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Reconsidering Elements of Research and Practice

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NOTES FROM THE FIELD

Mathematical Design Thinking in the Classroom through Graphic Art

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ABSTRACT This classroom study immersed high school geometry students in the creative and intellectually challenging design task of developing unique logos using mathematics and technology. The students applied and deepened their knowledge of transformations while using dynamic geometry software. One of the main aims of the task was to elevate student creativity and autonomy within the mathematics classroom while they engaged in mathematical design thinking to create their logos. The discussion provides insight into considering student work from a design perspective, which can offer students new ways to engage with mathematical concepts and make their thinking more explicit.

KEYWORDS *mathematical design thinking, logo design, modeling, prototyping, technology, geometry*

Introduction

The current emphasis on modeling within the Common Core State Standards encourages students to engage in mathematical thinking by creating a model and learning from the mechanics of the model simultaneously (CCSSI, 2010). In doing this, students are actively discovering, learning, and applying relevant mathematics to the model they are creating. Additionally, this provides them with opportunities to develop problem-solving skills that are applicable outside of the mathematics classroom.

Bringing mathematical design thinking into the classroom through design tasks, similar to those in Project Lead the Way (2017), provides opportunities for students to engage with mathematics in a unique way. In these situations, students have the autonomy to learn and do mathematics in ways that make sense to them. Dynamic Geometry Software (DGS) provides an interface for students to engage with and mathematize real-world situations. Such technology can help students represent and model natural phenomena while making the mathematical concepts an explicit focal point. Technology also promotes mathematical habits of mind and normalizes productive struggle.

In this classroom study, I sought to understand better how students engage in mathematical design thinking and how teachers can best support students to engage in this type of thinking. I repurposed an artistic logo design task to incorporate geometric transformations to encourage geometry students to mathematize the task within DGS. In this way, they would naturally engage in the design process by applying the relevant mathematical concepts they have learned. As a result, all students actively engaged in mathematical design thinking to create their unique logos, each demonstrating varying levels of awareness of the design process.

Review of Related Literature

Mathematical Design Thinking

The Common Core Standards for Mathematical Practice (SMP) have brought increased attention to problem solving and mathematical modeling across the K-12 curriculum (CCSSI, 2010). The SMP explicitly mentions modeling, perseverance, and reasoning; these three actions fall within the construct of mathematical design thinking, and, more generally, what is known as *mathematical knowing in action* (Schön, 1992). Additionally, implementing strategies that intentionally bring design thinking into the mathematics classroom can positively impact how students approach rigorous problems (Chin et al., 2019).

Design thinking is defined as a type of knowing in action characterized by a constant interplay between the design and the designer's thinking about the design (Schön, 1992). In this model, each depends on the other, and it is not possible to separate the actions within the space from the ways of knowing. Adapting Dym et al.'s (2005) definition of engineering design to mathematical design, mathematical designers generate, evaluate, and articulate mathematical concepts or processes. In doing this, the designers attend to human objectives while abiding by the constraints of the situation to which the design will be applied. The term situation includes instances where a problem is being solved and where one has not been posed.

Incorporating mathematical design thinking into the classroom can help students become stronger problem solvers and bring lower-achieving students to a level equal to their average-achieving peers (Chin et al., 2019). Design tasks give students the autonomy to explore, create, and do mathematics at the highest level (Smith & Stein, 1998). Consequently, students can apply and develop mathematical concepts within a context that is driven by their interests in relation to the task's requirements.

Principles Guiding Mathematical Design Thinking in the Classroom

While developing a task to engage students in mathematical design thinking, teachers must carefully consider their objectives and support students as they engage with this type of thinking. Kolko's (2015) principles of design thinking serve as a guiding framework for designing and implementing rigorous mathematical design thinking tasks within the classroom. These principles are:

1. Create models to examine complex problems.
2. Use prototypes to explore potential solutions.
3. Focus on users' experiences, especially their emotional ones.
4. Tolerate failure.
5. Exhibit thoughtful restraint.

