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# Teaching the History of Tracking Time with Technology

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**ABSTRACT** This article focuses on the mathematical value of teaching angles through the use of sundials in the classroom. The history of sundials and the mathematics embedded within them is discussed in detail. In addition, practical applications of angles are included, along with interactive practice telling time with the angles created on sundials. Time as a whole is explored from historical, mathematical and technological perspectives. Further, connections to middle and secondary grade mathematics, practical applications, and technological tools that aid student understanding are discussed. Teaching the history of tracking time through the study of sundials provides a rich learning experience for students in the middle grades; they are able to see mathematical concepts, such as angles applied to the real world, as well as learn new history about a topic they most likely are not very familiar with.

**KEYWORDS** *middle school, math, technology, angles, sundials, history*

## Introduction

The concepts of time and mathematics are inextricably linked. Segments of time—weeks, months, days, hours, minutes, seconds—are described through ratios and proportions. Time is also periodic—night, day, and the seasons repeat in predictable ways. In this paper, we discuss the value of teaching the history of tracking time using technology, focusing on the sundial and its connection to middle grades mathematics. We explore time from historical, mathematical, and technological perspectives. We highlight key discoveries throughout the history of tracking time with emphasis on the sundial. Furthermore, we discuss connections to middle and secondary grades mathematics along with practical applications and technological tools that aid student understanding.

**Mathematical Connections between Sundials and Angles**

*Figure 1.* Three-dimensional sundial (self photographed).

## History

Sundials are the earliest known tool for measuring time. As the sun rises in the east and sets in the west, it creates shadows on the sundial that can be used to determine the time of day. There are many different types of sundials. The most common is a flat plane with a gnomon attached, which is a vertical post that creates a shadow on the plane (Klein & Riskin, 2003). An example is illustrated in Figure 2.

Although the exact discovery of sundials is unknown, the oldest civilizations could have told time simply by "observing the natural world, perhaps by wedging sticks in the ground and monitoring the movements of the shadows" (Toothman, 2009, p.1). As civilizations and mathematics advanced, the knowledge of *conic sections* led to the discovery of the first known sundials. A conic section is the intersection of a plane and a cone, where the angle of the intersection of the plane will create different shapes when changed. The discovery of conic sections occurred in the fourth century B.C. and is attributed to the Greek mathematician, Menaechmus (Dolan, 1972). Knowledge of conic sections probably hastened the invention of sundials because of the resemblance between certain conic sections and the paths of the shadows created by the sun's rays when hitting the gnomon.

*Figure 2.* Sundial with gnomon (History of Watches, 2016).

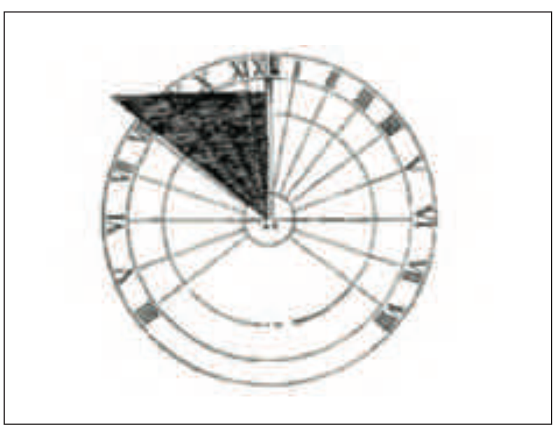

*Figure 3.* 45 degree angle shown on a sundial (Bryant, 1999)

Throughout history people have used many types of sundials, from a gnomon on a plane to an intricate 3D circle, like the one depicted in Figure 1. This specific paper will focus on sundials created from a flat plane with numbers and a gnomon that stands vertically, which are simpler to read. Figure 3 shows an angle created by the sun hitting the gnomon that is 45 degrees. We can then look at the Roman numeral that is being touched by the shadow and see that it is VIII, or 8 o'clock. Learning the history of sundials helps students to understand the use of mathematics in tracking time and how it has evolved into what we know today.

