JOURNAL OF
MATHEMATICS
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*A Century of Leadership in Mathematics and Its Teaching*

# Forward-Thinking Orientations for Mathematics Education

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# Gerrymandering in the High School Geometry Classroom

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**ABSTRACT** Teaching gerrymandering in our high school geometry classrooms provides students with a unique opportunity to use mathematics to describe, analyze and make sense of the world around them. Our purpose is to provide our students the opportunity to apply learned definitions and formulas of area and perimeter to a sociopolitical context. We present a unit that we designed for teaching high school students about gerrymandering and describe how teachers may implement this unit in their classrooms. In this unit, students discovered the mechanics of gerrymandering. They made calculations and observations about area, perimeter, and the ratio of area to perimeter. They considered compactness as a potential tool to indicate gerrymandering. They used proportional reasoning to measure the fairness of the partisan split for a given map. We provide supporting examples of student work and discourse and make recommendations for future iterations of this unit.

**KEYWORDS** *geometry, gerrymandering, equity, high school, projects, apportionment, proportional reasoning, compactness*

## Background

Throughout American democracy, gerrymandering has been perpetrated by all political parties and at all levels, from congressional districts to school districts (American Civil Liberties Union [ACLU], 2016). In most states, the legislature draws district boundaries and thus can manipulate them to gain legislative control. There are several states in which nonpartisan, bipartisan, or citizen commissions are used when redistricting. That number has grown in the past several decades (NCSL, 2020). In fact, there are over 200 proposals for redistricting reform being discussed by state legislatures for the 2021 redistricting process (Brennan Center, 2020). These proposals are motivated by an upswing in disproportionate representation after the 2010 census. Republicans gained 63 congressional seats and control of the United States House of Representatives. According to Wines (2019), the Republican party "had poured money and expertise into state legislative races with the specific aim of gaining control over redistricting; the Democratic Party had not" (para. 9). Partisan gerrymandering became a more prevalent part of the political news conversations, with cases in Pennsylvania, Wisconsin, and Maryland being elevated to the Supreme Court (Brenner, 2018). In an amicus brief for two of these cases, Charles et al. (2019) argued that "The Constitution protects an individual's right to an undiluted vote. The government acts unconstitutionally when it intentionally dilutes an individual's vote" (p. 7). The potential dilution of our students' votes and voices is at the core of our motivation to teach gerrymandering in the high school math classroom. We teach high school students from the South Bronx who will be eligible to vote within three years. When surveyed, our students shared that they do not feel the government is connected to them or is for them. We want our students to be informed about the redistricting process and the potential for disproportionate political power. This way, they can hold their government accountable to represent their interests fairly.

## Our Schools

Our schools are located in Districts 10 and 12 in the Bronx. Neither school requires testing for admission, and both are public schools. Neither school tracks classes by ability, and we are both integrated co-teachers for about half of our total classes. Students in both schools are roughly two-thirds Hispanic and one-third Black, as reported in the data for our schools. About 90% of students from both schools are eligible for free lunch. While the mission and vision of our schools are quite different, both schools are designed to meet the needs of our students and create a supportive community where students feel respected and seen. They are both small schools of 350–450 students. Since the 1990s, New York City has closed many large comprehensive high schools to create small schools where students are known well, that value smaller class sizes, or that organize around a specific discipline or theme. Bronx Theatre High School is modeled after a theatre company. Students learn to act as an ensemble and to support one another. By taking on a role, they learn empathy and critical thinking. Wherever possible, theatre is integrated into classrooms. For example, in the gerrymandering unit, students take on roles of politicians to debate competing map ideas. Fannie Lou Hamer Freedom High School is part of the New York State Performance Standards Consortium. Rather than standardized testing, students complete a portfolio of project-based assessments for graduation. Many of the student quotations in this article are from papers written by Fannie Lou students.

## Unit Outline

This unit builds on the concepts of area, perimeter, and proportional reasoning. Many of our students enter the classroom able to recite a definition for area or perimeter and substitute values into a formula. Our purpose is to provide an opportunity for high school geometry students to apply what they already know about these concepts to a sociopolitical issue. As stated in the Common Core Math Standards for High School Geometric Measurement, Dimension and Modeling, our students use properties of geometric shapes to describe objects and "apply geometric methods to solve design problems" (Council of Chief State School Officers, 2021, para. 1). We spent 4–6 weeks on this unit, but there are many opportunities for extensions that could extend the length at the teacher's discretion.

The unit consists of four parts. In *Contextualization,* we look at the population of each state, the number of seats in the United States House of Representatives for each state, and the partisan split. We ask questions about fairness in terms of how many representatives a state should have given its population and how many we expect should go to Democrats, and how many to Republicans. In *Play the Game,* students play a board game in which they compete to win districts. By playing the game, our objective is for the students to identify the number of votes needed for them to win a district and for them to practice minimizing their own spent votes while maximizing the spending of opponents' votes.

The third part of the unit is centered around *Squaretopia,* a state that gets divided into ten districts. Students learn that the number of districts won by each party entirely depends on how the districts are drawn. Here our goals are for students to draw districts that are contiguous and to identify and use the minimum number of votes needed to win a district in order to maximize the number of districts they can claim.