(Kolko, 2015, p. 68 – 69)

Modeling. Modeling is a way to visualize situations and explore the facets and constraints of a problem, also known as the problem space (Kolko, 2015). In contrast, Kolko (2015) distinguishes prototyping as exploring and experimenting within a problem's solution space. While engaging in design thinking, students can use modeling to help them understand and represent the problem or task in ways that make sense to them, then use prototyping to develop solutions to the task. In K-12 mathematics, students regularly create models to represent the problem space so they can set up and perform a brief calculation to arrive at a teacher's anticipated answer. In this context, students are engaging in basic modeling but never reach the prototyping stage of design thinking.

Prototyping. Prototyping provides students a creative space where they have the autonomy to develop one or more unspecified solutions to a problem or task; students assume responsibility for determining the appropriateness and effectiveness of these solutions based on the criterion they develop. Prototyping is a nonlinear process marked by student engagement in cycles of design and validation (Fountain, 1990). By discussing prototyping as a cyclic process, we can classify students' actions within these cycles to allow both students and teachers to identify and emphasize the significance of these actions. Rothenberg (1990) explains that students can engage in prototyping with the intent to generate and explore ideas *(generative prototyping)* or to determine what aspects of their prototype are meeting their expectations and goals *(evaluative prototyping)*. Fountain (1990) classifies the prototypes that students develop but later discard as *throwaway prototypes,* whereas those that students develop and then modify as *evolutionary prototypes.* These terms are referenced later in the discussion of the task.

Student's Experiences. A safe classroom environment is essential to the success of mathematical design thinking tasks. Mathematics classrooms need to be places where students are comfortable taking risks and sharing novel ideas so they can embrace the freedom, challenges, and unknowns that occur during mathematical design thinking. *Rough draft talk,* defined by Jansen et al. (2017), provides students a space to share in-progress thoughts and ideas without the stress of evaluation. Teachers can engage students in rough draft talk by acknowledging and honoring their mathematical work and emotional experiences with mathematics at all stages of the design process. In addition, teachers can seek to understand a student's work from the student's perspective without imparting their perspective by practicing what is known as *mathematical empathy* (Araki, 2015). Rough draft talk and mathematical empathy normalize the nonlinearity of learning and elevate the design process over the product.

Risk Taking. Finally, students engaged in mathematical design thinking must manage the complexity of their preferred designs with the risks that are required to pursue and create these designs (Kolko, 2015). In the task described here, students who designed an entirely new logo took more risk than those who decided to recreate and modify an existing logo; those working with complex logos risked not having the time to finish their designs. Each student had to find a balance between complexity and the associated risk.

In determining how to apply mathematics in a new way to make their logo, the students can be seen as "doing mathematics" as defined by Smith and Stein (1998). They explain that these high-level tasks free students from the limitations of finding a solution and provide opportunities to engage with and discover mathematics while exploring the solution spaces to problems. At the same time, Smith and Stein (1998) also discuss that students may find themselves experiencing some level of anxiety "due to the unpredictable nature of the solution process" (p. 348). By acknowledging and normalizing the risks and anxieties that some students experience while engaging in design tasks, teachers can support students in managing risk and ambition throughout the task.

Context and Task Details

Classroom Context

A total of twenty-one high school geometry students engaged in this mathematical design task by constructing a logo in DGS using geometric transformations. I began with the expectation that students had some prerequisite knowledge of geometric transformations since they learned how to recognize, represent, and perform translations, reflections, rotations, and dilations earlier that year. We reviewed these concepts immediately prior to the task during a two-day introduction where students also learned to navigate GeoGebra, the DGS they would be using. It was important for students to explore the software and familiarize themselves with various tools, including how to perform each type of transformation needed for the design task.

