JOURNAL OF

MATHEMATICS EDUCATION AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Beyond Teaching Mathematics

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JOURNAL OF **MATHEMATICS EDUCATION** AT TEACHERS COLLEGE | SPRING 2016 | VOLUME 7, ISSUE 1

TEACHERS COLLEGE | COLUMBIA UNIVERSITY

Some Thoughts on Educating More Able Students

Geoffrey Howson
University of Southampton, England

For the past three years, the Program in Mathematics and Education at Teachers College, Columbia University, has, in coordination with its colloquium series, held a Columbus Day Symposium. Each year, the all day event has been focused on a particular theme in mathematics education, and has brought in various speakers and leaders in the field to share their work and insights. This has included a presentation of an award to a scholar who has made significant contributions to the field. This year, Dr. Geoffrey Howson received this award. Throughout his career, Dr. Howson has written, edited, and contributed to many important books and papers on mathematics, education and mathematics education. Among other things, he has also served as President of the Mathematical Association of Great Britain, and Secretary-General of ICMI. The theme for the symposium this past year was the teaching of the mathematically gifted, and Dr. Howson has also made contributions in this area. The following is an edited version of the remarks he gave at the Columbus Day Symposium.

KEYWORDS *more able students, mathematically gifted students*

First I should like to say how sorry I am that I cannot be with you in person today. My only visit to Teachers College was in 1968. Then Howard Fehr and I talked about the Secondary School Mathematics Curriculum Improvement Study (SSMCIS) and the School Mathematics Project (SMP), the two curriculum development projects with which we were concerned, and their progress. So another visit seemed long overdue. In preparing this talk I was also reminded of a chat with Hans Freudenthal, the celebrated Dutch mathematician and mathematics educator, when he invited me to take part in his 80th birthday celebrations. I asked him how long he intended to keep on lecturing. He replied, 'As long as I think I am talking sense. But what worries me is if I didn't realise that I had stopped doing so. X (naming a world renowned mathematician) didn't.' I hope that you will not leave today thinking that I have fallen into the same trap as X!

To Learn More Mathematics

You will see that I speak of educating rather than teaching: the reason for this will, I hope, become clearer later.

I should like to begin by drawing your attention to a series of very interesting papers on this topic to be found in the De Morgan Journal (http://education.lms.ac.uk/wp-content/uploads/2012/02/DMJ-FS.pdf). These include papers on special schools in several countries and also a survey of what was happening in England. It was, in fact, the paper by Alexandre Borovik, who was student, teacher and selector at the Novosibirsk special school, which caused me, like those concerned in running that school, to avoid the word 'gifted.' To my mind, 'gifted' implies something much more permanent than what actually is the case in practice. My ability, when 13 or so, rapidly to polish off scores of routine exercises did not, I believe, qualify me to be called 'a *gifted* young mathe-

matician'—I was simply more able than my classmates at picking up routine skills.

There are also papers on that web site which bear on the pros and cons of 'acceleration' and 'enrichment'. The first is taken to mean moving a class or, less frequently, a pupil onto work which would normally be done in the succeeding school year. This has both advantages and disadvantages which I do not wish to discuss further here. However, it is perhaps worth my observing that many mathematicians whom I have known did, like myself, begin secondary schooling a year, or even more, before it was normally done. After this premature 'acceleration' schoolwork took its normal course. This did offer great advantages, for one then had extra time before entering university. However, in English state schools such opportunities would no longer appear to exist. What is now considered to be more in the able pupils' interest is to supplement what would appear to be a too meagre and simplistic a diet by enrichment. This is usually in the form of deeper and more demanding exercises on the topic currently being taught or general and diverse problems demanding greater mathematical thought and providing increased mathematical insight and understanding. I believe this option is commonly followed in other countries and this is exemplified by the materials available both on-line and in print form to facilitate such an approach. For example, twenty years or so ago I was on the steering committee of a project, NRICH, which aimed to produce such material for both primary and secondary school pupils. It has grown considerably since those early days and what it now offers on its website (http://nrich.maths.org), including examples to be found in the 1960s SMP texts to which I shall refer later, is of much more value to schools. Of course, such facilities are only of value if teachers feel confident enough to use them and to bring them to the able student's attention: for students attempting such work will require some supervision and, on occasions, help. Regrettably many teachers will lack that confidence and ability, particularly those in primary schools.

Such enrichment, in the various forms in which it appears, is, of course, enormously valuable as are the summer courses and competitions that exist, along with popular mathematics talks which can also be found. Yet, again, teachers have to be interested and confident enough to know of and to encourage pupils to take advantage of these.

However, enough of generalities and what is universally accepted. Let me put forward some slightly different, personal points of view of what enrichment might also encompass. These, I believe, only really concern students in secondary and special schools.

