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A Comparison of Mathematics Teachers' and Professors' Views on Secondary Preparation for Tertiary Calculus

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ABSTRACT This article compares the views of teachers and professors about the transition from secondary mathematics to tertiary calculus. Quantitative analysis revealed five categories where teachers and professors differed significantly in the relative frequency of addressing them. Using the rite of passage theory, the separation and incorporation phases were investigated by carrying out thematic analyses on these five categories. For the professors, the analysis revealed specific content within algebra and precalculus that they viewed as vital preparation for students' tertiary calculus success. For the teachers, the analysis highlighted the classroom environment realities of teaching in the separation phase. The rite of passage and professional turf theories are used to discuss and interpret the findings.

KEYWORDS *secondary precalculus, secondary calculus, tertiary calculus, teachers' and professors' views of preparation*

The Secondary-Tertiary Transition

The secondary-tertiary transition in mathematics is a complex issue, involving a wide spectrum of challenges and situations for students to navigate (Clark & Lovric, 2008). While there is no straightforward answer as to when the secondary-tertiary transition begins and ends, educational research that covers the period between two years before entering university and two years after most likely presents transition issues (Gueudet, 2008). Secondary mathematics teachers who teach senior-level students only rarely have the opportunity of examining how well-prepared their students are for subsequent mathematics courses in college, since only college professors can provide such feedback (and the occasional returning student). Yet, it has been hypothesized that there is a qualitatively different approach to mathematical thinking and instruction between the secondary mathematics and tertiary calculus levels and that there is a "great need for improved communication between the two sectors" (Hong et al., 2009, p. 877). Research about the secondary-tertiary transition indicates that the mathematical under-preparedness of students entering into tertiary mathematics is an issue that may compromise students' success in mathematics at colleges and universities (Hong et al., 2009; Hourigan & O'Donoghue, 2007; Selden, 2005).

Theoretical Framework

Clark and Lovric (2008) applied the *rite of passage* theory to the secondary-tertiary transition in mathematics. This theory originated in anthropology and has been used to describe the transition between major life stages where change results from interruptions or distortions of what was previously known (Clark & Lovric, 2008; van Gennep, 2011). A rite of passage was defined as a process that occurred during the transition from one well-defined situation (secondary school) to another equally well-defined situation (college or university) (Clark & Lovric, 2008; 2009). There are three stages within the secondary-tertiary rite of passage. The first is *separation,* which takes place while students are still in high school and includes anticipation of forthcoming university life (Clark & Lovric, 2009). During this time, senior-level students are often in precalculus or some level of secondary calculus, i.e., Advanced Placement (AP) Calculus or non-AP Calculus. Next is the *liminal* phase, which includes the end of secondary school and expands across to the first year at university (Clark & Lovric, 2009). The third phase is *incorporation,* which includes roughly the first year at university (Clark & Lovric, 2009). In this phase, professors develop views of how well-prepared their students are for tertiary calculus.

Whereas the rite of passage theory looks at the transition mainly from the subjects' perspective, i.e., in our case, that of the students moving from secondary to tertiary mathematics, this perspective can be usefully complemented by a turf analysis of the two professional groups situated at either end of the passage (Abbott, 1988), i.e., mathematics teachers and professors. The two professions of teachers and professors are in a sequential relationship defined by the flow of students through the passage. Turf wars are not uncommon between adjacent professions, and the migration of calculus into the secondary domain may be seen as part of a wider movement to place college material into high schools. This theoretical perspective suggests that some professors would tend to view this development critically. In this article, we explore the different viewpoints of teachers and professors in hopes of creating an even better mutual understanding between the two professions, which eventually might help smooth the passage for the students.