Design Task

I provided the instructions shown in Figure 1 for the logo design task. Students accessed the materials for the task (i.e., instructions, a reflection handout, and assessment rubrics) on a Google Classroom webpage. The instructions for the task are in Figure 1.

Figure 1

Logo Design Task

Design your logo using transformations in GeoGebra.

This is a time for your creative side to shine! Consider what your logo will represent. If you are not sure where to start, consider making a new logo for your favorite brand of shoes, clothes, fast food, etc. Bring it to life with color and design!

Your logo needs to:

1. Represent a company, activity, program, brand, or something else.
2. Use at least ***two transformations.***

The design task was crafted to engage students in mathematical design thinking, primarily prototyping. The entire task spanned three consecutive class periods, each of which were forty-nine minutes long. On the first day of the project, the students and I examined transformations in existing logos before they were given time to work on their own. Some students requested to use existing logos; this request was granted with the condition that they needed to modify the existing logo in some meaningful way. The students continued their work on the second day and submitted their work by the end of the day. Their submission included the logos and an accompanying handout where they discussed the transformations they used and how they engaged with the design process. On the third day, the students gave short presentations of their logos to their peers.

Implementation

The logo design task challenged students to blend creativity with problem solving while also requiring them to apply mathematical knowledge of transformations to their design. I provided them with the autonomy to choose the transformations they applied to make their desired logo. Throughout the task, the students were challenged to combine their understanding of geometric transformations with a working knowledge of the GeoGebra software. The short timeline required students to monitor their progress while balancing their ambitions and the associated risk.

I spoke with students to uncover how they were making sense of the task, using technology strategically, and persevering through the task. My aim was to understand their thought processes as they engaged with the task in relation to the components of mathematical design thinking and the SMP (CCSSI, 2010). However, it can be difficult to capture a student's mathematical knowing in action. Students are not always fully aware of or able to articulate their thinking because it is a compilation of the many small and often unconscious decisions they make while engaging in design. Therefore, in an attempt to make their mathematical knowing in action more explicit, I asked students to record their design process and any discoveries they made on paper. This provided me with insight into their thinking and served as a guide for students when they presented their logo to their peers on the third day.

Throughout the three days, I ensured the focus was on students' ideas and work. To do this, I monitored student progress and generated and maintained discourse with students about their work, primarily by engaging the students in rough draft talk (Jansen et al., 2017). I was intentional in allowing students to remain the authority on their process and encouraged students to discuss their ideas with each other at various points. Students had a great deal of freedom to discuss their work with others throughout the activity.

Data Collection

All of the class sessions were videotaped to capture student work in progress, including the conversations between the students and between the students and myself. Students submitted a digital copy of their logo along with a handout. The handout that students completed consisted of the following four questions regarding their design process and their use of transformations:

1. What does your logo represent? How does it represent that?
2. For every transformation that you created, fill in the following table. You may use an additional paper if you need more space. (In the table, students named each transformation, recorded the pre-image, image, and explained relevant properties.)
3. How did you design your logo? Walk me through the process you used.
4. What 'aha' moments or discoveries did you have while creating the logo?

The presentations students made included some of the same information; however, the students were asked to explain specifically: 1) the theme of the logo, 2) transformations they used to create the logo, and 3) their favorite part of the task, something that was challenging for them, and anything they would change if they had additional time to revise their work.

Narrative of Logo Design Project

In this section, I will discuss how students interacted with the design task. First, with a summary of students' engagement, followed by samples of student work and conversations that took place during the task.

Progression of the Design Task

Upon receiving the task, students generally took three different approaches. One group began by exploring logos found through searching Google Images to brainstorm ideas for their design. This inspired some students to modify an existing logo; for others, it helped them consider attributes of logos in general. A second group took time to create a mental picture of what their logos would look like before using GeoGebra to create the logo. The third group immediately engaged in exploratory trial and error, playing with the various DGS tools to see what logos they could create. Video recordings from the class showed many students restarting their logos at least once on the first day. Some of these students reused similar concepts in their next design, while others pursued entirely different directions after discarding their initial work.

