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*A Century of Leadership in Mathematics and Its Teaching*

**Reconsidering Elements of Research and Practice**

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# PROF. BRUCE RAMON VOGELI

JMETC Readers,

This issue of the journal begins on a somber note, as we reflect on the passing of Prof. Bruce Ramon Vogeli earlier this year. Given his immense influence on the Program in Mathematics at Teachers College, Columbia University, and on the revival of the Program's journal (JMETC), the editorial board decided it fitting to write a tribute in his memory.



Since the Teachers College community learned of Prof. Vogeli's passing in May 2020, there have been myriad tributes to him highlighting his contributions to the field of mathematics education and his half-century of service to Teachers College. In addition to these accolades, the editorial board felt it important to highlight his enormous dedication to, and impact on, students in the Program in Mathematics.

Prof. Vogeli was tirelessly devoted to the Program and its students—inevitably, he was the first to arrive, and the last to leave the office. He expended endless energy devising creative ways to ensure the program and all of its projects ran smoothly and effectively; he always had a new idea or scheme to attain the absolute best from students. And he was never too busy for even the most trivial of questions. Even a 30-second question might turn into a 30-minute conversation. He cared deeply for students' wellbeing, and relied on his decades of experience in the field to find creative solutions to difficult problems in students' academic, and even personal, lives. He incessantly encouraged students to achieve their very best and, in this, he demonstrated full confidence in their abilities—even for tasks students themselves thought impossible. His vocal support inevitably resulted in an assurance and trust in their own capabilities along the way.

Prof. Vogeli dedicated an immense amount of time to creating opportunities and experiences for students—ones that might advance their, not necessarily his, professional careers. He had a particularly special ability to incorporate students into his own projects, and to tap into individual students' interests and help them turn these into fruitful ideas on which to base papers and curricular materials. Consistently, he created projects that involved students in his own work, often giving the students ownership of projects he helped envision. Several of his edited volumes feature chapters that he invited his students to write. He also devised and led the effort to create handbooks of curricular materials, handing off the writing and editing to students and alumni, maintaining only an advisory role. Indeed, this journal itself, the Journal of Mathematics Education at Teachers College, was revitalized by Prof. Vogeli from an earlier departmental publication, specifically with the goal of providing students and alumni from the program more opportunities to research, write, and publish their work. He fostered leadership skills through this journal, by appointing students to serve as guest editors and allowing them to oversee the entire peer-review publication process. It is not hyperbole to say that this journal, from its inception, was centered around advancing the professional capacities, opportunities, and experiences for students in the Program. Prof. Vogeli's effort and dedication to students in these regards deserves equal praise as to that of his other works in the field; his legacy will continue to impact the field of mathematics education and its future generation of professionals and scholars for many years to come.

The Program in Mathematics, and its community, all owe a great debt of gratitude to Prof. Vogeli for the enormous opportunities he created and the scholars he nurtured.

JMETC Editorial Board

*Nick Wasserman (Chair), J. Philip Smith, Nicole Fletcher, and Hudson Gould*

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## On Professor Bruce Ramon Vogeli

*In addition to this short tribute from the editorial board, we also include a survey of Prof. Vogeli's professional life and career, written by his Program colleagues, Profs. Alexander Karp and Erica Walker. This piece, entitled "On Professor Bruce Ramon Vogeli" (pp. 1-5), opened a published volume that honored Prof. Vogeli.\* The following is a reprint from that volume. (It is important to note that since the publication of that volume, three more books edited by Prof. Vogeli were published: *Special Secondary Schools for the Mathematically Talented*, *Mathematics and its teaching in the Asia-Pacific Region (with John Mack)*, and *Mathematics and its teaching in the Muslim World (with Mohammed El Tom)*.)*

**Professor Bruce Ramon Vogeli** has been working at Teachers College, Columbia University since 1964. Over the last half-century, he has taught dozens of courses, written dozens of papers and books, and graduated not hundreds, but thousands of students. Indeed, just the number of doctoral students who defended their dissertations with him as advisor, has long passed one hundred.

Vogeli belongs to the generation born right after those who fought in the Second World War, but the experience of the war may nonetheless have been the most important factor in his development—direct opposition to evil, demanding courage, hard work, self-sacrifice, and education, was crucial for his formation as a human being. Moreover, the peace that came in 1945 was fragile, coming under threat frequently and everywhere in the world, from Berlin to Korea. It is therefore not surprising that Bruce Vogeli began his adult life by serving in the army, with which he was involved—counting his active service and reserve service—for nine years altogether, starting in 1948. Among other things, he served as a technical analyst, obtained his Bachelor's and Master's Degrees, and worked as a schoolteacher of mathematics during this time. In 1957, he enrolled in the graduate program at the University of Michigan (Ann Arbor). Here, his knowledge of the Russian language, which he had learned while in the army, proved useful: his dissertation, which he defended three years later, was devoted to Soviet mathematics education.