The advantage of our theoretical framework—rite of passage and professional turf—is that it allows us to connect across the professional turfs and identify specific factors that may influence students' experience in the rite of passage. These factors include the populations with which teachers and professors work and the secondary mathematics curriculum. First, the populations with which teachers and professors work are different because not all high school graduates proceed to college. For example, in 2009, 70% of students who completed high school enrolled in college the following fall, and 30% did not (National Center for Education Statistics [NCES], 2011). And of the college students, not all continue to tertiary calculus. Within social cognitive models, motivation is usually assessed for a specific subject area, such as mathematics, because some students are motivated to learn mathematics, whereas others are not (Linnenbrink & Pintrich, 2002). Students not planning to proceed to college level mathematics may not be as motivated to learn as those who are planning to do so. Yet teachers must plan, implement, and manage the classroom to carry out instruction in the separation phase, while seeking to prepare all students in precalculus or calculus courses for tertiary calculus success. Doing so, teachers may be subjected to many simultaneous and often contradictory challenges, difficulties, and necessities. By contrast, whereas not all students in college level precalculus and calculus courses equally enjoy learning mathematics, they have all made the commitment to be in those college level courses (and often are required to pass them for their intended majors). However, introductory college and university calculus courses may have fewer of those students who are the most well-prepared in high school mathematics classes. Students who score well on Advanced Placement or International Baccalaureate (www.ibo.org) exams may take the opportunity to jump ahead to more advanced STEM coursework in college (Sadler & Tai, 2007), although many choose to retake Calculus I (Hsu & Bressoud, 2015). This may color professors' views of the preparedness of their calculus students. Nonetheless, at the start of tertiary calculus, students generally exhibit high levels of enjoyment and confidence in mathematics (Sonnert & Sadler, 2015; Sonnert, Sadler, Sadler, & Bressoud, 2015).

Second, the mathematics curricula at the secondary and tertiary levels are different. High school teachers have little choice of the content they teach within precalculus or calculus courses. Either they follow state mathematics standards, the Common Core State Standards (CCSS), or the College Board AP Standards. The primary function of the mathematics curricula is, of course, to provide a feasible and meaningful order to the teaching of mathematics. It is the professors who may notice the discrepancy between the knowledge and performance of students from different regions of the nation. For example, before the 2010 adoption of the CCSS, 5 states required 4 mathematics credits for graduation, 26 states required 3, and 15 required 2. Now, 18 states require 4 mathematics credits, 24 states require 3, and 5 require 2 (Zinth, 2012). Even with the increase in course requirements across the nation for high school graduation, it remains most common for Algebra I, Algebra II, and Geometry to be required for high school graduation. In the separation phase, many students voluntarily move into precalculus and calculus, or it could be that precalculus is the student's fourth mathematics high school graduation requirement. Moreover, if students took Algebra 1 in the eighth grade, a trend since 2007 (Loveless, 2008), students would either take some level of calculus their senior year, or no mathematics class at all (unless courses such as statistics or discrete mathematics are offered). Making the disconnect between high school and college greater, tertiary calculus has become more theoretical, covering topics from limits up to derivatives in more depth (Bressoud, 2010), while the AP Calculus curriculum covers content from limits to derivatives and integral processes and applications. Bressoud (2010) argued that there was so much content to learn that students focus on learning procedures instead of understanding concepts. A less obvious and somewhat hidden function of mathematics curricula is the turf demarcation between two professions. Keeping this subtext in mind might be helpful in understanding some of the dynamics of curriculum debates.

Purpose and Research Question

The purpose of this paper is to advance understanding of the differences in teachers' and professors' views of secondary instructional practices that prepare students for success in tertiary calculus. Because the views presented here are from mathematics instructors across the United States (US), this article is situated in a US context. Our research question is: How do the views of secondary mathematics teachers and college mathematics professors differ about the preparation of students for success in tertiary calculus?

Methods

Source of Data

The Factors Influencing College Success in Mathematics (FICSMath) project (2009-2010) was the first national study of the secondary-tertiary transition in mathematics. Three sources of data were gathered for the development of the FICSMath survey items. One source was a broad literature review of current issues in secondary and tertiary mathematics education. Second, a panel was created from mathematics educators, mathematicians, and mathematics education researchers who discussed survey items cooperatively on two different occasions at Harvard University. The third source was an online survey sent to precalculus and calculus secondary teachers as well as to precalculus and calculus professors. While the professors' and teachers' surveys fulfilled the task of hypothesis generation for the FICSMath survey, we here considered the responses in their own right – as valuable information about instructional views across the secondary-tertiary transition. The fact that we focus on secondary mathematics should not be construed as implying that everything is fine in tertiary calculus. In fact, the calculus reform movement of the 1980s came about because of the need for instructional reform at the college level (for a review, see Tall & Ramos, 2004). For terminological clarity, the survey that was sent to teachers and professors was a qualitative survey used for the development of the FICSMath survey items, and then the FICSMath survey gathered quantitative data from more than 10,000 calculus students across the nation. Henceforth in this article, survey refers to the surveys that were sent to teachers and professors, and the FICSMath survey or survey items will be addressed as such.