During the first day, many students indicated that they began creating their logos before knowing what their logos represented. The video recordings revealed that students had a general conception of what a logo is, and they used this conception to guide their design. Instead of creating detailed arrangements or including random shapes that did not connect to each other in some way, the students generally focused on creating a cohesive design of transformations that approached their general conception. As they developed and refined their logos over the three days, students seemed to use their personal experiences with logos to determine what their creations represented. Interestingly, these students then created their own companies that matched their logos instead of matching their logos to existing companies. Once they decided what their logos represented, they considered different aspects of their logos and how these aspects provided meaning.

During the task, every student focused on how to use DGS to create the logo to fit their self-imposed expectations of what the logo could be. Some students used the grid provided in the DGS to create transformations, while others utilized the functions in the software instead. The videos revealed that as the students worked, they were also curious about what their classmates were creating; they looked at each other's screens, shared ideas, assisted each other with the DGS tools, and listened to nearby conversations.

Towards the end of the first day and throughout the second day, students began adding finishing touches to their logos; these included hiding points, lines, and labels in the DGS to make the logo cleaner and adding color to various shapes to make the logo stand out. Although I showed the students how to include these details, they had the autonomy to decide whether to use them. The students were not required to add these additional cosmetic elements; some students decided not to include these while others were not able to due to the time constraints. From here, students used their completed logos to fill out the reflection handout and bounced ideas off each other as they figured out how to record their design process. I encouraged students to sketch the pre-images and images to aid them in writing about transformations.

On the third day, every student presented their own logo to the class. There was a great deal of enthusiasm and excitement regarding the visual appeal of the logos. While sharing, students typically noted an area of improvement in their presentation. The students were also given opportunities to ask presenters about their logos. I asked questions to prompt discussion of particularly interesting characteristics of the students' design processes as well as to highlight noteworthy ideas. Each student enjoyed the support, enthusiasm, praise, and applause of their peers at the conclusion of the presentation.

Conversation About Student Designs

The sample work and conversations included below are chosen to highlight students' engagement with the design process. All student names in this paper are pseudonyms to protect student privacy. Two students made logos that incorporated their names; the data from their logos are included in the conclusions, but their logos are not pictured to maintain anonymity.

Leith's Star. The conversation in Figure 2 occurred on the first day of the project when Leith wanted to create a five-pointed star using rotations. In this conversation, Leith is trying to connect his conceptual knowledge of rotations to the procedural knowledge needed to transform a shape in DGS. Similar conversations about connecting the conceptual and procedural understandings of transformations occurred throughout, with a maintained focus on applying mathematics to achieve an aesthetic goal. Leith later assisted a peer in overcoming the same challenge.

Alana's Egg Brand. Alana had to make a decision about which transformation was the most appropriate to create rays for her "sunny side eggs" brand, which shows the "egg cracking at the break of dawn....which the sun is beaming (squiggles)" as she wrote in her reflection. The conversation and her work were captured on video

Figure 2

Leith Determining the Angle of Rotation

Leith (L): Asks how to rotate a shape to create a five-pointed star.

Simon (S): "There's a way to figure out the angle of rotation on a five-pointed star."

L: "How is that?"

S: "How many degrees are in a circle?"

L: "360"

S: "360 degrees, think of a rotation, you want to make it a complete circle, right?"

L: "Yeah"

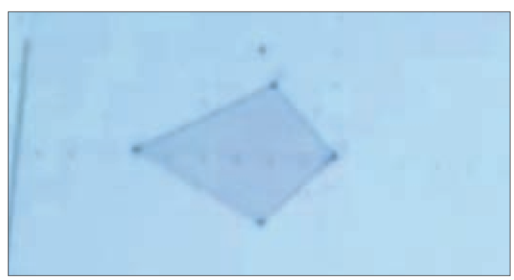
S: "So how many degrees would you make this rotation if it is 360 total?"