The young Doctor of Philosophy began his academic career as an assistant professor at Bowling Green State University, Ohio, becoming an associate professor in a few years. But now, the Russian language once again turned out to be useful. After Khrushchev's visit to the United States in 1959, a decision was made to organize a professor exchange—a Soviet scholar would come and teach a course in the United States, and an American professor would do the same in the Soviet Union. Bruce Vogeli became this American professor, traveling to Moscow in 1963.

It was exactly this period that witnessed the appearance of Soviet schools with an advanced course of study in mathematics, one of the most remarkable phenomena in the global practice of mathematics education. Vogeli observed their appearance and was the first to write about them for educators living outside the Soviet Union. His book, *Soviet Schools for the Mathematically Talented*, became an important milestone in international mathematics gifted education. It inaugurated a series of publications by Vogeli which addressed Russia, advanced study of mathematics, comparative education, and even more broadly, the development of curricula.

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\* Karp, A., & Walker, E. N. (Eds.) (2015). *In honor of professor Bruce Ramon Vogeli: Scholarship and leadership in mathematics education*. Bedford, MA: Consortium for Mathematics and its Applications (COMAP).

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**On Professor Bruce Ramon Vogeli, *continued***

These topics have continued to occupy Vogeli for much of his life, and perhaps no less importantly, many of his students started working on them as well. Shortly after returning to the United States, Vogeli became a professor at Teachers College, Columbia University, and naturally, his doctoral students became interested in subjects that interested their teacher—one can name dissertations written under his direction on mathematics schools in Hungary and Russia (Identification and Development of the Mathematically Talented: The Hungarian Experience; Schools for the mathematically talented in the former Soviet Union), or dissertations on education in Latin America (Relationships between mathematical education and economic production in six Latin American countries from 1960 to 1970), or dissertations on mathematics education in Africa (Computers in Africa: Survey of availability of computers, trained manpower, and computer education), or dissertations on how to develop and evaluate a curriculum (Transformational geometry in the junior high school: An evaluation of curricular units in 7th Grade; Game theory for the secondary mathematics curriculum)—the list could go on and on. One can say that a school of comparative studies in mathematics education formed at Teachers College, Columbia University—the “Vogeli School”. And in this respect, Vogeli became a worthy successor in the tradition initiated by David Eugene Smith, who founded the Program in Mathematics at Teachers College and established the field of comparative studies in mathematics education.

Vogeli also continued the other work of David Eugene Smith and subsequent generations of Teachers College professors. The writing of textbooks is sometimes considered a less meaningful pursuit for a scholar than the writing of research papers. It should not be forgotten, however, that it is precisely textbooks that are read by millions of students and used by thousands of teachers. Without minimizing the role or significance of research papers, we would argue that the experience of research studies and research findings, multiplied by pedagogical experience, and embodied in the form of a generally accessible school textbook, exerts an influence on the surrounding world faster, and often also more effectively than even the most striking scholarly articles, which are read only by a very few. Bruce Vogeli has authored and coauthored many dozens of textbooks—for elementary, middle, and high schools—which have been used by literally millions of children (and not just in the United States, since these textbooks have been translated into other languages.)

Lastly, and understandably, in over fifty years of working at Teachers College, Columbia University, Vogeli has done a great deal both for Teachers College, and for many other universities. Consultations, meetings, guest lectures, Fulbright professorships, and so on, and so forth—he has been invited all over the world to help with the development of mathematics education.

Still, the most important part of his work, of course, has been at Teachers College itself. Here, he has been a member of numerous committees and work groups, and he has chaired departments. But his most important contribution has been as head of the Program in Mathematics. Having at one time become the top place in the country for doctoral degrees in mathematics education, the Program has retained its leading position in the field—more doctoral dissertations are defended here than anywhere else.

There is yet another fact that must be noted. Over the last few decades, people's understanding of what is important for a specialist in mathematics education has changed time and time again. At certain stages,

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**On Professor Bruce Ramon Vogeli, *continued***

the importance of a deep knowledge of mathematics was effectively rejected: people simplistically argued that in the future graduates would unlikely need to teach any particularly recondite subjects, and hence had no need to understand them themselves. The fact that a teacher must nonetheless have a sound grasp of a subject in order to teach even its relatively basic sections somehow eluded people's understanding. The result is that now, when no one seems to doubt the importance of content knowledge, including pedagogical content knowledge, many institutions have neither the structure for instruction in the corresponding areas, nor people who are capable of carrying out such instruction, combining mathematical and pedagogical preparedness. The Program in Mathematics at Teachers College, Columbia University has preserved its character, and the credit for this enormous achievement belongs to Bruce Vogeli, who was able both to preserve and to expand important courses, and to find, invite, and educate collaborators.

The fact that Vogeli has had such a long career does not mean that there has been any diminution in his activity. In the last few years alone, he has published a number of books including *Russian Mathematics Education: Programs and Practices* (coedited with Alexander P. Karp) and *Mathematics and its Teaching in the Southern Americas* (coedited with Hector Rosario and Patrick Scott). These books both continue the research that Vogeli has carried out earlier, and mark out new paths for study. Bruce Vogeli continues to conduct seminars and teach courses for students, as he has done for decades, and to serve as their dissertation advisor.