Finally, in *Calculating and Analyzing Compactness Metrics,* students use four compactness metrics to reason about the gerrymandered Squaretopia maps. Students come to the realization that gerrymandered maps almost never result in compact shapes. This leads to the development and application of formulas that measure compactness. Because there is no single definition of compactness, students practice how to make a mathematical argument using a given metric. Throughout these four parts, students see that when they attempt to make a party win as many districts as possible, that they will likely create "weird" shapes. In the final part, they consider these strange shapes through the lens of compactness and question if the physical shape of a district indicates that gerrymandering has occurred.

### Part One: Contextualization

In Appendix A, we provide questions to help students build foundational knowledge of gerrymandering. If students have already taken a course in American History or Government, they may be able to answer questions about the House of Representatives, Senate, and apportionment. If not, we suggest assigning homework as a Google form or handout with the provided guiding questions. Alternatively, the teacher may choose to give a one-period presentation with context about the branches of government and apportionment of congressional representation.

Data from the 2020 Census will inform state apportionment counts and district maps for ten years. Deadlines for finalizing these maps are determined at the state level. Many of those timelines will need to be rushed or altered given COVID-19-related delays in collecting 2020 Census data (NCSL, 2021). We hope that as a result of engaging in this unit, students will be able to watch or engage in the redistricting process with a critical eye.

## Part Two: Play the Game

### *About the Game*

*Mapmaker:* The Gerrymandering Game, shown in Figure 1, is a board game developed in 2018 by three siblings from the Lafair family who live in a gerrymandered district in Austin, TX. Each student plays as one of 4 political parties.

The learning goals are for students to identify the number of votes needed for them to win a district and for students to think critically about simultaneously spending their opponent's votes while minimizing the use of their own. Students strategize about where to place their boundary pieces to win as many districts as possible for their party. They discover the four methods of gerrymandering outlined below and make observations about proportionate and disproportionate representation. Students play this game in groups of 3 or 4, either for one or two 45 minute periods. After students play the game, teachers may help students analyze the game collectively by taking pictures of each completed game board and analyzing the shapes and votes per district with the class.

**Figure 1**

*Mapmaker Game Board (Lafair, 2018)*


The *Mapmaker* board is divided into 74 hexagons. There are 40 circular chips in 4 colors to represent four political parties. Before the game begins, randomly place equal numbers of circular chips on the hexagons. Each color has ten chips numbered 1-10 to represent the number of voters who reside in that geographical hexagon. The players take turns drawing boundaries along the existing sides of hexagons to win the most districts for their party. Districts can have between 4 and 7 hexagons. The player with the greatest number of votes within the boundary wins the district. As a scaffold before playing MapMaker, teachers may select several online (e.g., gametheorytest.com/gerry/) or paper options with small arrays of squares in two colors.

### *Playing the Game in the Geometry Classroom*

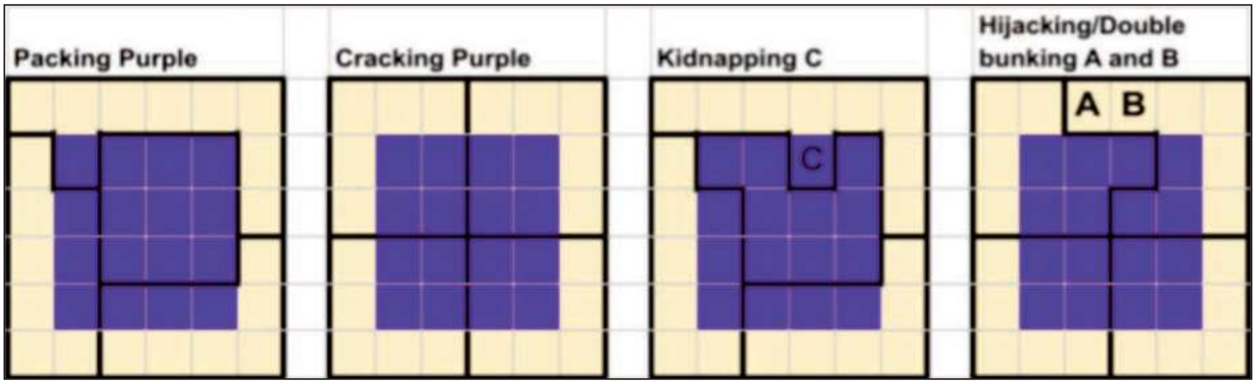
While the game itself is designed for each player to fight for their party, we recommend that students play the game a few times with varying objectives. In one iteration, players try to make their party win the most districts. In another, all players attempt to maximize the districts for one party. By playing this game, students will discover the following four ways that a party's power can be diluted.

