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Assessment of a Problem Posing Task in a Jamaican Grade Four Mathematics Classroom

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ABSTRACT This paper analyzes how a teacher of mathematics used problem posing in the assessment of the cognitive development of 26 students at the grade-four level. The students, ages 8 to 10 years, were from a rural elementary school in western Jamaica. Using a picture as a prompt, students were asked to generate three arithmetic problems and to offer their solution to one of these problems. The teacher used both formative and summative assessments to evaluate students' levels of cognition in mathematics. Formative assessment revealed that some students' mathematical ability were higher than their reading ability. Summative assessment showed that more advanced students generated more complex questions. The teacher was able to use the information gathered from the problem posing activity to create better lesson plans for the class.

KEYWORDS *problem posing, assessment, open-ended problems, fourth grade students*

Introduction

Problem posing is a high-level cognitive activity that has recently gained popularity in the field of mathematics education. This strategy is seen as being able to foster higher order thinking skills (Cankoy, 2014) and to develop better problem solving skills in students (Brown & Walter, 2014). The problem poser can develop a problem based on the solution of a previous problem or from personal experience. Problem posing in mathematics education has been researched in many countries including the US, Australia, Japan, UK and Singapore (Cankoy, 2014; Nohda, 1999; Silver, 1994; Singer & Voica, 2013). While previous researchers have looked at problem posing as a way of enhancing students' problem solving skills (Kar, Özdemir, İpek, & Albayrak, 2010; Silver, 1997), Singer and Voica (2013) have considered problem posing as a generative activity from which information about the students' levels of mathematical

thinking, competencies, and areas of weakness can be drawn.

A fourth grade teacher in Jamaica decided to use problem posing as an assessment tool to obtain information on her students' abilities in mathematics. The aim was to gather information from these assessments to enhance lesson plan and delivery. The teacher carried out summative assessment by comparing questions generated by students with her prepared rubric of expected responses. Also, by supporting each student during the problem posing activity the teacher was able to formatively assess what level of support each child needed.

In this paper, I analyze the process undertaken by the teacher to assess students' mathematical ability levels using the problem posing activity. First, I report on the planning and implementation of the lesson; second, on the teacher's reflection on the process; and third, on the high and low points of the entire activity.

Literature Review

Problem posing can be defined as creating a new problem from a given problem (Cai & Hwang, 2002) or from a given situation or experience (Kar, Özdemir, İpek, & Albayrak, 2010). Many studies have shown the positive effects of problem posing on the learning of mathematics. Silver (1997) opines that problem posing can serve as a means of enhancing creativity in students. Singer (2009) encourages problem posing for developing mathematical aptitude and increasing the ability to transfer knowledge. Brown and Walter (2005) recommend problem posing as a means of assisting students to develop better problem solving skills. Singer and Voica (2013) argue that problem posing per se can be used to determine students' comprehension levels in mathematical operations and understanding. To this effect, problem posing can be thought of as both a teaching and an assessment tool.

Assessments using the problem posing method are better at providing the teacher with detailed information on students' understanding of mathematical concepts than assessments using closed questions, such as multiple choice and one-solution questions. Advocates for problem posing, such as Pehkonen (1997) and Singer and Voica (2013), argue that problem posing allows students to interact with multiple problems, methods, and solutions simultaneously. This interaction is thought to more greatly increase the possibility of developing creativity in students. Here, the teacher is provided with important insights into children's understanding of mathematical concepts. The teacher, acting as a facilitator, guides students to improve on their strengths and to identify those areas in which they are weak (Lobato, Clarke, & Ellis, 2005). Assessing students' strengths and weaknesses can be a difficult task even in problem posing activities. Due to the breadth and depth of problem posing, only a vague general assessment criteria can be given; however, Silver and Cai (2005) state that the teacher should decide on how to assess students by considering how the problem posing task is related to the objective of the lesson.

