical factor for economic growth. But, once educated, where will the top students go? If they believe that there are few top jobs for them inside the country, they will leave for attractive jobs outside the country, thus creating a “brain drain.” If, when asked, “What profession do you plan to pursue?” a significant proportion of the country’s best mathematics, physics and engineering students reply, “Teach public high school,” then the country will not lose so much of its top talent to jobs abroad. Those who want jobs in technology, computer engineering, finance, or other highly specialized fields will leave for countries with the infrastructure to provide those jobs. But, by using high salaries as an incentive and recruiting the best students to teach in public high schools, the country will create a top tier profession which will maintain and regenerate itself by educating more than enough students to counter the “brain drain” of engineers and financiers going abroad. A growing, educated population will attract capital and companies to build infrastructure, which will slow and eventually reverse the trend of students leaving for jobs abroad, and a positive feedback loop will result. Just as it did in post-war Japan, a common sense, incentivized teacher salary schedule will lead the developing country’s education to a virtuous cycle in which the strong K–12 mathematics curriculum taught by teachers with the best mathematics backgrounds produces high school graduates who are similarly strong. Some of the best of them then become future teachers who continue to receive the best preparation the country can offer in mathematics and then they teach and the cycle continues.⁸

References

- Abdulkadiroblu, A., Angrist, J.D., & Pathak, P.A. The Elite Illusion: Achievement Effects at Boston and New York Exam Schools. (2011) National Bureau of Economic Research. NBER Working Paper No. 17264 Accessed on February 2, 2012 at <http://www.nber.org/papers/w17264>
- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., Tsai, Y. (2010). Teachers’ Mathematical Knowledge, Cognitive Activation in the Classroom, and Student Progress. *American Educational Research Journal*. March 2010, Vol. 47, No. 1, pp. 133–180. Accessed on February 2, 2012 at http://library.mpib-berlin.mpg.de/ft/jb/JP_Teachers_2010.pdf
- Berliner, D. C., & Biddle, B. J. (1995) *Manufactured Crisis*. Cambridge, Massachusetts: Perseus Books.
- Blum, W., Baumert, J., Neubrand, M., Krauss, S., Brunner, M., Jordan, A., & Kunter, M. (2005). *COACTIV—A Project for Measuring and Improving the Professional Expertise of Mathematics Teachers*. Paper presented at the 15th International Commission on Mathematical Instruction (ICMI) Study Conference. Accessed on February 2, 2012 through <http://stwww.weizmann.ac.il/G-math/ICMI/strand2.html>
- Carnoy, M., Beteille, T., Brodziak, I., Loyalka, P., & Luschei, T. (2009). *Do Countries Paying Teachers Higher Relative Salaries Have Higher Student Mathematics Achievement?* Report prepared for the TEDS-M project under the auspices of the International Association for the Evaluation of Educational Achievement (IEA). Accessed on February 2, 2012 at http://www.iea.nl/fileadmin/user_upload/Publications/Electronic_versions/TEDS-M_Cost_Study.pdf
- Chetty, R., Friedman, J.N., & Rockoff, J.E. (2011). *The Long-Term Impact of Teachers: Teacher Value-Added and Student Outcomes in Adulthood*. National Bureau of Economic Research Working Paper No. 17699, December 2011.
- Cole, Jonathan R. (2010) *The Great American University: Its Rise to Preeminence Its Indispensable National Role Why It Must Be Protected*. New York: Public Affairs.
- Eide, E., Goldhaber, D., & Brewer, D. (2004). The Teacher Labour Market and Teacher Quality. *Oxford Review of Economic Policy*; Summer 2004; 20, 2; ABI/INFORM Global p. 230. Accessed on February 2, 2012 at http://www-bcf.usc.edu/~dominicb/pub/The%20Teacher%20Labour%20Market%20and%20Teacher%20Quality_Oxford%20Review%20of%20Economic%20Policy.pdf
- Friedman, J. (2012). School Finance 101. Accessed on February 2, 2012 at <http://schoolfinance101.wordpress.com/2012/01/19/follow-up-on-fire-first-ask-questions-later/>
- Gross, M. L. (1999). *The Conspiracy of Ignorance*. New York: HarperCollins Publishers.
- Gifu University, Faculty of Education. (1996). In Yasuda, T. (2003) *Nyuushi Suugaku Dentou no Ryoumon 100*. Tokyo: Koudansha.
- Kinney, C. (1998) Teachers and the teaching profession in Japan. In *The Educational System in Japan: Case Study Findings*. (pp. 183–253) U.S. Department of Education.
- Krauss, S., Neubrand, M., Blum, W., & Baumert, J. (2008). *The Professional Knowledge of German Secondary*

⁸ Rephrasing of quote from “Breaking the cycle: An International Comparison of U.S. Mathematics Teacher Preparation.”

