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Conceptualizing Student Responsibilities in Discourse-Rich Classrooms

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ABSTRACT While a growing body of research examines teachers' facilitation of discourse-rich classrooms, surprisingly little research is devoted to learners' responsibilities in such classrooms. In this paper, we share a theoretical model for explaining students' responsibilities in yielding mathematical learning in discourse-rich classrooms. These responsibilities consist of the following: (1) determined listening and striving to understand others' contributions; (2) proactive contribution; (3) maintaining equal positioning; (4) willingness to resolve incommensurability; and (5) on-task talk. Each of the responsibilities is interdependent, suggesting that failure to meet one responsibility decreases the likelihood that another responsibility will be met. The model suggests important implications for supporting learners in discourse-rich classrooms.

KEYWORDS *discourse, mathematics, collaboration, student responsibilities, collaborative learning*

Discourse-rich, student-centered classrooms dominate current trends in mathematics education policy, research, and practice. Advocates of discourse-rich classrooms suggest that students work with classmates to solve cognitively demanding tasks, while teachers act as facilitators, guiding students to co-construct knowledge with their peers (e.g., Jackson et al., 2012; Stein et al., 2008; Van de Walle et al., 2016). Curriculum and policy documents in the U.S. (e.g., Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000, 2014); Department of Education, 2014) consistently recognize the influence of discourse-rich classrooms and suggest that "mathematical discourse among students is central to meaningful learning of mathematics" (NCTM, 2014, p. 35). Further, empirical research suggests that discourse-rich classrooms increase opportunities for authentic engagement and equitable mathematical participation (Brown, 2007; Esmonde & Langer-Osuna, 2013; Jarosz et al., 2017; Summers, 2006). Still, there is much to learn regarding how discourse-rich classrooms should operate.

A growing body of research has examined classroom norms (Partanen & Kaasila, 2015; Yackel & Cobb, 1996), teaching practices (Herbel-Eisenmann et al., 2013; O'Connor & Michaels, 2019), and the role of tasks (Henningsen & Stein, 1997; Jackson et al., 2012) in discourse-rich classrooms. However, there is surprisingly little research devoted to learners' responsibilities in such classrooms. Several questions persist regarding student accountability, such as the following: How should students communicate with one another to ensure an optimal environment for learning? What communicative behaviors enhance equal participation structures in collaborative environments? How should students resolve conflict when talking about mathematics? Answering these and other related questions requires a student-level examination of discourse-rich classrooms. To provide theoretical insight into student-level factors of collaborative classrooms, it is constructive to conceptualize first what students should be doing.

In this paper, we aim to share a theoretical model explaining students' responsibilities for yielding mathematical learning in discourse-rich classrooms. The model was refined through an iterative process of generating cross-cutting themes from literature and our own experiences researching and facilitating discourse-rich classrooms in middle-grade and university settings. Taken together, we report a model that is informed by research and corroborated by experience. We make no claims regarding the exhaustive nature of our model. Rather, we identify several student-level factors that, based on our research and experience, explain how students learn in collaborative environments. In the next section, we briefly describe our conceptualization of discourse-rich classrooms through a discursive perspective on learning.

Discourse-Rich Classrooms

While discourse-rich classrooms vary in structure, they exhibit two defining characteristics: (1) students actively participate and communicate with others, and (2) learning is presumed to occur through mutual communication. First, students in discourse-rich classrooms actively participate by communicating with their peers and teachers. Depending on the classroom structure, this may occur in a variety of ways. Stein and colleagues (2008) suggested that discourse-rich classrooms proceed in three phases: launching a mathematical task, exploring a problem in small groups, and discussing and summarizing the problem through whole-class dialogue. They summarized a typical discourse-rich classroom as follows:

> During this 'launch phase,' the teacher introduces the students to the problem, the tools that are available for working on it, and the nature of the products they will be expected to produce. This is followed by the 'explore phase' in which students work on the problem, often discussing it in pairs or small groups. As students work on the problem, they are encouraged to solve the problem in whatever way makes sense to them and be prepared to explain their approach to others in the class. The lesson then concludes with a whole-class discussion and summary of various student-generated approaches to solving the problem. (Stein et al., 2008, p. 316)

While not all discourse-rich classrooms proceed in a similar manner as envisioned by Stein and colleagues, students in discourse-rich classrooms actively communicate about mathematics rather than passively listening to the teacher.