The Sample

The FICSMath research group contracted with the Market Data Retrieval (MDR) Company to email surveys to a nationally representative random sample of precalculus and calculus mathematics teachers and precalculus and calculus professors. MDR sent emails to 2,193 mathematics teachers and 2,166 professors at two- and four-year universities, asking them to participate in a brief online survey. The online format made it possible to seek a nationally representative sample of precalculus and calculus teachers and professors in a time and cost efficient manner (Wright, 2005), and the random sample increased the likelihood of receiving representative responses (Bouma & Atkinson, 1995). The mathematics teachers responded to the question, "What do you do, as a high school mathematics teacher, that you think prepares students for college calculus success?" The response rate was 3.8%, with 84 mathematics teachers (52% male) returning the surveys. The professors' survey asked, "What can high school teachers do to prepare students for success in college calculus courses?" There was an 8.5% response rate, with 185 professors (62% male) returning the surveys.

An obvious disadvantage of the online survey was the low response rate from teachers and professors. Baruch and Holtom (2008) and Wright (2005) reported that emailed surveys historically have lower response rates than surveys that are delivered by other methods. MDR reports their average response rate as click-through rates, which measures if the recipient actually opened the email and clicked on the link for the survey. Their click through rate was 3% for the 2009-2010 school year (Rigol & Ziemnicki, 2011). This was lower than the 3.8% and 8.5% return rate from teachers and professors, respectively, meaning the response rates for our survey were somewhat higher than expected, when compared with the rates that MDR reports.

Analysis

The surveys requested up to three opinions from teachers and professors about what best prepares students for college calculus success. Some respondents provided a list of three statements, but many replied in paragraph form rendering more than three statements. Following Saldana's (2013) initial coding guidelines, statements were grouped into categories with other responses that shared similar characteristics. Statements within the categories revealed patterns that merged into themes. For clarity, the categories are capitalized while the themes are capitalized and italicized. There were 388 statements from professors in 13 categories and 199 statements from teachers in the same 13 categories.

Figure 1 compares the proportion of statements from professors with the corresponding proportion of statements from teachers for the 13 categories. The categories are presented in reference to the 95% confidence interval, called the Cone of Similarity. The categories within the Cone (including Conceptual Understanding and Proofs on the outer boundary) were not significantly different in terms of the proportion of statements in which they were mentioned by the professors and teachers. The five categories outside of the lower and upper bounds of the Cone had significantly different proportions of statements provided by the professors and teachers. Figure 1 shows that professors mentioned content (Algebra and Precalculus) more frequently than did teachers, and teachers mentioned pedagogy (Classroom Environment, Real World Problems, and Textbooks) more frequently than did professors. For example, professors listed many types of functions students should know, while teachers presented classroom practices used to teach functions, such as small groups or classroom discussions.

The proportions of statements made by teachers and professors within a particular category provide insight into the extent to which teachers and professors view the category as important to the preparation of successful calculus students. The particular contents of their views are captured in the themes. Within the Cone of Similarity, one finds categories that were of similar concern to teachers and professors and that, additionally, had similar contents. The categories outside of the Cone, by contrast, constitute a strategic research site for gaining insight into *differences* in opinion and outlook across the passage from secondary to tertiary mathematics—differences that may be at the root of some of the problems that students experience in the passage. In this article, we therefore concentrate on the categories outside of the Cone and expand on the themes that teachers and professors expressed within those categories.