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- Mathematics Teachers: Investigations in the Context of the COACTIV Project*. Paper presented at the 11th International Congress of Mathematical Education (ICME-11) Topic Study Group 27. Accessed on February 2, 2012 at <http://tsg.icme11.org/document/get/397>
- Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: validation of the COACTIV constructs. *ZDM Mathematics Education*, 2008, 40: 873–892. Accessed on February 2, 2012 at <http://www.springerlink.com/content/t86vvlh11481tv82/fulltext.pdf>
- Michigan State University the Center for Research in Math and Science Education. (2010) *Breaking the Cycle: An International Comparison of U.S. Mathematics Teacher Preparation*. Accessed on July 4th, 2010 at <http://www.eduuuc.msu.edu/content/sites/usteds/documents/Breaking-the-Cycle.pdf>
- Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. (2003). *Elementary and Secondary Education*. Accessed at <http://www.mext.go.jp/enblish/org/formal/05d.htm>.
- Ministry of Health, Labour and Welfare of Japan. (2001). <http://www.mhlw.go.jp/english/database/db-1/1310fr/1310c1re.html>
- Naigai Kyouiku Kenkyuukai. (2002a). *Ippan Kyouyou no Jisshi Mondai*. [Actual Past Problems on “General Knowledge (Expected of College Graduates)”. Tokyo: Jijitsuushinsha.
- Naigai Kyouiku Kenkyuukai. (2002b). *Kyoushoku Kyouyou no Jisshi Mondai*. [Actual Past Problems on “General Knowledge of Education”]. Tokyo: Jijitsuushinsha.
- Neubrand, M. (2008). *Knowledge of Teachers—Knowledge of Students: Conceptualizations and outcomes of a Mathematics Teacher Education Study in Germany*. Symposium on the Occasion of the 100th Anniversary of ICMI (Rome, March 5–8, 2008). WORKING GROUP 2: The professional formation of teachers. Accessed on February 2, 2012 at <http://www.unige.ch/math/EnsMath/Rome2008/WG2/Papers/NEUBR.pdf>
- New York Times. (2010). Factory Jobs Return, but Employers Find Skills Shortage. *New York Times*, July 2, 2010. Accessed on July 2, 2010 at http://www.nytimes.com/2010/07/02/business/economy/02manufacturing.html?_r=1&partner=rss&emc=rss&src=igw&pagewanted=print
- New York Times. (2011). Grading the Digital School: In Classroom of Future, Stagnant Scores. *New York Times*, September 3, 2011. Accessed on February 2, 2012 at <http://www.nytimes.com/2011/09/04/technology/technology-in-schools-faces-questions-on-value.html?ref=technology>
- OECD. (2011). *Divided We Stand: Why Inequality Keeps Rising*. Accessed on February 2, 2012 through http://www.oecd.org/document/51/0,3746,en_2649_33933_49147827_1_1_1_1,00.html
- Osaka Teachers College. (1995). In Yasuda, T. (2003) *Nyuushi Suugaku Dentou no Ryoumon 100*. Tokyo: Koudansha.
- Ravitch, D. (2000). *Left Back: A Century of Failed School Reform*. New York: Simon & Schuster.
- Roudou Hourei Kyoukai. (2001) *Basic Survey on Wage Structure 2001*.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), pp. 3–14.
- Trelfa, D. (1998). The Development and Implementation of Education Standards in Japan. *The Educational System in Japan: Case Study Findings*. (pp. 23–78) U.S. Department of Education.
- U.S. Department of Education. (1987). *Japanese Education Today*.
- U.S. TEDS-M. (2010). Accessed on March 31, 2013 at <http://www.educ.msu.edu/content/sites/usteds/documents/USTEDS-FAQ.pdf>
- Waseda University. (2003). *Teacher Training Course*. Accessed at http://www.waseda.ac.jp/schl/edu/i_english/index-e.html
- West, K. I. (2013). Strengthening a country by building a strong public school teaching profession. *Journal of Project Research* (8).