Another defining characteristic and by-product of the first characteristic is that learning is presumed to occur through mutual communication. Proponents of discourse-rich classrooms assume that students learn collaboratively by contributing mathematical ideas and listening to others' ideas (Scardamalia & Bereiter, 2006). This is in stark contrast to teacher-centered classrooms, wherein learning is presumed to occur through passive participation.

Mathematical Learning as Changing Discourse

In this paper, we assume that the goal of discourse-rich classrooms is to produce mathematical learning. Aligning with Sfard (2008), we define learning as a lasting change in discourse. Accordingly, communication in discourse-rich classrooms may be considered *productive* if it leads to changes in students' discourse that are *durable* and *desirable* (Sfard & Kieran, 2001). Changes in student discourse are durable if they are likely to continue in future communication, while changes in student discourse are *desirable* if they align with accepted discourse practices of the broader discourse community. Mathematical discourse is distinguishable according to four features: word use (e.g., keywords related to numbers and shapes), visual mediators (e.g., operators, coordinate plane), narratives (e.g., theorems and definitions), and routines (e.g., repetitive patterns) (Sfard, 2008). Therefore, we will consider discourse-rich mathematics classrooms to be *productive* if they often lead students to exhibit a lasting change in the way they communicate about keywords in mathematics, visual mediators, narratives, and routines.

To illustrate *productive* communication, consider a scenario wherein two learners, Aaron and James, discuss the area of a square. Aaron conjectures that the area of a square is always larger than the side length. James contradicts this assertion and shares a counterexample (e.g., side length = 0.4). After listening to James' counterexample, Aaron agrees and suggests that if the side length is between zero and one, the area is smaller than the side length. In this exchange, Aaron changed his discourse in a *desirable* way. If this change in discourse persists in future communication, we may deem Aaron and James' communication as *productive*.

For our model, we perceive Sfard and colleagues' (Sfard, 2008; Sfard & Kieran, 2001) conceptualization of mathematical learning and *productive* communication as a primary goal for discourse-rich classrooms. Therefore, our model is designed to describe student responsibilities for engaging in such communication. In the following section, we describe five student responsibilities that we find integral to promote learning in discourse-rich classrooms. Then, we explain how these five responsibilities interact to form a model of student responsibilities.

Student Responsibilities for Discourse-Rich Classrooms

Based on prior research and our experiences as educators, we suggest five responsibilities for which students are accountable in discourse-rich classrooms to promote mathematical learning: (1) determined listening and striving to understand others' contributions, (2) proactive contribution, (3) maintaining equal positioning, (4) willingness to resolve incommensurability, and (5) on-task talk. We discuss each of these responsibilities by reviewing relevant literature and examining episodes from our research (Campbell & Hodges, 2020; Campbell & King, 2020; Campbell et al., 2020) and teaching experiences.

Determined listening and striving to understand others' contributions

To communicate in ways that lead to learning, students must be determined to listen to peers and actively strive to understand their peers' contributions. Scholars refer to this type of engagement as aligning frames (van de Sande & Greeno, 2012), discussing proposals (Barron, 2003), and communicating effectively (Ryve et al., 2013; Sfard & Kieran, 2001). van de Sande and Greeno (2012) suggested that students working in groups align their frames by either mutually drawing on common knowledge or actively listening to other participants with relevant knowledge to complete the task. Complementing van de Sande and Greeno's (2012) study, Barron (2003) and Sfard and Kieran (2001) found students must become active rather than passive while other group members are talking to generate learning opportunities. In short, *determined listening and striving to understand others' contributions* refers to listening actively and seeking to understand others' mathematical strategies by asking clarifying questions, building off others' contributions, and using other communicative behaviors that reveal a motivation to understand.

Though active listening may seem straightforward, it requires intense determination and often does not come naturally to learners. In our research with middle school students (Campbell & King, 2020; Campbell et al., 2020), we noticed the rarity of active listening amongst students working in groups. For instance, consider the following transcript between Josh and Amber as they attempted to create an argument for the claim "the sum of two odd numbers is even." (utterances 1 – 3).

1. *Amber:* In every single-digit number, that is odd if you know they will be even added together, then adding an odd to a two-digit number that is odd, then the answer will be even like the single-digit number was.