Background of Terminology

Many theories have linked assessment of problem posing with problem solving and have considered the assessment of both to be somewhat similar in nature.

Summative Assessment

Summative assessment, also referred to as *assessment of learning*, is used to provide grades for the students and to ensure accountability on the part of the teacher. This is conducted after a unit of study, at the end of a school year, or at prescribed periods throughout the education cycle. The summative assessment of an open-ended question is more detailed and more complex than that of a closed ended one. There are several ways to evaluate students' responses to problem posing tasks. Becker and Shimada (1997) suggested the three-part criteria of *fluency*, *flexibility* and *originality* to assess open-ended problems. Similar criteria can be adopted to assess problem posing tasks due to their open-ended nature. Silver and Cai (2005) identified *quantity*, *originality* and *complexity* as the three criteria that may be used to assess problem posing tasks. The summative assessment used in this paper was adopted from Silver and Cai (2005) with comparison to the Becker and Shimada (1997) criteria.

Quantity. Similar to fluency (Becker and Shimada, 1997), quantity (Silver and Cai, 2015) is obtained by counting the number of correct responses generated by the student. For example, in the lesson under review, each mathematical question written by a student was counted as a correct response and received one point. Quantity can provide information on the students' levels of mathematical and creative thinking because students generate problems within their level of mathematical capability (Nohda, 1999).

Originality. Originality refers to uniqueness or insightfulness of ideas generated by the students. Originality measures quality of the mathematical thought, creativity, and level of critical thinking. Examples of questions that may be considered in the originality category of this prompt are those with words such as dozen, or those with fractions or decimals. Becker and Shimada (1997) state that the teacher should give a high score for originality as it requires cleverness and high mental ability to create something new or insightful. Silver and Cai (2005) added that originality is not always present in students' responses as this is more difficult for students to attain.

Complexity. Mathematical complexity refers to the cognitive demands of the generated question. For the lesson under review, mathematical complexity was assessed based on the number and type of arithmetic operations in the proposed question. According to Hembree (1992), better problem solvers can create different approaches

and more complex problems. In the activity presented here, questions that require the use of subtraction or division were considered to be more difficult to generate than questions requiring addition or multiplication.

Formative Assessment

Formative assessment is *assessment for learning*; it is conducted during the teaching and learning process with the purpose of evaluating the student's level of understanding so that the teacher can adapt or modify instruction to better engage the student in learning. The focus of formative assessment is not to administer a grade but to help the student in developing conceptual understanding and mathematical thinking skills. Formative assessment, in general, has been shown to be a particularly effective pedagogical strategy, as it helps the teacher to determine the student's level of mathematical thinking and conceptual understanding and to modify teaching strategies to better support students in learning (Black, 2010).

Method

The participants were 26 grade-four students, ages 8 to 10 years, in an elementary school in rural Jamaica. Students were from low socio-economic background and had different reading abilities. The teacher had been teaching for over 13 years and had a Diploma in Teaching. In Jamaica, qualified teachers may hold a three-year teachers' college diploma or a four-year university degree. The latter is equivalent to the four-year degree in the United States.

The lesson presented in this article is one of four problem posing lessons given by a teacher over a four-week period. The task, given in the form of a picture prompt, is shown in Figure 1. The rubric used for formative assessment was aligned to that suggested by Silver and Cai (2005) in which students' responses were assessed under the criteria of *quantity* and *complexity*. The researcher observed the process of planning and implementation of the lesson and conducted an interview with the teacher after the lesson. The problem posing task was introduced to students as follows: "Write three mathematics problems using the information given in the picture and solve one of the problems you have written."