L: "360 divided by 5."

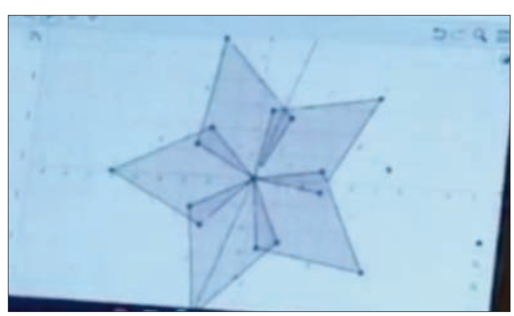
S: "Yeah"

L: (Thinking)

L: "So that would be, 70 something."



Pre-image



Image

Figure 3

Alana Identifying the Appropriate Transformation

Alana (L): "It's like an egg brand." (Top picture)

Simon (S): "To go on with the theme?"

A: "Yeah, and this is like the light, basically. But I need to make this (squiggles) better."

S: "Could you make one really, really good one and just use transformations to make the rest?"

A: "That sounds good."

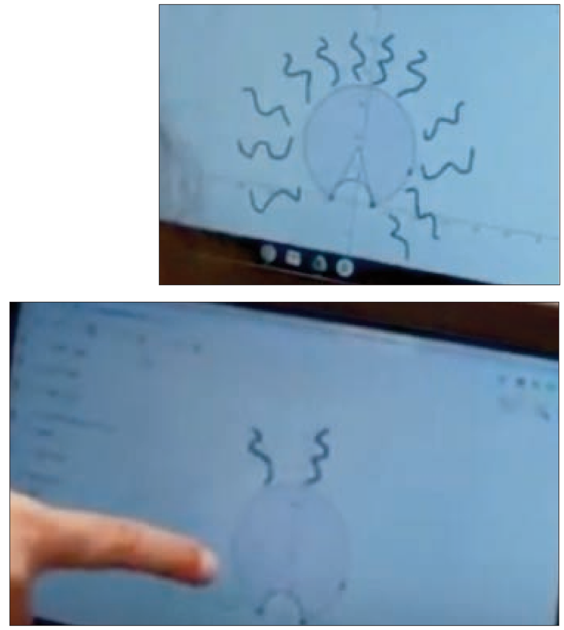
S: "Yeah? Which transformations?"

A: "Um, translation? … or rotation?"

S: "So if you translate, you start with a squiggle on the right, you could translate it down here, it would still be a squiggle here. So which one would be more sun like? Translations or rotations?" (Bottom picture)

A: "Rotations."

S: "Yeah!"



while she determined which transformation would be most fitting to create the sunbeams. This is another example of mathematizing a problem to reach an aesthetic goal.

In her presentation, Alana explained that "my favorite part was probably doing the squiggles. Um...I don't know why. Like all these are the same (pointing at the squiggles) except these two because I couldn't get them perfectly in the center, so, like, I had to redraw two of them. Or (actually) just one of them, and I reflected it." It is notable that the squiggles, although challenging, were her favorite part.

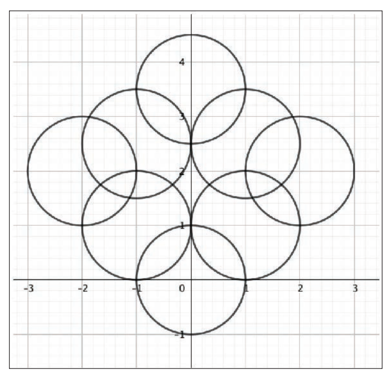
Problematizing the Task

The logo design task emphasized aesthetics, with mathematics serving as the means to create them. In their reflections, most students recognized one or more mathematical problems or challenges they overcame to create the logo in DGS. Two student reflections appear alongside their completed logos in Figure 4. I chose these to illustrate how students used transformations explicitly and implicitly to create their logos.