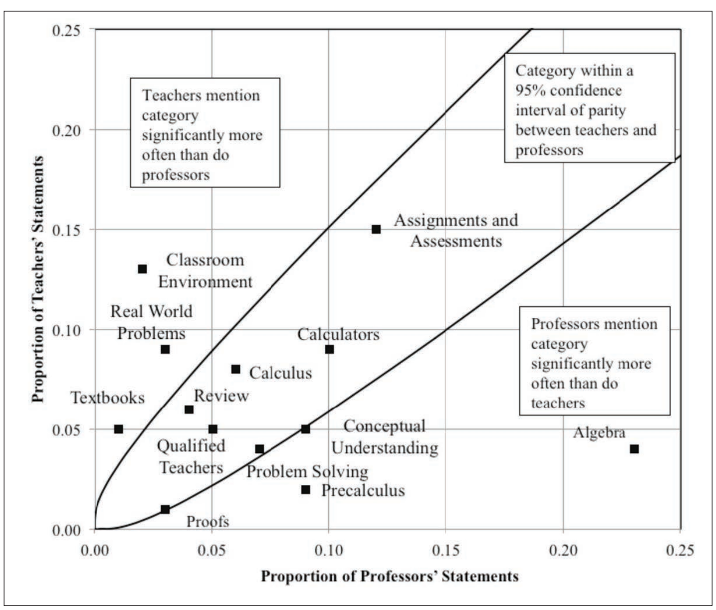

Figure 1. Cone of Similarity created from the proportion of statements made by the teachers and professors.

Reliability

The reliability of the coding was established through inter-rater reliability. The first rater created definitions of the themes generated from the analysis of the categories outside of the Cone of Similarity (see the definitions used in the reliability study in the Appendix). The second rater used these definitions to determine into which theme the statements should be placed. This was done for both teachers and professors. The Cohen's Kappa was 0.897, i.e., the adjusted agreement (accounting for chance agreement) between the two raters was 89.7%. This is considered good agreement between the raters (Landis & Koch, 1977).

Results

The focus of this paper is to better understand teachers' and professors' views of the secondary-tertiary transition by investigating the categories outside of the Cone of Similarity and the themes within these categories. The proportion of statements for each category is provided in the Appendix.

Algebra Category

The Algebra category was created from statements that addressed algebra content and/or pedagogy, either broadly or specifically. Professors made 93 statements that were coded into the category of Algebra; hence, 24% of the statements made by professors addressed algebra. Within this category, professors expressed the view that high school mathematics should focus on algebra and not on calculus. One illustrative statement within this category made by a professor was, "Our students who struggle as they progress through the calculus sequence are generally not struggling with calculus, but with their limited algebra skills. The students with the weakest algebra skills are the students to most likely drop out of the calculus sequence." For teachers, there were 8 statements in the Algebra category; thus, only 4% of the 199 statements addressed algebra. The teachers' statements within the algebra category focused on how they review algebra during calculus instruction. For example, one statement was, "I tell my students that the concepts of calculus are relatively easy but they produce very difficult algebra problems. Therefore, mastery of algebra is essential, especially rational equations and rational exponents."

The theme with the most disparity of statements between the groups was the *Essential Content* theme (professors: 28 statements; teachers: 3 statements), followed by 6 themes from professors only: *Functions* (14 statements), *Teach Algebra not Calculus* (11 statements), *Graphical Conceptual* (9 statements), *Mathematical Literacy* (8 statements), *Procedures* (5 statements) and *Problem Solving* (4 statements). Professors stressed that students need to work fluently with linear and quadratic functions, rational functions, polynomial functions, exponential and logarithmic functions, and trigonometric functions. A deeper focus on functions was considered by professors as better preparation for success in tertiary calculus than being placed in secondary calculus. Professors saw algebra as standing alone, not as content that should be taught and strengthened through the instruction of upper level mathematics. For example, one professor stated, "A low level understanding of calculus benefits them much less than a high level of algebraic ability or understanding when they get to a college course."

Precalculus Category

The Precalculus category was generated from statements that addressed either precalculus content, the importance of learning this content, or pedagogical strategies used to teach precalculus. The proportion of statements made by professors was 9%, whereas only 2.5% of the teachers' statements fell into that category. The professors' themes included *More Time on Precalculus* (12 statements), *Teach Precalculus not Calculus* (9 statements), *Unit Circle and Graphs* (6 statements), and *Identities* (3 statements). Overall, professors opined that teachers needed to spend more time on precalculus content. One professor responded, "Give students a greater depth of knowledge of precalculus topics. Many students recognize topics discussed in class but do not know how to work with them or what they are used for." Providing content specific information, one professor noted, "Students need a deeper understanding of trigonometric and circular functions beyond right triangle trigonometry." Another professor added, "Teach the trig identities including proofs and be sure that the students understand them."