- *Packing* is defined as pushing all of the opposite parties' votes into one district, thus minimizing the number of districts they win.
- *Cracking* is slicing up opponent votes into districts already won by your party, thus rendering those votes meaningless. (Daryl et al., 2020) Essential to the cracking strategy is the identification of a bare minimum number of votes your party needs to claim the district. Once those votes are used, no further votes for your party should be spent in that district.
- *Kidnapping* is when boundaries are drawn to move the home of an incumbent into another district where they are unlikely to win.
- *Hijacking* or double-bunking occurs when boundaries are drawn to place two incumbents in the same district so that one must lose, thus losing a seat for that party (Caltech, 2008).

These four practices are illustrated in Figure 2 (next page). For a helpful audiovisual tool that demonstrates these four concepts, see Propublica's *The Redistricting Song* on YouTube (2011). The teacher may walk around

**Figure 2**

*Four Gerrymandering Strategies*



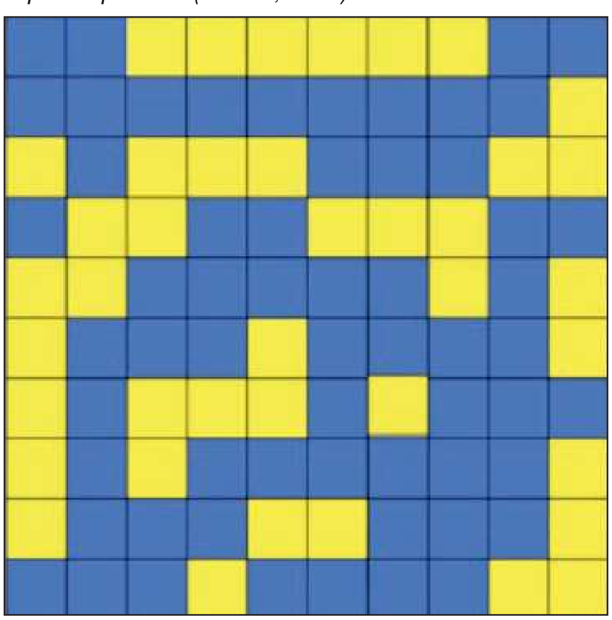

to identify instances where students are implementing these strategies. After the game, invite students to share their strategies and then name them.

### Part Three: Squaretopia

This activity was created by the Metric Geometry and Gerrymandering Group (MGGG, 2017). Figure 3 shows a ten-by-ten grid that represents a fictional state, Squaretopia. Imagine a state with 100 voters, 60 voting for the blue party and 40 voting for the yellow party. The legislature of this state decides how to create ten districts with equal numbers of voters per district. In our classrooms, students create a district map for each of the four criteria.

**Figure 3**

*Squaretopia Grid (MGGG, 2017)*



1. Create ten compact districts. We define "compact" in this context as the district being as closely packed as possible, minimizing the distance across in any direction. Compactness is discussed in depth in Part Four.
2. Create ten proportional districts. This means to create districts such that the number of districts won by each party is proportional to that party's share of the general vote.
3. Gerrymander for the blue party. There are 60 blue squares in Squaretopia. Each district could theoretically have six blue votes, assuming the physical constraints allow. Try to make the blue party win as many districts as possible.
4. Gerrymander for the yellow party. The yellow party gets 40 votes, so it could win six districts with at least six yellow votes in each of those districts. The learning goals for Squaretopia are as follows:
   - Students define and create contiguous districts. All blocks in one district are connected to the next by at least one full side of a square.
   - Students will persevere in problem solving. Creating districts that meet given criteria will require many drafts. Through this process, students learned the importance of approaching problems without having the final solution in mind.
   - They will utilize proportionality as a way to measure fairness. In Squaretopia, 40% of the voters are in the yellow party, and 60% the blue party. Therefore when students created their proportional maps with ten districts, four were yellow, and six were blue.

- Students will identify the number of votes needed to win a district.
- Students will utilize packing and cracking practices to maximize the desired party's number of districts and minimize the other party's number of districts.
- They will discuss if a district is compact or not compact using the vocabulary of area and perimeter.

Students discovered many ways to create these ten districts. The blue party can win anywhere from 3 districts to all ten districts, depending on how these districts are created. There are 60 blue votes, and a minimum of 6 votes per district is needed to secure a majority. Theoretically, there are enough blue votes for the blue party to win all ten districts. Similarly, the 40 yellow votes are enough for yellow to win up to 6 districts. It is neither fair for the blue party to win all ten districts nor fair for the yellow party to win six districts. Students consider what they think is a fair amount of districts for each party to win. Should the number of districts won by each party be proportional to the total number of votes that each party received? Or, is fairness achieved by making compact districts? We explore these questions in the activities.

### *Student Work*

We use a ten-by-ten grid of blue and yellow squares cut into ten districts of ten squares each. Districts with ten squares can result in a 5 to 5 tie. Thus, instructors may consider using 9x10 or 7x10 grids with an odd number of either 9 or 7 squares per district, which cannot result in a tie. The students had several options for creating their district maps. Ideas include using a Google drawing annotated with color labels or district numbers, drawing district boundaries using a tablet or written annotations or highlights on a color or grayscale handout. There are likely many other ways, and we look forward to exploring them in the future. The resulting maps consisted of ten individual shapes. Some students benefit from scaffolds to help visualize the individual shapes. For example, individual districts may be physically cut apart. Or, in a digital version of the activity, one can grey out all districts except one to draw focus.