Assessment Criteria

The teacher created a rubric with possible questions students could pose and classified them according to the

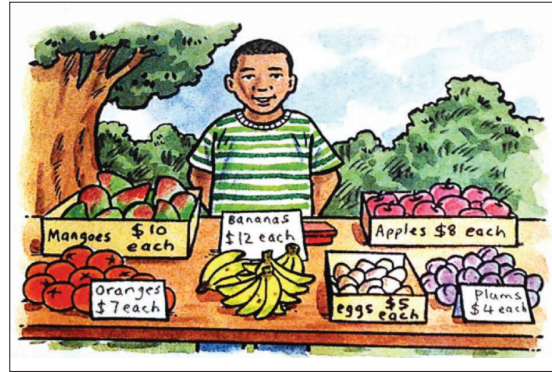


Figure 1. Picture given to students during problem posing activity. (Adopted from Trotman & Severin, 2005)

type and number of arithmetic operations that they required. The categories were labeled using the arithmetic verbs—*Add*, *Subtract*, *Multiply* and *Divide*—as well as the possible combinations of these. Mathematical complexity was judged based on the number of operations needed to obtain a correct solution. Calculations requiring more operations were deemed to be of higher complexity and hence awarded a higher score. A question such as "If one banana costs \$12, how much will 5 bananas cost?" was classified in the *Multiply* category. A question such as "A girl bought 5 mangoes and 2 oranges from \$100, how much does she have left?" was placed in the *Multiply, Add and Subtract* category. Student-generated questions were categorized as mathematical questions or statements. The mathematical questions, given in either words or diagrams, were compared to the created rubric, and points were awarded accordingly. Students were not penalized for the order of the operations nor for the method used to solve the problem unless it was mathematically incorrect. For example, consider the question "If one banana costs \$12 what is the cost of 5 bananas?" In this case, both solutions $\$12 + \$12 + \$12 + \$12 + \$12 = \60 and $5 \times \$12 = \60 were deemed to be acceptable.

Results

Teacher's Analysis of Student-Generated Responses

The teacher's analysis of student-generated responses is presented in two sections. The first provides a summary of students' problem posing responses, and the second provides information on formative assessment strategies that the teacher employed during the lesson.

Quantity: Number of student-generated responses.

More than 80% of the students were able to generate 3 or more problems. The class produced a total of 77 responses, 72 of which were classified as solvable mathematical questions. Table 1 shows the number of student-generated questions in each category.

Table 1 shows that more than 44% of the generated questions were about multiplication only, and 34% were about addition only. These were simple questions that most students could solve mentally or by writing. Four questions (5.5%) required subtraction only. Eleven (15%) of the generated questions had two or more operations. Only 5 of these 11 questions required subtraction.

Complexity. All students were able to generate at least 1 question (see Table 2). Twenty-two students were able to generate 3 questions. Ten students generated all 3 questions in the same category. Thirteen students generated questions in 2 categories, and 3 students had questions in 3 categories. Questions generated by students focused on calculations with 3 items or less.

Table 2 shows that most students posed at least 1 question in the *Add* (15 students) or in the *Multiply* (13 students) category. The two students who posed questions in the *Multiply, Add and Subtract* category also posed questions in the *Subtract* and *Add* categories. All other students generated questions in 1 or 2 categories.

The information in Table 2 suggests that students were more comfortable posing questions about addition and multiplication. Fewer students produced questions that required subtraction. This probably means that students had more difficulty with subtraction. Most students did not combine 2 or more operations in 1 question. Questions with 2 or more operations were considered to be more complex and required greater calculations. Singer and Voica (2013) suggest that students of higher intellect create more complex problems. Failure to create complex mathematics questions could indicate a low mathematical processing level. That is, students may have created only questions that they were able to solve themselves.

Formatively assessing students with problem posing and problem solving.

Black (2010) stated that teachers should use students' solutions on summative tests to assist them in providing meaningful support to students. In an effort to harness more information on students' level of cognition, the teacher ranked students from Level 1 to 5 based on the type of problems they created. The ranking system shown in Table 3 was constructed by the teacher and recorded in her evaluation notes.