To Learn to Become Mathematicians

In 1982 I visited, in short succession, special schools in two different countries. My visits were short ones and so the imbalances I observed may not have persisted over a longer period. However, there seemed to be an essential difference in the lessons I observed.

In one country the aim seemed to be to teach more mathematics, for what I observed were well delivered university-type lectures—essentially a kind of acceleration. In the other, there was greater evidence of trying to produce mathematicians. There a lesson began with two pupils explaining how they had tackled two particularly tricky problems recently given to them for homework. The importance of a mathematician being able orally to explain his or her work to others cannot be overstressed. It is not only an invaluable attribute, but practising it is likely, from my experiences, to increase one's own understanding of the mathematics involved. The teacher then gave a brief introduction to a new topic together with a small number of examples, and this was followed by the class being asked to tackle certain problems with a marked increase in complexity. But the class were not expected to do this individually; groups of students rapidly formed and the tasks were tackled cooperatively. Dealing with problems was seen as a group activity and not as a race between individuals. Of course, there would be many occasions for working individually but problem-solving was not seen solely as a well-defined individual activity. This illustrates two other forms of enrichment, however ones not available to the lone able student in the class. Of course, he or she could be asked to explain some mathematics to the class, but I believe that this would separate them even further from their less able classmates which could easily exaggerate already existing potential social problems. Asking them to explain matters to an individual student might be more useful should the opportunity arise.

Yet *how* one learns mathematics is an important factor and I fear that in England these days far too little attention is paid to the way in which mathematics can be learned by reading—one does not always have to have a teacher or lecturer. When the School Mathematics Project began in the 1960s we were concerned with pupils drawn from the top 25% of the ability range: ones who had reached their selective secondary schools by passing examinations in mathematics (mainly arithmetic), gen-

eral knowledge, and English. The books, then, were written so as to be *read* by students. Moreover, they contained much mathematics that was not on the examination syllabuses. These extras, and the 'puzzle corners' which were also included, were intended to be always at hand for the student who had finished the set work. Unfortunately today's students aged 11–16 in England may well not have textbooks, and if they do they will almost surely be simply recipe books for solving standard examination problems. Catering for the more able does not always take a high priority—they will pass the national examinations anyway and so it is more financially beneficial for a school to concentrate on getting borderline pupils through these examinations. Here I believe that being asked to read and to attempt the exercises in some of the SMP chapters on material which does not feature in the present examination syllabuses would be more beneficial to students than relying solely on an enrichment diet consisting simply of problem solving. There are a host of possibilities, varying considerably in length and difficulty, for example, Farey sequences (including some continued fractions), polyhedra (including model making and Schlegel diagrams), perspective, linear programming, and chapters on proof and on symmetry and the seventeen wallpaper patterns. I still have hopes that some of this old SMP material will find its way onto the internet and facilitate this approach. Here I am influenced by my own learning when at school. In my final years at school I had to study as separate subjects 'pure' and 'applied' mathematics, the latter being Newtonian statics and dynamics plus some hydrostatics. My teacher for 'Pure' was extremely competent, that for 'Applied' less so. In my final year, having covered most of the examination syllabus already, the teacher for Pure Mathematics would spend some time teaching me some more advanced work, but much of my time was taken up reading a book he gave me on projective geometry, working the problems in it and explaining to him what I had read, understood and done. Although this might seem an odd topic to choose, I found it fascinating, the 'what if…?' was a new idea and meeting an axiom system with duality which offered 'prove one theorem, get one free' was a pleasing foretaste of the hypermarket offers. But it also gave me confidence in my ability to learn through reading and to explain what I had learned to another person. The former proved particularly valuable in applied mathematics, for my teacher was weak and showed no signs of being totally in command of the subject herself. I had to rely on a good textbook in order to learn the subject. My being awarded a coveted State Scholarship to university in the examinations I then took was, in fact, due principally to the marks I attained in applied mathematics. This might be used as proof of the value of good textbooks, but, on the other hand, it could, regrettably, be used to demonstrate that a restricted examination-inspired diet is likely, in most educational systems, to prove more 'beneficial' than a truly educational one. Oddly enough, I later learned that a mathematician, who probably gave more to SMP than anyone else, had also been asked to read the same projective geometry text in his sixth form days and had found similar inspiration in the task.

To Learn More About Mathematics

Yet, in retrospect, I think I might have been given a book to read that not only taught me mathematics, but taught me more *about* mathematics. Let me emphasise this last point. At the moment in dealing with able pupils we try to teach them more mathematics. What we do not do is to teach them more about mathematics, how it has evolved and how it is used.