## Mathematics of Sundials

### Connection to Standards

The concept of time is one that students encounter every day. Students begin to read an analog clock in first grade. Based on the Common Core Standards, first graders should be able to "tell and write time in hours and half hours using analog and digital clocks" (CCSSI, 2010, 1.MD.B.3). Time is a familiar concept for students because they use it to track their days. The concept of time and clocks can also be addressed in mathematics when students learn about angles through the study of clock hands and shadows. Angles are a new concept for students in the early grades. To help their students grasp the idea of an angle, teachers can connect angles concretely to everyday objects. According to the Common Core, 4th graders should be able to "recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement" (CCSSI, 2010, 4.MD.C.5). Additionally, Common Core State Standards in 7th grade and in high school mathematics include knowledge and application of angles. It is crucial that students develop a solid base understanding of these concepts, as they build upon them throughout their schooling.

### Real World Application of Angles

Where are angles seen in students' everyday life?

*Figure 4.* Roof of a house (Types, 2015); Piece of Pizza (Frozen, n.d.); Corner of a football field (A corner, n.d.)

One of the most common places they probably see angles is in the hands on a clock.


*Figure 5.* Analog clock (Istock, n.d.)

Students can start to practice identifying and measuring angles using an analogue clock, as this is more familiar than a sundial. Students should be able to measure angles created by the two hands with a protractor. In addition, they should be able to identify the number of degrees in between every hour on a clock as 30° (Pagni, 2005).

### Connection between Analog Clocks and Sundials

Not only can students use an analogue clock to measure angles, you can challenge students by introducing sundials to them as the original way that people tracked time. Students should first take time to notice that the numbers on a sundial are different from numbers on a clock. The numbers generally used on a sundial are called Roman numerals. Students may need to be reminded how to read Roman numerals. They may also notice that the numbers do not go all the way around the circle, but rather start at 4 and end at 8. A discussion about why this is the case would be beneficial for students to understand that, since sundials need the sun and its shadow to operate, the numbers do not include the times at night when the sun is not out. Students can then be asked to calculate the angle measures on a sundial. For example, they could be asked for the angle measure made by this shadow and the time of day.

*Figure 6.* Sundial casting shadow on a specific time (Fort, n.d.)

They will be able to see that at 6:00 pm at night, a 90° angle is made (Figure 6). Introducing or discussing benchmark angles with students would be beneficial as well because, "by thinking about the amount and direction of rotation in terms of the benchmark angles of 90 degrees, 180 degrees, and 270 degrees, students can estimate other angle measures" (Olson et al., 2008, p. 26).

### Interactive Practice

Now try this for yourself! What is the time and shadow depicted on the sundial in Figure 7? *(Answer below picture.)*

Answer: 50 degrees, just before 8:30 am

*Figure 7.* Sundial casting shadow on a specific time (Fort, n.d.)

Now compare the difference in the angle made on an analogue clock at 5:00 pm and the angle made on a sundial at 5:00 pm. How and why are these angles different?

*Figure 8.* Analog clock (Istock, n.d.)

*Figure 9.* Sundial casting shadow (Fort, n.d.)

**a.**

| Time | Angle |
|---|---|
| 4 | 130 |
| 5 | 110 |
| 6 | 90 |
| 7 | 70 |
| 8 | 50 |
| 9 | 36 |
| 10 | 22 |
| 11 | 8 |
| 12 | 0 |
| 1 | 8 |
| 2 | 22 |
| 3 | 36 |
| 4 | 50 |
| 5 | 70 |
| 6 | 90 |
| 7 | 110 |
| 8 | 130 |

**b.**

| Time | Angle |
|---|---|
| 4 | 130 |
| 5 | 110 |
| 6 | 90 |
| 7 | 70 |
| 8 | 50 |
| 9 | 36 |
| 10 | 22 |
| 11 | 8 |
| 12 | 0 |
| 13 | 8 |
| 14 | 22 |
| 15 | 36 |
| 16 | 50 |
| 17 | 70 |
| 18 | 90 |
| 19 | 110 |
| 20 | 130 |

*Figure 10.*
**a.** Time related to angle shown on sundial (self photographed).
**b.** Military time related to angle shown on sundial (self photographed).

Identifying angles on analog clocks and sundials gives students a deeper understanding of how angles have been used in different ways throughout history to track time.

### Fitting a Function to the Sundial

Creating a function that relates the time of day to the angle shown on the sundial provides older students with an interesting challenge. To construct such a function, we began by creating a table of values with times and corresponding angles, as shown in Figure 10(a). Since no shadows are cast without daylight, the table starts at 4:00 am and ends at 8:00 pm.