Table 3 shows that 35% of students were placed at Level 3. Questions generated by these students focused on only 1 mathematical operation. A total of 8 students

Table 1
Student Solutions in Each Category

Category	Add	Subtract	Multiply	Multiply and Add	Multiply and Subtract	Multiply, Add and Subtract	Total
Number of Responses	25	4	32	6	3	2	72

Table 2
Number of Students Who Generated Questions in Each Category

Category	Example	Number of Students	Category	Example	Number of Students
Add	What is the cost of a banana, an apple and a mango?	15	Multiply and Add	What is the cost of 2 bananas and 3 mangoes?	6
Subtract	I have \$50. I bought 1 orange. What is my change?	4	Multiply and Subtract	What is my change if I bought 7 apples from \$100?	3
Multiply	If 1 banana costs \$12, what is the cost of 5 bananas?	13	Multiply, Add and Subtract	I have \$100. I bought 2 bananas, 3 apples and 4 eggs. What is my change?	2

Table 3

Mathematical Level Ascribed to Students

Level	Number of Students	Example of Generated Question	Reason
1	3	See Figure 2.	Student drew pictures showing one type of fruit.
2	6	See Figure 2.	Student drew pictures showing more than one type of fruit.
3	9	If one banana costs \$12, how much will 5 cost?	Student was able to write questions with one mathematical operation.
4	6	What is the cost of 2 eggs and 5 apples?	Student generated at least one question with two operations.
5	2	I have \$100. I bought 2 apples, and 4 bananas. How much change should I get?	Student was able to write at least one question with three operations.

(31%) were placed at Levels 4 and 5; these students generated at least one question with 2 or 3 operations, respectively. The 9 students at Levels 1 and 2 used diagrams to represent their questions. Examples of these questions are shown in Figure 2.

Based on students' response to previous tests, the students at Levels 1 and 2 were initially placed at the same level due to their poor reading ability. However, during classroom activities, the teacher realized that some students focused on only 1 item (i.e., type of fruit) for each question while others had more than 1 item in the same question. The teacher asked students to verbalize their questions so as to match the verbal statement with the respective diagram. Using formative assessment, the teacher placed students who combined 2 or more items in their diagram on a higher level than those who

focused on only 1 item. Table 3 is a modified version of a previous table created by the teacher.

Teacher's Actions

Preparation. Conducting a problem posing lesson for the first time, the teacher researched the topic and had discussions with colleagues about possible questions that could be used for the lesson. The teacher thought it best to use a question from the grade-three textbook, as students were in their second month of grade four. The problem was deliberately chosen because it reflected an everyday activity and was flexible enough to challenge both fast and slow learners in the class. The teacher wrote possible responses students could create and categorized them in a rubric. The teacher also wrote down different types of questions and strategies she

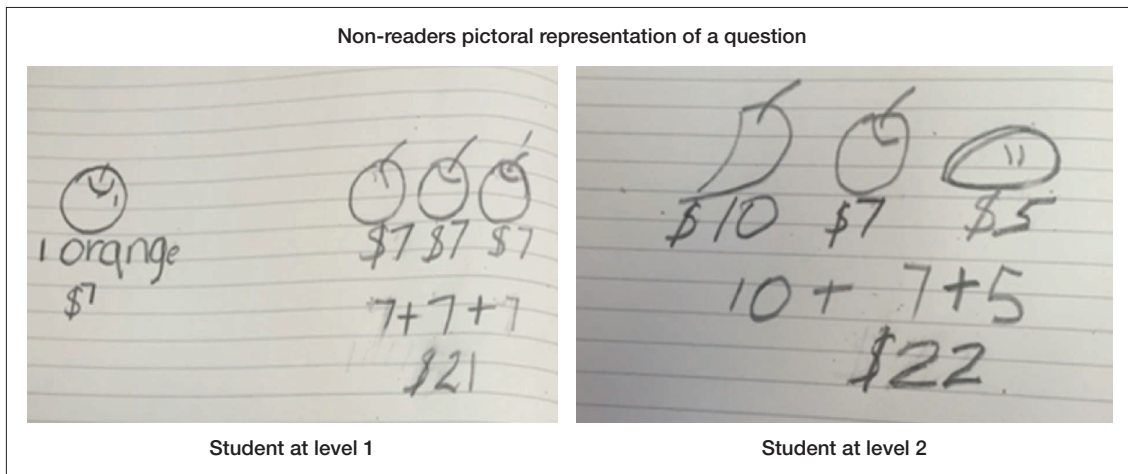


Figure 2. Responses of students assigned to Levels 1 and 2.

could use to guide students as they thought about, wrote, and solved their questions in the class.