How it Has Evolved

I am not familiar with all the books that have been published in recent years and perhaps better examples now exist. However, I shall describe a book from the 1960s which I think would have been a better choice for an able sixteen or seventeen year old to have read. It is Annita Tuller's (1967) *A Modern Introduction to Geometries.* Some of you may know this book, but for those who do not, it tells, concisely and straight-forwardly, how geometries have developed over the centuries: Euclidean, Cartesian, projective, non-Euclidean, transformation, Hilbert-inspired and finite. The reader would appreciate how mathematics and mathematical thought is always developing. It does so in a way having a more mathematical than purely historical bias and so would be a good choice for the reader I have in mind. I hope there are more recent and more readily available books that serve this purpose. I have, however, regrettably not met one. Certainly there are 'popular' books on, say, the recent solutions of great mathematical problems that could be recommended, but none that I have read would, I feel, be more suitable. Again, there are popular lectures that are given which in their own way demonstrate the evolution of mathematics—but valuable as they might prove, I believe that a book has demonstrably much more to offer and is not so necessarily compressed.

How It Is Used

How does the student learn about the way in which mathematics is used away from university mathematics departments? This was a problem that a colleague, Ron McLone, and I attacked in the early 1980s. With the help of a grant from British Petroleum we commissioned a series of chapters and of companion lectures throughout the country by their authors which resulted in a book, *Maths at Work* (Howson & McLone, 1983). The fifteen chapters illustrated how mathematics—at very widely varying levels of depth—was being used by different people. These included academics from different disciplines such as biology and archaeology; from industry—industrial safety, retail trade, insurance, banking; from government agencies—the Transport and Road Research Laboratory, the Meteorological Office, the Central Electricity Generating Board (and even NASA, although the work reported had been done in the UK!). One contributor told of a contract to recommend ways of increasing the shipping capacity of the Suez Canal. (After disposing of various alternative shipping patterns, the solution that more of the canal should be made 'dual carriageway' was eventually acted on and became effective only in August this year). Southampton colleagues told of how they had been contracted to devise statistical sampling programmes for testing consumer preferences for new apple pies which could be produced with or without five variables (giving 32 possibilities in all), or had given evidence in a court case which depended on the probability of the defendant's case being reasonable. Quite how widely the book was used in schools I do not know. However, a few years ago I mentioned its existence when talking to two Southampton University applied mathematics professors and found to my delight that both had been given the book to read when in their last years at their schools and were greatly appreciative of it. Sadly, a book of this nature will become outdated very quickly and will have little appeal in countries other than that in which it is written. It also entails a great deal of work on the part of the editors and authors. Yet I believe that informing able (and not so able pupils) and their teachers about the varied uses to which mathematics is now being put is an important part of what should be their mathematical education. Perhaps the preparation of a similar book is something which, say, the NCTM might consider. It also helped guide a weekly Wednesday afternoon course that my department mounted for mathematically able sixteen and seventeen year-olds from schools in the neighbourhood. The afternoon was split between lectures which introduced some new mathematics to the students and group problem-solving. The lecture topics were chosen to illustrate the use of mathematics applied to social problems: models for population growth and genetics. A major aim was to induce interest in mathematics; for, sadly, competence and interest are not always combined.

To Increase Interest in Mathematics

When thinking of really gifted mathematicians I recall how in the late 1970s I asked several colleagues and friends to name the ten British university mathematicians whom they considered to be the most creative. I received about thirty different names. Of these, if I remember correctly, only two had been at schools which in the late 1970s were still in the state public-funded sector. The other schools were now all independent, fee-paying schools—although at the time that these mathematicians were students in them some had received state money to pay for the education of boys who won scholarships to them. (It was a sign of the times, and would not be the case now, that all thirty were men.) All except, I believe, one had taken their first or postgraduate degrees at Oxford or Cambridge. (In the De Morgan Journal to which I have referred the French contributor, who describes his days in the special classes at the school Louis le Grand in Paris, speaks not only of the way in which leading mathematicians are increasingly drawn from the upper middle classes, but also how Grothendieck, who has since died, was the only leading French mathematician who had not studied at the Ecole Normale or the Polytechnique.) What distinguished the British mathematicians was that they had almost all attended schools with excellent mathematics teachers, most of whom I knew personally or by repute, would have worked in small classes constituted of the more able of their year-mates, and would have had high mathematical attainments expected of them. Unfortunately, they are the exceptions. In the UK it is the more able student who is in a state sector school in a large class and without the competition provided by similar students who demands our special attention.

This has been a somewhat rambling talk. However, to sum up, I see the needs of the students in a special school, a class of high attainers in a privileged school, and the sole more able student in a less well-staffed school as requiring very different kinds of attention—although our aims in educating them must be identical.

I also believe that in addition to ensuring that more-able students can be enabled both to learn more mathe-

matics and to develop greater mathematical skills than their less mathematically competent classmates, we should also develop ways in which they can better 'learn to become mathematicians'. There is also a need for them to learn more *about* mathematics and how it is used in the world today and to develop an *interest* in the subject rather than merely the ability to obtain high marks in it: perhaps these two aims go hand in hand.

Thank you.

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