In the Precalculus category there were also three themes brought up by teachers. These were: *Precalculus to Prepare Students for Secondary Calculus* (3 statements), *Limit the Curriculum* (2 statements), and *Students Not Prepared for Precalculus* (2 statements). For example, one teacher stated, "A lot of my choices about what to teach in precalculus are influenced by my experience as a calculus teacher. I focus on students graphing trig equations and understanding what the graphs mean." Another teacher wrote, "Rather than trying to skim over a broad range of topics I choose to teach a few topics well." Finally, a teacher reminisced, "When I started teaching precalculus I assumed that students understood what open circles and solid circles mean on a discontinuous graph. They don't. They also think all graphs are continuous."

Classroom Environment Category

The proportions of statements of teachers and professors in the Classroom Environment Category were 12.6% and 2.6%, respectively. This category was clearly more important to teachers, who mainly addressed pedagogical strategies used to teach precalculus and calculus content. There were 2 statements from teachers and 3 from professors that were grouped together into the *Support for Learning* theme. For example, one teacher responded, "I vary my teaching style to incorporate a lot of group-oriented activities. This allows students to learn from each other and also allows for those less likely to ask questions or talk math to participate in a discussion." Likewise, one professor recommended to "build the confidence level of students pertaining to doing math." For the *Classroom Dynamics* theme there were 14 statements from teachers and 5 statements from professors. One teacher stated, "Students work with others rather than in isolation allowing them to talk about and write about their understandings. They are free to share ideas, help each other with concepts and problems, and share freely with the class." Aligning with this, one professor stated, "Help students learn how to form productive study groups in and out of class to develop attributions about mathematics and what makes one good at it." By contrast, one professor expressed, "Require more rigor and problem solving on their [students'] own without the aid of others."

There were three themes unique to teachers and one unique to professors. The teachers' themes were *Role Play a College Class* (3 statements), *Provide Explanations* (2 statements) and *Various Strategies* (2 statements). For example, a teacher reported, "I do a unit called 'College Role Play' where I role play a professor, moving through material much faster than normal." One teacher stated, "Give lots of examples. I put different problems on the board and ask the students how they approached the problem to solve it." Another wrote, "I provide individualized help, guided practice that bleeds into independent practice as part of the class period, direct instruction with modeling and feedback." The one theme unique to professors was *Individual Accountability* (2 statements). For instance, one wrote, "Teach them [students] that learning mathematics requires work—it isn't enough to 'understand' when the teacher explains a solution. They must work through it themselves."

Real World Problems Category

The proportion of statements in the Real World Problems category was 10% for teachers and only 2.6% for professors. The statements in this category were merged because they addressed either real world problems or strategies used to teach contextual problems, such as modeling or discovery. The *Connect to Other Courses* theme (teachers: 4 statements; professors: 2 statements) stressed the importance of connecting mathematics to other courses, such as physics. The themes unique to teachers were *Hands-on-Learning* (10 statements) and *Use Models* (2 statements). Teachers reported that they provided tactile learning experiences during instruction of real world problems. For example, one stated, "I use a lot of hands-on, manipulatives, and visual activities to help students internalize concepts." Another wrote, "I bring in models that can help the students visualize concepts." *Discovery* was a theme unique to professors (2 statements). Expressing a view different from most, one professor noted, "The technique of 'discovery' is useful but it uses a lot of class time, which can be more efficiently used with teacher directed development of concepts."

Textbooks Category

Of the teacher statements, 4.5% fell into the Textbooks Category; of the professor statements, only 0.8%. This category was among the smallest for both professors and teachers, as seen in Figure 1. Nonetheless, the Textbook category was analyzed because of the statistically significant difference in the proportion of statements. In the *Teach How to Read a Textbook* theme (teachers: 4 statements; professors: 3 statements), for example, one teacher said, "Students need to be able to read their book and self-teach." There was one theme unique to teachers, *Multiple Resources* (5 statements), where teachers expressed their concern that textbooks have become less rigorous over time. One teacher stated, "I find most Precalc textbooks don't include enough algebra," and another stated, "We have kept our expectations high and have not let the rigor leave our classes as our texts have become more watered down over the years. We continually supplement our classes with meaningful, challenging materials."