In Squaretopia, students were able to explain and demonstrate packing and cracking. When discussing their grid of yellow and blue voters (Figure 4), a student wrote, "There is a way for the yellow to win. The trick is to minimize the blue; to use them up when making the groups. There are about two to three groups that are made out of 100% blue tiles. Then the other groups are made of mainly six yellows and four blues. This is how you can make a minority a majority." This student successfully cracked the blue party by finding the number of votes needed for yellow to win, which was 6; the number needed for blue to win; finding the remainder in the district, 4; and then spending four yellow votes in as many districts as possible, in this case, Districts 1, 3, 5, 7, 8 and 10. They also packed blue into Districts 4, 6, and 9.

Another student noted, "This is how you make yellow win: You pack four districts, all full of blue. The blue is winning the four districts by 100%. If you pack 40 blue into four districts, you have 20 blue left and 40 yellow. You use 40 yellow to spread out those remaining districts, and the remaining blue is just the minority over the yellow." Because there were 40 yellow voters and six needed to win a district, the maximum number of districts that yellow could win was six, since 40/6 = 6 with a remainder of 4. This, in turn, meant there would be a minimum of four blue districts. The student described packing blues into those four districts.

In general, student maps that are gerrymandered will yield very non-compact districts. Figure 5 (next page) is a student's map of a gerrymander for the yellow party.

**Figure 4**

*Student Work Sample #1*

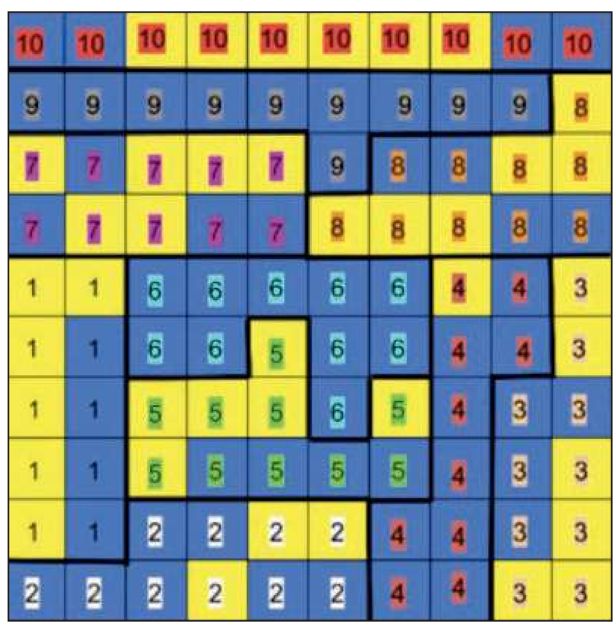
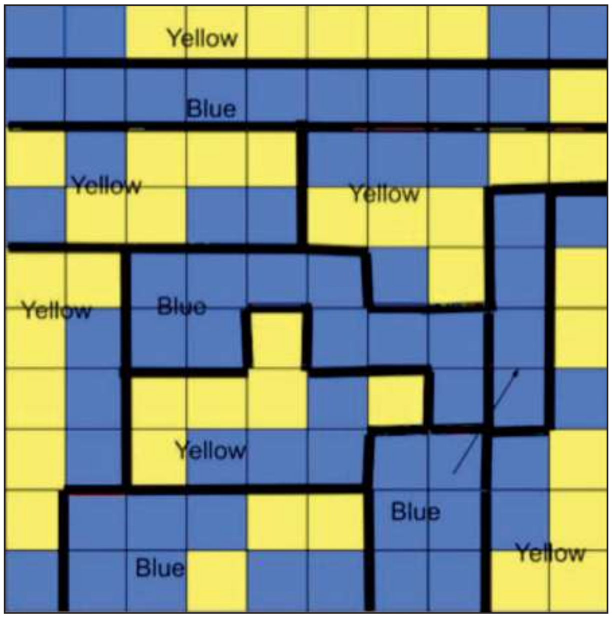


**Figure 5**

*Student Work Sample #2*




They note in an end-of-unit project that:

> Congress is causing this problem by gerrymandering each district state map so that the minority political party wins over the state and they get more seats in the House of Representatives. In the imaginary state called Squaretopia, blue is supposed to win because it makes up 60% of the state. My blue districts are really weirdly shaped because my objective was to make yellow win. Once I secured the win for the yellow, I just started grouping the remaining squares into districts. Congress does the same, and that is how gerrymandering happens.

This brings us to a key question of this unit: How can we best measure fairness? As we show in the next section, one possible metric is compactness.

### Part Four: Calculating and Analyzing Compactness Metrics

In this section, the compactness metrics were presented to students, who then computed them for the districts they created in their Squaretopia maps. Our learning goals here are the following.