Implementation. The teacher created a food stand with fruits and egg-shaped objects inside the classroom (similar to the one in Figure 1). She asked different students to purchase items from her makeshift shop. The teacher did not discuss the answers to the students' questions; she wrote the prompt on the board and asked the class to read it. Selected students were asked to give an interpretation of what they thought the prompt was asking them to do.

Being satisfied that students understood the problem, the teacher allowed them to generate their own questions. The teacher occasionally looked at her lesson plan and the list of questions for guidance. She verified students' statements by repeating or rephrasing them. She encouraged students to use different operation(s) for each question, intermittently initiated pair and group discussion, and challenged students to solve their questions in more than one way so as to verify their answers. The teacher walked among the desks and carefully observed students at work. She provided feedback by explicitly informing students about what should be corrected. For example, a student wrote "a boy selling fruits" as his response, and the teacher advised him to write a question that uses adding, subtracting, multiplying, or dividing. Another student wrote a question about the cost of 5 bananas, but solved it by adding 12 and 5. The teacher took the manipulatives to the student's desk and asked her to use the manipulative to show her solution. The student was asked to give the cost of 1 banana, 2 bananas, 3 bananas, 4 bananas, and 5 bananas. The teacher then asked this student to show her question on the board during presentation of work. While most students suggested $\$12 \times 5 = \60 as the answer, a student at Level 2 presented the solution $\$12 + \$12 + \$12 + \$12 + \$12 = \60 . This question was used to reinforce the relationship between addition and multiplication. At the end of the lesson, the teacher collected students' notebooks for summative assessment.

Analysis and Discussion

Some students had difficulty understanding the problem even though it represented a common daily activity of buying and selling. This could indicate students' failure to relate mathematical activities in the classroom to daily life experiences or that the teacher was wrong in thinking that the question was easy enough for each student

to understand. Asking some students to explain the prompt in their own words helped others to understand the problem. This was a good contingency plan on the part of the teacher.

Using numerous ways to elicit responses from the students and observing their reactions and struggles during seated work helped the teacher to understand the students' mathematical proficiency. Moving about the class and assisting individual students provided the teacher with valid information on students' mathematical understanding. In the interview the teacher stated that she evaluated students during this time to confirm or refute the level at which she had initially placed them. The teacher provided guidance according to the ability of the students and took mental notes on the level of support each student needed. She also observed each student's level of participation in class discussion. The teacher commented that the discussion about $12 \times 5 = 12 + 12 + 12 + 12 + 12$ revealed to her that reading level and mathematical thinking level are different. Students with low reading level do not always operate at a similar level in mathematics. The teacher also voiced her regret in mistakenly telling two students how to solve a question. In one situation the student needed the answer for 8×5 in order to add it to 35. The teacher told the student, " 8×5 is 40." In the other situation, the student was subtracting 7 from 50 vertically, but placed the 7 under the 5. The teacher told him that the 7 should be in the ones column under the zero. In the interview, she explained that she should have given students counters and a place value chart, respectively, and guided them to the solution rather than simply telling them what the answer was. Due to the fact that this was a student-centered lesson, the need for the teacher to periodically look at the lesson plan did not affect the flow of the lesson. There were two instances, however, where there were long pauses (more than 30 sec) as the teacher checked her notes for the appropriate question to ask. These long pauses could have caused some students to lose their focus on the discussion.