2. *Josh:* Alright, Amber. So... Alright, so. Um, if you go back down to the basics, seven plus five um, is twelve. Yeah, it is. OK. And seven plus three is ten. And all the basic, tiny numbers—the one-digit numbers. They all equal evens, so that means, uh, because it just depends on the last number in the number, uh, to make it an even. So, since all of the one-digit numbers are even, it just comes down to the one-digit numbers in the big number. You guys get what I'm saying? Do you want to write something down?

3 *Amber:* No, you can write something down, but I don't get what you're saying. Write in good handwriting, please.

Interestingly, in this exchange, Amber and Josh shared nearly identical arguments (utterances 1 and 2). Josh did not try to connect any of Amber's ideas and, instead, presented his argument as a new strategy. Neither student asked clarifying questions nor attempted to engage with the other's idea, resulting in ineffective communication. As a result, Amber implored Josh to simply write his answer on the task sheet ("I don't get what you're saying. Write in good handwriting, please"). While not all instances of passive listening are so obvious, it seems that learners often do not actively listen to understand one another's contributions. Determined listening requires listeners to ask clarifying questions or build upon others' responses. For instance, consider the transcript below extracted from a previous research project (Campbell & Hodges, 2020) between three college-aged learners discussing the meaning of the index of terms in a set (utterances 4 – 10).

4. *Katrina:* Yeah. OK, *j* is the set of—and *m*, or *j*—and *m* is—would be like the index of that set.

5. *Danielle:* Is the index of *j*.

6. *Katrina:* Index of *j*, OK.

7. *Danielle:* Now, let's make sure we can all understand this before we write it down.

8. *Hayden:* I don't understand index.

9. *Danielle:* It's the little numbers.

10. *Katrina:* It like defines, yeah. So, it defines the location in the set. So, like if you said *j* sub *m* and you wanted to find *j* sub 4, it would be 7 [showing example on paper].

Danielle displayed active listening by clarifying Katrina's initial proposal ("Is the index of j."). Hayden suggested that she did not understand what Katrina meant by the word index ("I don't understand index."), resulting in the group further explaining their use of the term (utterances 9-10). Danielle's suggestion, "Now, let's make sure we can all understand this before we write it down," clearly portrays a propensity for engaging with others' thoughts. While Katrina's final explanation was not mathematically complete (utterance 10), the group came to a collective understanding of how they would use indexes to define a number in a set. In this exchange, all group members were determined to listen and respond to one another. As a result, they generated opportunities to change their working mathematical definition of and discourse related to the term *index*. Productive communication is at least partially dependent on learners' abilities to actively listen for understanding while others are talking.

Proactive contribution

The second student responsibility for discourse-rich classrooms is *proactive contribution*, which refers to learners' willingness to offer their mathematical insight while collaborating with others. Authentic participation is central to learning mathematics (Cuoco et al., 1996; Lave & Wenger, 1991), so students must actively participate by writing and sharing their problem-solving strategies. Additionally, when students proactively contribute, they allow for a diverse range of ideas to be heard, increasing learning opportunities. Barron (2000) suggested mutuality, or the potential for all group members to contribute, is essential for effective group problem-solving. In our research with college-aged learners, we found a positive relationship between mutuality and *productive* group engagement (Campbell & Hodges, 2020).

In comparison with teacher-centered classrooms, it is simple to recognize why *proactive contribution* is necessary for productive discourse-rich classrooms. In teacher-centered classrooms, a minority expert (i.e., teacher) offers most of the mathematical discourse. Similarly, in discourse-rich classrooms without *proactive contribution*, a minority of students offer most of the mathematical insight. In both situations, the majority of students are passive observers. If one is forced to participate in such a way, it seems advantageous to observe the most experienced and knowledgeable contributor. Since the teacher is the most experienced and knowledgeable contributor in most classroom settings, teacher-centered classrooms seem a preferred environment for learning compared with discourse-rich classrooms with low student participation.