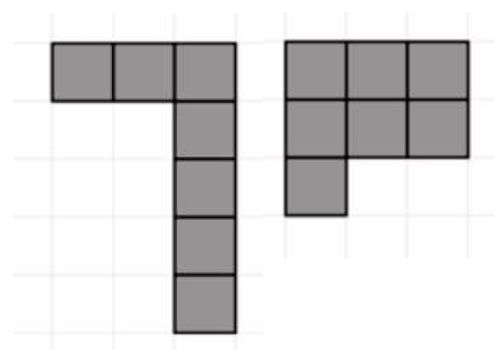
- Students will appropriately identify geometric features (area, perimeter, length, width) and evaluate compactness metrics in terms of these features.
- Students will discuss both abstractly and using numeric values what each metric is capturing. For example, the Skew Metric captures only a ratio of length to width, while Square Reock illustrates how close a shape is to being a square. (These metrics are explained at length in Appendix B.)
- Students will review each metric critically to analyze how accurately each metric captures compactness and discuss this analysis with peers and in writing.

Here are two cases where we see the Supreme Court directly reference compactness: "I know it when I see it" Supreme Court Karcher v. Daggett (1983) and "bizarrely shaped and far from compact" Supreme Court Bush v. Vera (1996).

In the previous section, we discussed the four Squaretopia maps which students created. The most unfair map was arguably the gerrymandered map for the yellow party since the party with the fewest overall votes should not win the majority of the districts. Students considered whether it was possible to tell by looking at the shape of the districts that the map is unfair. We introduced the concept of "compactness." Like the Supreme Court, students expressed that the gerrymandered maps have districts that look less compact. But what does that mean? Figure 6 shows an opening activity that we created for students to formulate their thoughts on what it means for a shape to be compact.

**Figure 6**

*Classroom Activity*



Argue that one of these districts is more "compact" than the other. Try to use specific geometric ideas to make your argument.

Argue that one of these districts is more "compact" than the other. Try to use specific geometric ideas to make your argument.

The following statements represent ways in which students intuitively defined compactness:

- They compare the length to the width.
- They identify the amount of "free space" and state that greater amounts of free space mean a shape is less compact.
- "The right one is two boxes away from being a square while the one on the left is eight boxes away from being a rectangle."
- "They both have seven boxes and three columns. There are less rows on the right, so it is more compact."

*Compactness Metrics.*

We now transition from students' intuitive arguments about compactness to the metrics that have been defined and used in gerrymandering legal cases. Over 24 compactness metrics have been used in the literature about gerrymandering. The metrics Skew, Isoperimetric, Square Reock, and Convex Hull are based on actual compactness metrics among the most widely used (Barnes & Solomon, 2020). These metrics were adapted for classroom use by MGGG (2017). In Appendix B, there are definitions of each metric. Each metric is designed to give higher values to shapes that are more compact and are in the form of a ratio comparing the area to the perimeter. The range of scores is (0, 1] with 1 being perfectly compact. The most compact shape is a circle, meaning that a circle has the most area for the least perimeter and minimizes the longest distance across the shape. The districts in Squaretopia have boundaries along the edges of a square grid, and thus the most compact district on the grid is a square. These compactness metrics have been altered to reward square shapes.

Students gather data for each of the four Squaretopia maps they created in Part Three: compact, proportional, gerrymander for blue, and gerrymander for yellow. The chart shown in Figure 7 provides a simple template of the information the students will gather.

The teacher may choose to provide four copies of this chart, have students choose their own organizational structure, or provide spreadsheets with built-in formulae to help ease some of the tedious number crunching. For a class on $n$ students, this results in $40n$ values for each compactness metric which can be analyzed as a class. Having this information for each of the four maps can lead to a rich analysis of which features of the shape each metric is capturing.

Students then find evidence of gerrymandering. They had already computed one or all of the compactness metrics for a proposed map. Now, they look for deviations from the ideal result of 1.0. The map with metrics that are overall closer to 1.0 can be argued to be the least gerrymandered. We can see these justifications made by a student calculating and comparing Isoperimetric and Square Reock measures in Figure 8 (next page). They stated:

> Compactness metrics are mathematical methods used to distinguish if the shape is closer to being a square or not. A shape is considered compact when closer to a square. After getting the results for shape 1 (0.62) and shape 2 (0.81), we can say that shape two is more compact because shape 2 is closer to 1.0 than shape 1. Since shape 1 was an example taken from the gerrymandered map, we can easily say that if one district in that map isn't compact, then neither are the rest. Using the Square Reock method as well, we have another proof that shape two is more compact than shape one because it is still closer to 1.0.