The teacher commented that summative assessment was not as tedious as she previously thought. This could be due to the extensive preparation for the lesson and the resulting detailed rubric. From among the students who created more than 3 questions, the best 3 questions were used in her analysis. In the teacher's view, no student generated questions worthy of points for originality. In her rubric, questions in the category of *Originality* were those that required thinking beyond the four basic operations. Examples of these questions as

pre-recorded by the teacher were: "Which would cost more, buying 4 oranges or buying 4 apples?" or "What is the cost of $\frac{1}{2}$ dozen eggs?" These questions required the use of comparisons and fractions, respectively. No student generated questions of this nature. In the interview, the teacher said that she believed the marking scheme for originality may have been too difficult for the class. She also had second thoughts about whether or not she was able to guide students to think deeply about the concepts so that they could produce unique questions during the lesson. Setting high standards and guiding students towards achieving these standards is better than lowering the standards to the perceived cognitive level of the students. However, these standards should be realistic and attainable for students.

The findings show that most students (69%) were only able to generate questions with 1 operation. This could indicate (1) students' reluctance or (2) inability to write complex questions.

- (1) Students may have been unwilling to write elaborate and difficult questions as they were required to solve the questions themselves. Students may have written more difficult questions if their classmates or someone else were required to solve them. As for the cases where students were unable to correctly solve their own question, they may have thought that they were capable of solving the question or trusted the teacher to assist in solving. This rare case was observed among three members of the class.
- (2) Another reason that most students only generated questions with 1 operation could be the inability of students to write more complex questions. This would suggest that students wrote questions based on their proficiency in mathematics. Ellerton (1986) corroborates this correlation between the number of operations in students' posed problems and their mathematical abilities. The higher the students' mathematical abilities, the more operations they are able to manage in a problem. Following this line of argument, the teacher was correct in using students' generated questions to assess their proficiency.

It was observed that most students repeated the same type of question with different items. For example, all three questions generated by one student were about the cost of different amounts of bananas. Another student asked for the cost of 2 bananas in the first question, the cost of 3 apples in the second question, and the cost of 4

mangoes in the third question. These students appeared to generate their second and third responses by using the first response as a cue. Silver (1994) suggested that this type of problem posing by students reflects a lower level of understanding. Students with higher mathematical ability are able to plan and produce more difficult problems. This was also reflected in students' diagrams; some students focused on 1 item while others were able to combine 2 or more items in a question. The students who produced the most complex questions generated 3 questions in 3 different categories. For example, the 3 questions of one of the top students were:

- (1) What is the cost of 3 oranges and 4 apples?
- (2) How much change should I get if I buy 4 oranges from \$50?
- (3) I have \$100. I bought 2 apples and 4 bananas. How much change should I get?

This may suggest that students with higher mathematical ability are able to see more relationships among mathematical concepts and are better able at refocusing their thinking to consider multiple solutions paths than students with lower mathematical ability.

Conclusions

A well-formulated rubric made it easier for the teacher to carry out a summative assessment of the problem posing task. While a good rubric reduced the tediousness of summative assessment, formative assessment proved to be more difficult than anticipated. The teacher had difficulty in creating a balance between guiding students towards an answer and actually helping them to solve the problem. Additionally, the need to refer to notes during the lesson had the potential of negatively affecting the flow of the lesson and hence its outcome. Overall, the task helped the teacher to better understand students' mathematical ability.

The findings of this study contributed to the understanding of the correlation between students' problem posing and their mathematical knowledge. Problem posing helps teachers to learn more about their students' mathematical understanding and increases teachers' experience with formative assessment. Problem posing provides teachers with the opportunity to get insights into students' understanding of mathematical concepts and to pinpoint misconceptions within students' reasoning. Formative assessment of problem posing provides

information for designing and delivering more effective lessons. Through problem posing, teachers can gather more knowledge on students' understanding and create better lesson plans to facilitate learning.

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