Discussion and Conclusion

Student learning across the rite of passage stages is complex (Clark & Lovric, 2008; van Merrienboer, Kester, & Paas, 2006). Clark and Lovric (2009) referred to the passage as being disconnected because of the "far from satisfactory" communication between secondary teachers and university mathematics professors (p. 762). We believe the investigation of the categories outside of the Cone of Similarity can provide useful information across the stages of the secondary-tertiary transition.

The main result of our analysis was that the professors' and teachers' views divided along a content-pedagogy cleavage. Whereas the teachers had a stronger focus on the teaching process, the paramount concern for the professors was what students knew (or did not know) when they arrived in tertiary calculus. The professors voiced that students must have foundational content knowledge and preferred that algebra and precalculus be the focus of instruction, instead of secondary calculus. The content (Algebra and Precalculus) vs. pedagogy (Classroom Environment, Real World Problems, and Textbooks) cleavage aligns with high school teachers working in a setting in which pedagogy is of vital concern, while the professors' setting is more content knowledge oriented. The difference between the focus on pedagogy and that on content across the secondary-tertiary transition has been long recognized (Graeber & Tirosh, 2008; Klein, 2003; Shulman, 1986). According to our theoretical framework, defining such differences is at the very heart of how professions demarcate their turf. In addition, research has indicated that college mathematics professors, on average, possess relatively low levels of pedagogical understanding about how to make complex mathematics comprehensible to others (Bass, 1997). Likewise, it has been shown that a deep understanding of mathematical concepts may enable teachers to access a broad repertoire of pedagogical strategies that can explain and represent mathematical content to students (Krauss et al., 2008). A more precise distinction between professors' and teachers' outlooks may reside in what Hill, Ball, and Schilling (2008) referred to as knowledge of content and students. According to these authors, knowledge of content and students is a construct that defines how content knowledge is "intertwined with knowledge of how students think about, know, or learn particular content" (p. 375). Together, content and pedagogy constructs contribute to an explanation of the teachers' higher, and the professors' lower, concern with pedagogy used to teach high school mathematics.

In conclusion, this research probed the opinions of both teachers and professors about the secondary to tertiary transition in mathematics education. The research question was first investigated by identifying the differences in the frequency of statements between the teacher and professor groups. A thematic analysis of the categories was then presented only for those with significantly different frequencies for high school teachers and professors (Figure 1). The Cone of Similarity was a visual tool that clearly distinguished the content focus of professors, on the one hand, and the pedagogical focus of teachers, on the other. While there was also some disparity revealed by the thematic analysis, we should emphasize that, on the whole, teachers' and professors' views were quite similar. There was agreement among professors and teachers that: Students need to be supported in the learning of algebra and precalculus; pedagogy that incorporates group work can be beneficial, but individual student accountability for content knowledge is important; placing mathematics in context can support learning; and students need to know how to read a textbook. The main area of disparity was that professors opined that there should be more focus on algebra and precalculus without teaching secondary calculus, while teachers thought that calculus provided a means of reviewing algebra and precalculus concepts.

More research is needed to find out which strategies and interventions described actually predict success in tertiary calculus. Teachers and professors must work within their coupled domains, and it is only reasonable that they have a deep commitment and concern for the quality of students' mathematics education. In this article, we focused on the categories outside of the Cone of Similarity rather than inside the Cone because, from a practical perspective, it is more important to learn about the differences between the two groups. The problem areas in the passage can be addressed only when this gap between the teachers' and the professors' outlooks is better understood, and when teachers and professors realize more clearly that they are part of one and the same rite of passage process.

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References

Abbott, A. (1988). *The system of professions: An essay on the division of expert labor.* Chicago: University of Chicago Press.

Baruch, Y., & Holtom, B. C. (2008). Survey response rate levels and trends in organizational research. *Human Relations, 61*(8), 1139–1160.