**Figure 7**

*Collecting Compactness Data*

| District | Skew | Isoperimetric | Square Reock | Convex Hull |
|---|---|---|---|---|
| 1<br>2<br>–<br>–<br>–<br>10 | | | | |
| Average | | | | |

**Figure 8**

*Student Work Sample #3*

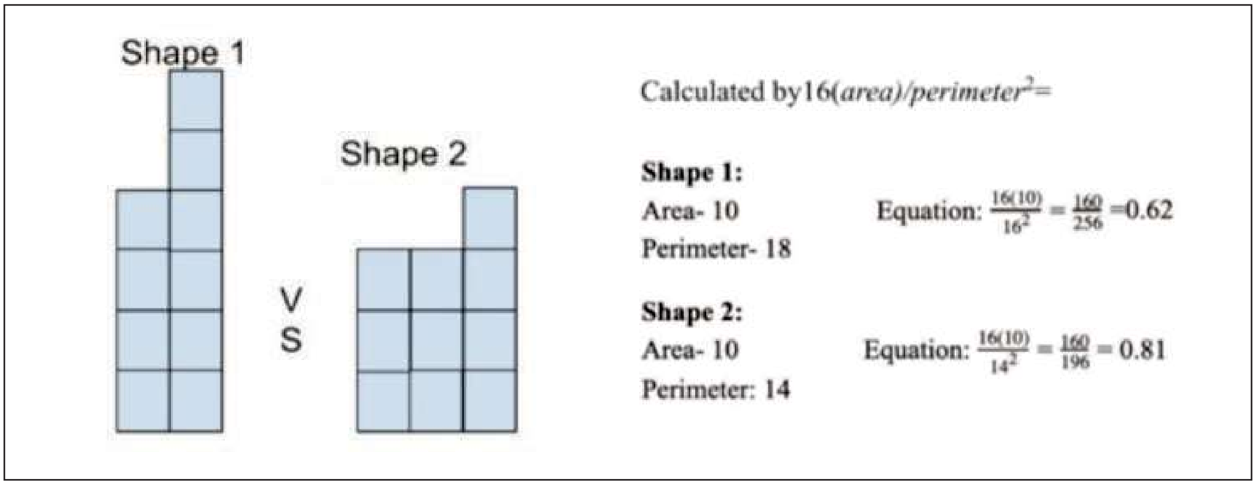


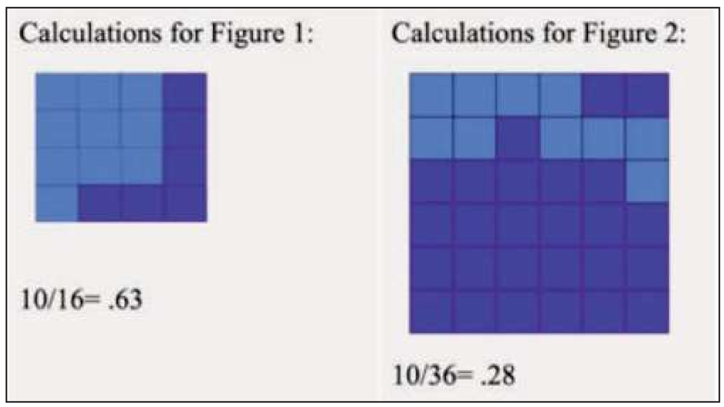

Figure 9 illustrates another student's analysis using Square Reock.

They shared that:

> The formula A/S (A = Area and S = Area of the smallest square containing district) Figure 1 is also proven to be more compact with the Square Reock method because Figure 1 is 63% compact and Figure 2 is 28% compact. This method really shows how gerrymandered Figure 2 is because it's only 28% compact.

**Figure 9**

*Student Work Sample #4*



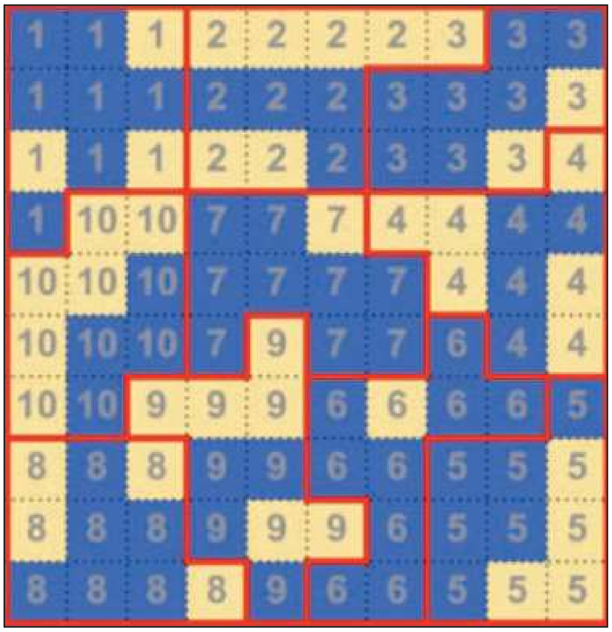

There is no communally agreed-upon way to prove gerrymandering (Barnes & Solomon, 2020). This realization came from the students' exploration and discussion with each other. We provided students with the opportunity to calculate the compactness metrics. Then, we facilitated a critical discussion about which measurement best proves gerrymandering, using the data from the chart in Figure 7. Students identified districts they had purposefully packed and cracked and then named the compactness metric that was furthest from 1.0 for those districts. Results were compared to come to a consensus about the best compactness metric. It is key to note here that consensus may not be achieved, and if it is, the decision about the best compactness metric is likely to be different for every class of students. Our true objective is the critical discussion taking place.