Maintaining equal positioning

Maintaining equal positioning is another student responsibility that is important for promoting equitable mathematical participation structures. The literature suggests that students position or label (van Langenhove & Harré, 1999) themselves and others while working collaboratively in ways that increase or inhibit opportunities for authentic mathematical participation (Barnes, 2004; Bishop, 2012; Campbell & Hodges, 2020; Wood, 2013; Wood & Kalinec, 2012). Often, this act of positioning is tacit, while other times, learners purposely create hierarchies while working with others (van Langenhove & Harré, 1999). Barnes (2004) found evidence for fourteen different positions assumed by eleventh-grade learners in Introductory Calculus classrooms. Learners assumed positions of expert, audience, manager, helper, and so on. Some of the positions, such as the position of expert, allowed students to participate in mathematics authentically. In contrast, other positions, such as helper, relegated students to perform menial tasks without engaging in meaningful mathematics. Similarly, Bishop's (2012) analysis revealed the influence of positioning on mathematical participation. In their analysis, two seventh-grade girls constructed a hierarchy amongst themselves, resulting in one girl being labeled as smart and the other girl labeling herself as a less competent doer of mathematics. These positions influenced the roles each girl assumed while problem-solving. For instance, the girl positioned as a competent doer of mathematics often controlled problem-solving activities. Positions, whether tacitly or purposely assigned, often result in hierarchical classroom structures, privileging meaningful mathematical access for some and denying access for others.

In our work with college-aged learners, we found that students working in groups tend to assume positions that fall on a spectrum, with passive observers on one end of the spectrum, dominant controller on the other end, and balanced negotiator in the middle (Campbell & Hodges, 2020). Passive observers mostly listen to group communication, while dominant controllers dominate group discussion. Balanced negotiators both offer their contributions and actively seek to negotiate with others. Students are often forced into passive observer positions by other group members who dominate group discussions. Conversely, students are sometimes forced into dominant controller positions if other group members refrain from contributing to group conversations. In short, each group member's positioning influences mathematical participation. For collaborative engagement to be *productive*, learners should seek to position one another as equals. For instance, students should take turns assuming positions of *expert* (Barnes, 2004) or other positions that provide access to participation. Additionally, learners should actively seek to monitor their positions and realize when they become dominant or passive. By maintaining equal positioning and reflecting on their positions, students create equitable participation structures in discourse-rich classrooms

Willingness to resolve incommensurability

Incommensurability, or conflicting discursive rules related to a similar topic, often occurs between learners working in a collaborative environment (Sfard, 2019). For instance, in the hypothetical scenario presented in a previous section ("Mathematical Learning as Changing Discourse"), Aaron and James suggested two mathematical strategies that contradict one another. When two or more participants exhibit incommensurable discourses related to a similar topic, they may resolve the conflict through discussion, argue without a resolution, or avoid conflict altogether. Avoiding conflict altogether reduces opportunities for learning since students do not experience opportunities to change their discourse if their ideas are unchallenged. Instead, to engage in *productive* communication, students must be determined to resolve incommensurability by discussing opposing strategies (Chiu, 2000, 2008a, 2008b; Jarosz et al., 2017; Orme & Monroe, 2005; Sfard, 2007, 2019) studies with ninth-grade Algebra students revealed that polite disagreements, or respectful arguments about mathematical strategies, were significantly positively correlated with success and creativity in group problem-solving. However, rude disagreements were negatively correlated with group success, indicating that the nature of conflict, whether polite or rude, influences the likelihood of resolving incommensurability. Other studies similarly corroborate the influence of argumentation on learning (e.g., Jarosz et al., 2017).

In our work with middle-school and college-aged learners, we have come to learn that students are often reluctant to resolve incommensurability. Instead of arguing about the viability of their approaches, they often 'agree to disagree' or refrain from engaging in conflict at all. For instance, consider the following exchange between two middle-grade learners who were working in a group to construct an argument for the claim 'the sum of two odd number equals an even number' (utterances 11 – 12; data obtained in a prior research project [Campbell & King, 2020]).

11. *Brittany:* I just like added all the odd numbers 1 – 9, and they all became even because they're all divisible by 2. And all numbers end with a 1 – 9, so if it's odd, then it will, then you just add it with another odd number, and it's divisible by 2.
12. *Felicia:* Yeah, that's...OK. So, I'll write out what I put out, you write out what you put, and then you write out whatever you put.

During this exchange, instead of discussing their arguments to decide which was viable or most efficient, Felicia suggested the group simply compile all their strategies on the task sheet (utterance 12). From our experience, this avoidance of critique and argumentation is evident across the grade levels.

While students often avoid conflict, there are times that they willingly seek to resolve incommensurability. Danielle, Katrina, and Hayden's (three college-aged students) interaction on a mathematical proving task portrays the benefits of deliberating about the viability of differing mathematical strategies (utterances 13-18; data obtained in a prior research project [Campbell & Hodges, 2020]).

13 *Katrina:* We multiply 3, 5, and 7 just for kicks. It creates 105, and when you make—which it sounds dumb. Would we be able to talk about 105 in terms of what it means to be prime? Because then the only factors of 105 will then be 3, 5, and 7. I don't know if it's helpful.