Bass, H. (1997). Mathematicians as educators. *Notices of the American Mathematical Society, 44*(1), 18–21.

Bouma, G. D., & Atkinson, G. B. J. (1995). *A handbook of social science research.* Oxford: Oxford University Press.

Bressoud, D. M. (2010). The changing face of calculus: First-semester calculus as a high school course. In C.L. Diefenderfer & R.B. Nelson (Eds.), *The calculus collection: A resource for AP and beyond,* 54. Washington, DC: Mathematical Association of America.

Clark, M., and Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. Mathematics *Education Research Journal, 20*(2), 25–37.

Clark, M., and Lovric, M. (2009). Understanding secondary-tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology, 40*(6), 755–776. DOI: 10.1080/00207390902912878.

Graeber, A., & Tirosh, D. (2008). Pedagogical content knowledge: Useful concept or elusive notion. In P. Sullivan & T. Wood (Eds.), *The International Handbook of Mathematics Teacher Education: Volume 1 (Knowledge and Beliefs in Mathematics Teaching and Teaching Development)* (117–132). Rotterdam, The Netherlands: Sense Publishers.

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics, 67*(3), 237–254.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Hong, Y. Y., Kerr, S., Klymchuk, S., McHardy, J., Murphy, P., Spencer, S., Thomas, M.O.J., & Watson, P. (2009). A comparison of teacher and lecturer perspectives on the transition from secondary to tertiary mathematics education. *International Journal of Mathematical Education in Science and Technology, 40*(7), 877–889. doi:10.1080/00207390903223754

Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make successful transition to tertiary mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology, 38*(4), 461–476.

Hsu, E. & Bressoud, D. (2015) Placement and student performance in Calculus I. In D. Bressoud, V. Mesa, & C. Rasmussen (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus. MAA Notes.* (59–67). Washington, DC: Mathematical Association of America.

Klein, D. (2003). A brief history of American K-12 mathematics education in the 20th century. In J.M. Royer (Ed.), *Mathematical Cognition* (175–224). Greenwich, CT: Information Age Publishing Inc.

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., & Jordan, A. (2008). Pedagogical content knowledge and content knowledge of secondary mathematics teachers. *Journal of Educational Psychology, 100*(3), 716–725.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics, 33*(1), 159–174.

Linnenbrink, E. A., & Pintrich, P. R. (2002). Motivation as an enabler for academic success. *School Psychology Review, 31*(3), 313–327.

Loveless, T. (2008). *The misplaced math student: Lost in eighth-grade algebra.* Washington, DC: Brookings Institution, Brown Center on Education Policy.

National Center for Education Statistics. (2011). Fast facts. Retrieved from https://nces.ed.gov/fastfacts/display.asp?id=51

Rigol, G.W., & Ziemnicki, C. (2011). *Email trends in the education market 2011: A comprehensive analysis of the 2009–2010 school year.* Shelton, CT: Market Data Retrieval.

Sadler, P. M., & Tai, R. H. (2007). Advanced placement exam scores as a predictor of performance in introductory college biology, chemistry and physics courses. *Science Educator, 16*(2), 1–19.

Saldana, J. (2013). *The coding manual for qualitative researchers* (2nd ed.). Los Angeles, CA: Sage.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 193*(3), 4–14.

Selden, A. (2005). New developments and trends in tertiary mathematics education: or, more of the same? *International Journal of Mathematical Education in Science and Technology, 36*(2), 131-147.

Sonnert, G., Sadler, P. M., Sadler, S. M., & Bressoud, D. M. (2015). The impact of instructor pedagogy on college calculus students' attitude toward mathematics. *International Journal of Mathematical Education in Science and Technology, 46*(3), 370-387.

Sonnert, G. & Sadler, P. M., (2015) The impact of instructor and institutional factors on students' attitudes. In D. Bressoud, V. Mesa, & C. Rasmussen (Eds.), *Insights and Recommendations from the MAA National Study of College Calculus. MAA Notes.* (17-29). Washington, DC: Mathematical Association of America.