Using compactness metrics to argue about their Squaretopia maps, students were able to use prior knowledge about area and perimeter to explore a real-world topic. We saw in Figures 8 and 9 that gerrymandered districts are likely to be non-compact. However, the inverse is not true. Non-compactness does not imply gerrymandering, despite the use of this argument in the public gerrymandering conversation. We can see in Figure 10 (next page) that a proportional map is not necessarily compact.

This map may appear to have evidence of *hijacking* and *kidnapping* (in which an incumbent's address is carved out of their district or two incumbents are pushed into the same district). In Districts 6 and 9, we see concavity and lasso-like boundaries that could support this theory. But in reality, the boundaries of this map were drawn to result in proportional representation. This leaves us with the question: If compactness metrics cannot definitively prove gerrymandering, what other tools can we use? This is a question still to be answered.

**Figure 10**

*Student Work Sample #5*




## Conclusion and Student Reflections

Our goals for this unit are that students (a) discuss how to quantify fairness, (b) analyze a formula and evaluate whether it captures what it claims to capture, and (c) discuss compactness and features of districts both generally, by using mathematical vocabulary, and specifically, by computing numeric calculations. Through their written arguments, students quantified fairness. Students made the connection between disproportionality and unfairness, pointing out that a party with 40% of the vote should not get 60% of the representation. For further exploration, we suggest a lesson about the Efficiency Gap, which compares the number of wasted votes for each party (See Bernstein & Duchin, 2017). We saw students identify their choice for the strongest compactness metric in the materials presented here by seeing which metric deviated most from 1.0 for the gerrymandered districts they had created. In future classroom iterations, we hope to invite students to develop their own methods to measure compactness that can be applied to their Squaretopia maps. We are curious to find out if the methods developed by students might be quite similar to some of the accepted compactness metrics. If students are engaged in creating the metric, they will also be thinking about how to best capture the minimization of the perimeter for a given area. Having authored a formula themselves, they will be better equipped to critique existing formulas. In the short classroom activity in Figure 6, students used area and perimeter generally and numerically by pointing out how some shapes used lots of extra perimeter for the same area. We believe this activity could be expanded to allow students to define such a metric themselves.

We hope that when new district maps based on 2020 Census data are presented in the media, our students will be able to engage in an informed discussion about them. Because of our unit, students can observe that gerrymandering leads to a disproportionate partisan split. They can look at maps, identify compactness and discuss implications. As math teachers, we are accustomed to our students having math anxiety or assuming that class content will not be used outside of the classroom. A student reflected, "Usually when I learn math, the questions that go through my head constantly are 'When will I ever use this in real life?' and 'How does this relate to anything in the real world?'" Math-phobia extends all the way to the Supreme Court. There have been several cases where the highest court has pushed back against mathematics presented in cases. In response to mathematical presentations on gerrymandering, Chief Justice Roberts said, "It may be simply my educational background, but I can only describe it as sociological gobbledygook." Justice Breyer asked, "Is there a way of reducing it to something that's manageable?" (Roeder, 2017, para. 8, 14). Our purpose here is to unpack the 'gobbledygook' and to provide students with opportunities to reason and think critically about problems that affect all of us. As a student wrote, "We got to bring what's going on in the real world into the classroom. It was pretty interesting that we could use math to solve problems that are going on around the country." Our students engaged in dialogue about the workings of our government and utilized quantitative reasoning to identify unfair representation. The following quotations were chosen to highlight students' reflections about engaging with this unit:

> "You can have a state that is 35% blue, yet gerrymandering can find a way to make blue the winning party... it rigs elections and destroys the idea of democracy, by citizens not getting what they asked for."

> "I learned a lot about how it affects me and the offices that speak up for me… I now truly understand how district maps are made and how the House of Representatives works. I didn't know that the number of seats each state is given depends on the population."

"I think it will help me a lot in my life because this is a real-life problem that affects all of us, meaning the minorities. It was great to learn about this, to be aware of what is happening out there, and to join other people that are trying to stop this from happening because it is not fair."

"You have no idea how many times I've explained gerrymandering to people since I got to college [...] you'd be genuinely surprised how many people are completely unaware of gerrymandering and how greatly it affects our political climate."

Teaching gerrymandering allows students to reason through mathematical concepts of area, perimeter, and proportionality within a meaningful and relevant context of equity and representation. We have not yet solved the problem of gerrymandering. We need more mathematical thinkers to be a part of the conversation and contribute to creating fair representation in our government. By creating, analyzing, and critiquing Squaretopia maps, students learn that math is not about getting the right answer but using mathematical tools to communicate and make arguments. This process allows students to better understand what mathematics is and creates opportunities for using math as a tool in a myriad of ways.

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## Appendix A

### Guiding Questions for Part One: Contextualization

1. What is a district? What types of districts exist in the United States for federal, state or local government?
2. In terms of population, which are big states, and which are small?
3. How should each state be represented in the federal government? Should we give the same number of representatives to each state or should the number depend on the population of the state?
4. In the House, where the number of representatives is determined by population, how many should each state get?
5. How many total seats are there in the House? Is this number fixed? Has it always been?
6. What is the partisan split of the state?
7. How should the number of Democratic and Republication representatives relate to the partisan split of the state?
8. What is the population density of each district and state and how is this shown in maps?