14 *Hayden:* That's not the only factors. Those are the prime factors.

15 *Katrina:* Those are the only ones.

16 *Danielle:* 21 is also a factor of 105.

17 *Katrina:* Well, yeah, but then that factors out to be 3 and 7.

18 *Danielle:* Right, which would make them prime factors.

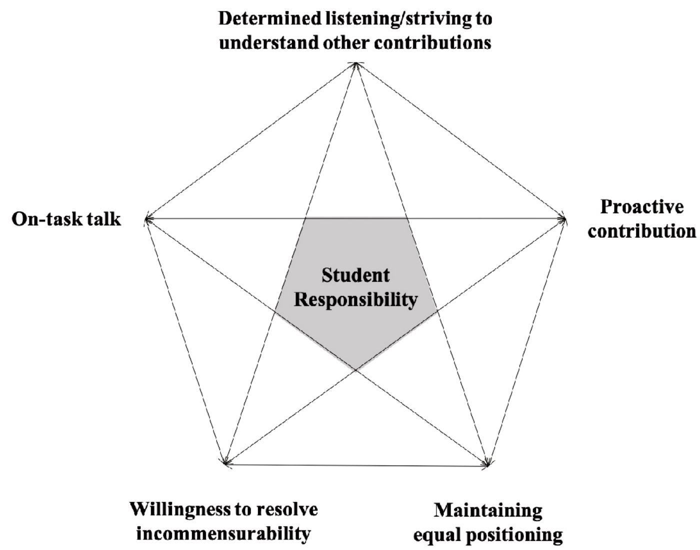
The group deliberated about Katrina's claim that 3, 5, and 7 were the only factors of 105. By deliberating with one another, the group came to the understanding that 3, 5, and 7 were the only *prime* factors of 105—not the only factors, as Katrina originally suggested. As evidenced by this interaction, willingly engaging in conflict to resolve incommensurability provides students with opportunities to change their discourse in desirable ways.

On-task talk

The final student-responsibility is *on-task talk*. For discourse-rich classrooms to be *productive*, students are responsible for ensuring the majority of their talk is related to mathematics. Some researchers found occasional off-task communication can aid non-dominant students in gaining power and agency in mathematics classrooms (e.g., Esmonde & Langer-Osuna, 2013). Without disregarding such findings, empirical research also suggests that *on-task talk* is highly predictive of successful collaborative problem-solving (Chiu, 2008; Jarosz et al., 2017). For instance, Jarosz et al. (2017) investigated predictors of successful group problem-solving for college-aged learners in an introductory statistics course. They found that successful groups utilized a lower proportion of off-task talk than less successful groups. Research does not suggest that groups should never engage in off-task talk. Indeed, such environments may become unauthentic or unenjoyable for students. Rather, learners should limit distractions and ensure the *majority* of their talk is related to mathematics. In the next section, we share our theoretical model of student responsibilities in discourse-rich classrooms and explain how the five responsibilities interact and influence one another.

Figure 1

Theoretical Model of Student Responsibilities in Discourse-Rich Classrooms



Note: This model shows the connection amongst student responsibilities in a discourse-rich classroom.

Model of Student Responsibilities

Thus far, we have described five student responsibilities that promote learning in discourse-rich classrooms. We claim that the five responsibilities do not operate in isolation. Instead, each student's responsibility is intertwined with the others, influencing the likelihood that another responsibility may be upheld. Figure 1 shows the connection amongst the responsibilities.

The double-sided arrow signifies their interdependence. To illustrate the interdependency, consider *proactive contribution* and its relationship to the other four responsibilities. *Proactive contribution* and *determined listening and striving to understand others' contributions* are interdependent because, when few students proactively contribute, there are few opportunities to listen actively with the purpose of understanding. Likewise, if learners do not believe others are actively listening to their contributions, they will be unlikely to contribute proactively. *Proactive contribution* and *maintaining equal positioning* are interdependent since students' willingness to communicate in groups influences how they position themselves and others. Likewise, students' positions influence how compelled they feel to communicate in groups. *Proactive contribution* and *on-task talk* reveal a trivial interdependency. Finally, *proactive contribution* and *willingness to resolve incommensurability* are interdependent since students can only resolve conflict if multiple group members provide mathematical contributions. Likewise, students will only be willing to resolve conflict if there is a group norm of proactive communication amongst all group members. Indeed, each student's responsibility reveals an interdependency on other responsibilities. Such an interconnected model suggests learning in discourse-rich classrooms is an intricate process. Failure to maintain one responsibility could inhibit the potential to maintain other responsibilities, which is detrimental to creating learning opportunities in a collaborative environment. The care required for the successful implementation of discourse-rich classrooms is well-documented in the literature (e.g., Sfard & Kieran, 2001; Webel, 2013). Students must be supported in meeting their responsibilities to create learning opportunities.