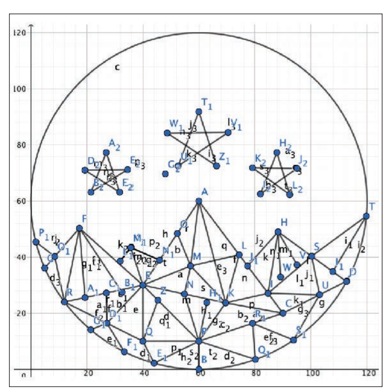
Figure 4

Problematizing the Logo Design Task

Jacob: "First, I had to find the best size for the circle. Then I had to find where to translate the circles to. This took the longest because it was hard to find where it would fit best. I figured out, and then once I finished the side, I reflected it to make the other side match."



Haley: "I made a circle big enough for the mountains I was going to put. I pressed segment lines, and from there I made the middle, so the main mountain, and from there I was making the 2 smaller mountains on both sides of the main mountain. After making those 3 mountains, I started making smaller triangles of different sizes facing different directions into the mountains so it wouldn't look so empty…"



Results from Student Work

The Role of Transformations in the Design Task

In their reflections, students identified and described the transformations they used to create their logo. From these reflections and video recordings of conversations, I identified why students used the particular transformations they did. Some students, including Leith and Alana, realized that they needed to use rotations to create the symmetry they were looking for in their logos. Others, including Jacob, found that translations and reflections helped them to move objects across the screen in different ways. Haley was one of many who applied dilations and translations to make larger and smaller figures. This demonstrates that students learned and understood which transformations were appropriate to reach their objectives in the aesthetics of their logo. Students did not use each transformation equally as shown in Table 1.

Prototyping Within the Design Process

Students regularly engaged in prototyping throughout the task. The students' prototyping actions captured in the video were categorized in Table 2 based upon their intent, as either throwaway or evolutionary (Fountain, 1990), and their purpose, as generative or evaluative (Rothenberg, 1990).

One third of students created a throwaway prototype with the intent and understanding that they were simply generating ideas, and that their first design may not be their final logo. The students who began by researching logos or experimenting in GeoGebra engaged in generative prototyping; they were developing ideas of what to create before moving into DGS (Fountain, 1990). This is a stark contrast from traditional mathematical tasks where the objective is to obtain a correct solution. In these tasks, students are comfortable with experimenting and tolerating setbacks, which is consistent with design thinking (Rothenberg, 1990).

Due to the eventual evaluation of students' work (students were required to turn in a product that they presented and I assessed), all students naturally transitioned to evaluative prototyping by the end of this activity (Rothenberg, 1990). Many shared plans to revise their logos if they had additional time. Additionally, ninety percent shared at least one insight they had while creating their logos. Only two students denied having any insights; the videotaped conversations with these two students reveal otherwise; however, these students did not view those moments as significant in their reflections.

These results show that students engaged in prototyping as a method to help them incorporate transformations within their logos, even if their awareness of the design process varied. The data also revealed that students can engage in mathematics and graphic design simultaneously and that their logo designs influenced the mathematics they chose to incorporate within their logos.

Conclusion

The Standards for Mathematical Practice establish the mathematical and critical thinking skills that all students should develop while learning grade-level content (CCSSI, 2010). The current modeling standard can be expanded to encompass all types of mathematical design thinking, including prototyping. By expanding this, we can create a space for the attributes and language of mathematical design thinking in the classroom and promote student autonomy and creativity in the learning process. Through mathematical design thinking, students can engage in cross-curricular activities from a mathematical perspective, and these activities can include but are not limited to graphic design.