## Appendix B

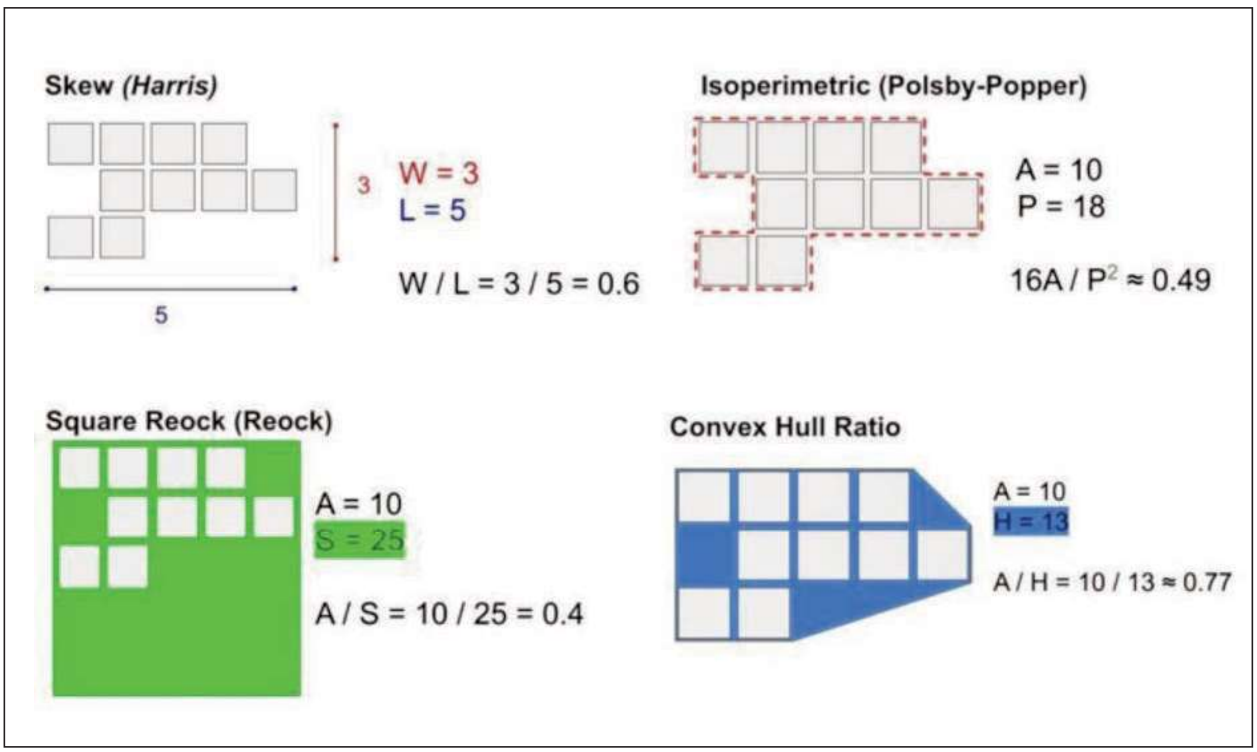
### Compactness Metrics

The diagrams in Figure 11 were created by Bernstein, a founding member of the Metric Geometry and Gerrymandering Group. These metrics were introduced to us at the MGGG conference (2017).

- **Skew**
  **W / L, where W = shorter dimension, L = longer dimension**
  - Adapted from Harris, in which L = longest axis, W = greatest width perpendicular to that axis
  - It can be argued that this measurement rewards maps where districts are as compact as possible. However, there are some tricks, like X or L shapes.
- **Isoperimetric**
  **$16A / P^2$, where A = area, P = perimeter**
  - Adapted from Polsby-Popper: $4\pi A / P^2$
  - Consider a circle. We know the circle is the shape which provides the maximum area for a given perimeter. Duchin points out, "plumper things have more area" (MGGG, 2017). The Polsby-Popper metric establishes the circle as the most efficient shape.

Figure 11

*Compactness Metrics, Bernstein, 2017*



- $4\pi A / P^2 = 4(r^2) / (2r)^2 = 1$
- In Squaretopia, we are constricted by right angles, and thus our Isoperimetric calculation is adapted to demonstrate a square as the most efficient shape.
- $16A / P^2 = 16 \times side^2 / (4 \times side)^2 = 1$

- **Square Reock**
  **A / S, where A = area, S = area of smallest square containing district**
  - Adapted from Reock: A / C, where A = area, C = area of smallest circle containing district
  - Like Isoperimetric and Polsby-Popper, this measurement rewards maps where districts are as close to the most efficient shape as possible.

- **Convex Hull**
  **A / H, where A = area, H = area of convex hull**
  - When imagining the convex hull for a shape, it may be helpful to use physical or virtual geoboards (a plastic grid with pegs placed at each intersection of vertical and horizontal lines) and have students snap rubber bands around the district.
  - This measurement rewards maps where districts have minimized divots and pivots that usually accompany the practices of kidnapping and hijacking.