Discussion

In this paper, we offered a theoretical model revealing students' responsibilities for *productive*, discourse-rich classrooms. The model consists of five components: (1) determined listening and striving to understand others' contributions, (2) proactive contribution, (3) maintaining equal positioning, (4) willingness to resolve incommensurability, and (5) on-task talk. Each of the components are interrelated and influence one another, suggesting that neglect of one component of the model decreases the likelihood that another responsibility will be maintained.

The five student responsibilities help learners engage in *productive* communication in discourse-rich classrooms to experience lasting mathematical discourse changes (i.e., mathematical learning; Sfard, 2008). From a theoretical standpoint, the responsibilities provide learners with opportunities for their current discursive rules to be challenged, which can result in *durable* and *desirable* mathematical discourse changes (Sfard, 2008). For instance, active, determined listening and willingness to resolve commensurability promote opportunities for learners to confront others' mathematical ideas. Similarly, *on-task talk* ensures that learners' mathematical contributions remain the focal point of deliberation, which is necessary for learners to experience opportunities to change their mathematical discourse. Each responsibility creates opportunities for conflict resolution, which can lead students to change their mathematical discourse in ways that are *durable* and *desirable*.

Our model suggests several implications for future research. The model leaves room for theoretical and empirical refinement. Future research might uncover other student responsibilities that are integral for *productive* discourse-rich classrooms or might determine more precise linkages between responsibilities. For instance, while we suggest that all responsibilities are interdependent, it is possible that some linkages are stronger than others. Therefore, some responsibilities may carry more weight in determining the productivity of collaborative engagement than others. Scholars might also seek to design a pedagogical model for aiding learners in meeting their responsibilities. Current literature on teacher facilitation of discourse-rich classrooms mostly focuses on teacher moves (e.g., revoicing) that promote a positive classroom culture (O'Connor & Michaels, 2019). Scholars might extend research on teacher moves by empirically investigating strategies to aid students in meeting their responsibilities in discourse-rich classrooms.

Another extension of our work is the consideration of student responsibilities for meeting different goals in discourse-rich classrooms. Based on Sfard and Kieran's (2001) conceptualization of productivity, we defined discourse-rich classrooms as *productive* if they often lead students to change their discourse in *durable* and *desirable* ways. This is an important outcome of education, but it does not capture all the potential goals of social perspectives on learning. For instance, some scholars suggest that discourse-rich classrooms help learners engage in important social skills such as argumentation and explanation (Hmelo-Silver et al., 2007). The proposed model does not take into consideration other potentially important outcomes of discourse-rich classrooms. Future research might expand, combine, or create new models of student responsibilities for meeting various goals.

In relation to practice, teachers might explicitly teach learners their responsibilities for maintaining a *productive* learning environment. However, unlike other strategies for improving teaching and learning, our model should be considered a whole unit. That is, it may be unproductive for learners to practice responsibilities one after another until mastery is reached. The five responsibilities work in tandem, and increased maturity in one responsibility is likely to enhance other responsibilities. Therefore, we suggest that practitioners introduce students to their responsibilities and work as a community towards maturation. Strategies for teaching the responsibilities and making them normative in a classroom community are beyond the scope of this paper. Still, reflection seems a promising tool for increasing student awareness of their actions (Wagner, 2007). By continually reflecting on their progress, students might become more aware of their abilities to meet their responsibilities for discourse-rich classrooms.

In closing, the field still has much to learn regarding how discourse-rich classrooms *should* operate. Current research is unbalanced, with most studies examining teacher facilitation while placing little emphasis on student-level factors. To understand supportive actions in discourse-rich classrooms, the field might further examine how students communicate with their peers and teachers. Analyzing such communication from the student-level can reveal desirable or undesirable communicative behaviors, suggesting further implications for pedagogical design. This paper might act as a starting point for future empirical analyses of student-level research in discourse-rich classrooms.

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