Tall, D., & Ramos, J. (2004). *Reflecting on post-calculus reform* [PDF document]. Retrieved from David Tall's website at the University of Warwick: http://homepages.warwick.ac.uk/staff/David.Tall/pdfs/dot2004b-tall-meija-icme.pdf

van Gennep, A. (2011). *The rites of passage* (reprint ed.). Chicago: University of Chicago Press.

van Merriënboer, J.G., Kester, L., & Paas, F. (2006). Teaching complex rather than simple tasks: Balancing intrinsic and germane load to enhance transfer of learning. *Applied Cognitive Psychology, 20*(3), 343-352.

Wright, K.B. (2005). Researching internet-based populations: Advantages and disadvantages of online survey research, online questionnaire authoring software packages, and web survey services. *Journal of Computer-Mediated Communication, 10*(3).

Zinth, J. D. (2012, March). *50-State mathematics requirements for the standard high school diploma* [PDF document]. Retrieved February 14, 2016 from http://www.ecs.org/clearinghouse/01/01/28/10128.pdf

Appendix

Definitions of Categories with Counts of Statements in Each Theme Made by Professors and Teachers

Theme	Number of Professors' Statements Within That Theme	Number of Teachers' Statements Within That Theme	Theme Description
Algebra Category Professors: 24.0% total statements Teachers: 4.0% of total statements			
Essential Content	28	3	Specific algebra content and concepts that should be taught in high school algebra.
Functions	14	0	Students need a better and broader understanding of functions.
Teach Algebra not Calculus	11	0	Students should receive more algebra instruction in high school instead of being placed in a high school calculus course.
Graphical Conceptual	9	0	Provide graphic representation of algebraic relationships to help students understand and apply algebra concepts.
Mathematical Literacy	8	0	The language and symbolic manipulation of algebra.
Procedures	5	0	The use of procedures and rules in solving algebra problems.
Algebra Course	4	3	Specifics mentioned about algebra within the context of the secondary algebra course or class.
Problem Solving	4	0	Students should be taught to stick with hard problems, not to give up when problem solving.
Calculator	3	2	Students should be able to solve algebra problems without a calculator yet they may be beneficial for demonstrating concepts.

Continued

Definitions of Categories with Counts of Statements in Each Theme Made by Professors and Teachers (Cont.)

Theme	Number of Professors' Statements Within That Theme	Number of Teachers' Statements Within That Theme	Theme Description
Precalculus Category Professors: 9.0% of total statements Teachers: 2.5% of total statements			
More Time on Precalculus	12	0	Students need a greater depth of knowledge of precalculus topics.
Teach Precalculus not Calculus	9	0	More focus on precalculus content instead of teaching calculus in high school.
Unit Circle and Graphs	6	0	Students need a deeper understanding of unit circle and trig graphs.
Identities	3	0	Teachers need to spend more time on trig identities.
Precalculus to Prepare for Secondary Calculus	0	3	Teach precalculous to prepare students for high school calculus
Limit the Curriculum	0	2	Limit the curriculum to focus on specific content in a deeper way.
Students not Prepared for Precalculus	0	2	Students do not have foundational knowledge needed for for learning Precalc.
Classroom Environment Category Professors: 2.6% of total statements Teachers: 12.6% of total statements			
Classroom Dynamics	5	14	Views of how students should (or should not) interact in the classroom while learning mathematics.
Support for Learning	3	2	Students should be supported in the learning of mathematics
Individual Accountability	2	0	Students need to take personal responsibility of their learning.
Role Play a College Class	0	3	Provide examples of what it will be like to learn mathematics at the college level.
Provide Explanations	0	3	Provide examples and explanations when teaching mathematics.
Various Strategies	0	3	Provide various type strategies when teaching mathematics.
Real World Problems Category Professors: 2.6% of total statements Teachers: 10.0% of total statements			
Place in Context	6	4	Place mathematics in context for learning
Connect to Other Courses	2	4	Mathematics should be connected to other classes
Discovery	2	0	Limit discovery of teaching to focus on covering content
Hands on Learning	0	10	Use hands-on projects and activities when teaching mathematics.
Use Models	0	2	Use model in class when teaching mathematics.
Textbook Category Professors: 0.8% of total statements Teachers: 4.5% of total statements			
Teach How to Read a Textbook	3	4	Students need to learn how to read a mathematics textbook
Multiple Resources	0	5	Use various resources to supplement the textbook.