Teachers who wish to integrate mathematical design thinking within their classroom must provide students

Table 1

Percentage of Students Who Used Each Transformation Within Their Logo

Transformations	Reflections	Dilations	Translations	Rotations
Percentage of Students	77%	45%	32%	32%

Table 2

Percentage of Students Engaged in Each of the Prototyping Actions

Design Thinking Actions	Throwaway	Evolutionary	Generative	Evaluative
Percentage of Students	33%	100%	52%	100%

with complex situations, contexts for students to apply and expand their understanding of mathematics. In mathematical design situations, teachers must create and maintain a classroom environment where students have ownership over their work and are comfortable engaging in challenge, intellectual risk, and productive struggle. When experiencing the design process through prototyping, students can learn that what initially appears to be a setback is instead a valuable conceptual gain that they may have otherwise not experienced.

Teachers can also do more to help each student be aware of and express their mathematical design thinking both verbally and in writing. One way of doing this is to teach the language of prototyping (generative, evaluative, throwaway, and evolutionary) so students can identify how their thinking and work fits within mathematical design (Rothenberg, 1990; Fountain, 1990). Student reflections provide insights, but their clarity and detail are dependent on students' abilities to verbalize or write their ideas after completing a design task and what the students view as significant. By recognizing the merits of their work and ideas through the language of prototyping, students may find themselves better able to record their design processes in detail.

Design tasks must also be developed and implemented to honor the creativity that emerges during the design process, which makes each student's work unique. Thus, incomplete and complete work must be able to exist side-by-side and elevated as equally valued contributions, communicating to students that their work is important and valued at all stages within the design process.

Implications for Further Research

Research is needed to develop additional methods that engage students in mathematical design thinking and to further investigate the connection between mathematical design thinking and mathematical habits of mind. In conjunction, research of this type would provide additional support for teachers to expand on the mathematical modeling that already occurs within classrooms to include all types of mathematical design thinking, including prototyping. Consequently, this can strengthen student learning and understanding of mathematical concepts and deepen students' awareness of the design process. Such an expansion provides an opening for educators to develop diverse and innovative learning opportunities that empower students to be creative, take risks, and problem solve, thus supporting students to become active doers of mathematics.

References

Araki, M. E. (2015). *Polymathic leadership: Theoretical foundation and construct development,* [Masters dissertation, Pontifícia Universidade Católica do Rio de Janeiro]. Social Science Research Network.

Chin, D. B., Blair, K. P., Wolf, R. C, Conlin, L. D., Cutumisu, M., Pfaffman, J., & Schwartz, D. L. (2019). Educating and measuring choice: A test of the transfer of design thinking in problem solving and learning. *Journal of the Learning Sciences, 28*(3), 337–380. https://doi.org/10.1080/10508406.2019.1570933

Common Core State Standards Initiative. (2010). Common core state standards for mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Dym, C. L., Agogino, A. M., Eris, O., Frey, D. D., & Leifer, L. J. (2005). Engineering design thinking, teaching, and learning. *Journal of Engineering Education, 94*(1), 103–120. https://doi.org/10.1002/j.2168-9830.2005.tb00832.x

Fountain, H. D. (1990). *Rapid prototyping: a survey and evaluation of methodologies and models.* Naval Postgraduate School; Monterey, CA.

Jansen, A., Cooper, B., Vascellaro, S., & Wandless, P. (2017). Rough-draft talk in mathematics classrooms. *Mathematics Teaching in the Middle School, 22*(5), 304–307. https://doi.org/10.5951/mathteacmiddscho.22.5.0304

Kolko, J. (2015). Design thinking comes of age. *Harvard Business Review, 93*(9), 66–71.

Project Lead the Way. (2019). Retrieved on June 26, 2019 from https://www.pltw.org/.

Rothenberg, Jeff. (1990). *Prototyping as modeling: What is being modeled*. Santa Monica, CA: RAND.

Schön, D. (1992). Designing as reflective conversation with the materials of a design situation. *Knowledge-Based Systems, 5*(1), 3–14. https://doi.org/10.1016/0950-7051(92)90020-G

Smith, M. S. & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344–350.