After some initial exploration, it becomes evident that the data in Figure 10(a) are difficult to analyze since several individual times are mapped to more than one angle measure, yielding a non-function. For this reason, we decided to change the independent variable to military time. A revised table is shown in Figure 10(b).

With the revised table, we entered the data into Geogebra and generated a graph as shown in Figure 11.

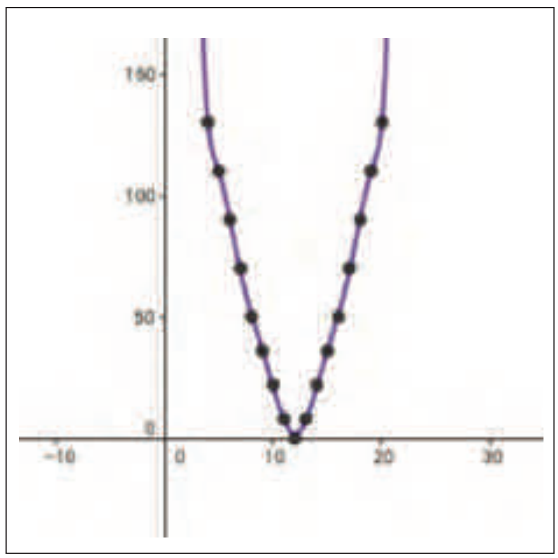


*Figure 11.* Graph of time and angles created on sundial (self created).

From this, we determined an approximate fit function from the data set, namely, f(x) = 0.01$x$^8 – 0.23x^7 + 4.37$x$^6 – 56.04$x$^5 + 482.24 $x$^4 – 2745.67$x$^3 + 9884.1$x$^2 – 20313.66$x$ + 18243.96. With Geogebra, we used a high-degree polynomial to fit a function to the data. Weirstrass's Polynomial Approximation Theorem states that, "Any continuous function on a closed and bounded interval can be uniformly approximated on that interval by polynomials to any degree of accuracy" (Weisstein, 1988). So although the function is not a perfect fit, Weirstrass's

Polynomial Approximation notes that a high enough degree polynomial will fit any data with arbitrarily close accuracy. This theorem allowed us to create our very own function to calculate the angle shown on a sundial depending on the time of day. This challenge allows educators and students to think more in depth about the mathematics aligned with the sundial.

## Technology

Examining sundials outside and in action is exciting; however, opportunities for such adventures are limited thanks to the sun's turtle-like speed and the vagaries of the weather. Technology can give us great resources to teach students how to read sundials without everyday dilemmas. One website that allows students to use an online sundial is http://www.fortpittblockhouse.com/sundial/. Ask students to follow this link and the image in Figure 12 will appear on their screen.


*Figure 12.* Sundial casting shadow on a specific time (Fort, n.d.)

Prior knowledge needed for this activity is how to read Roman numerals and that the sun rises in the east and sets in the west. In the bottom right hand corner of this screen, we see the sun at 'Dawn.' Ask students what angle the shadow is creating at dawn and what time it is. They should tell you that it is creating a 135-degree angle and it is 4:30 am. Then ask students to tap on the right-hand side of the screen to create a 40-degree angle, and ask what time that would be. They should tell you that it is 9:00 am (Figure 13).

This website allows you to drag the sun all the way up the right side and down the left side to resemble sunrise to sunset, providing many times/angles to play around with and answer questions.

Another resource available is an app that can be downloaded on an iPhone or iPad. This app is free in the app store and is called "Pocket Sundial Lite" (Figure 14). It allows you to take your device outside and lay it on the ground, facing either north or south, and it will move to the time of day. This is a great app for advanced students to use when they are finished with the other activity to further their understanding.


*Figure 13.* Sundial casting shadow on a specific time (Fort, n.d.)


*Figure 14.* Pocket Sundial Lite App (Pocket, 2011).

## Conclusion

Overall, teaching the history of tracking time through the study of sundials provides a rich learning experience for students in the middle grades. They are able to see mathematical concepts, such as angles applied to the real world, as well as learn new history about a topic they most likely are not very familiar with. The use of technology can be easily integrated into this lesson, which adds yet another way for students to interact with this topic. Teachers interested in future research could look into how the original sundial was created, in addition to how students can create their own sundial in the classroom. While creating their own sundial, students will have to learn which way it should face, the angle of the gnomon, and where to place the Roman numerals.

In conclusion, engaging students through an exploration of the mathematics, history and technology of tracking time will provide a meaningful and beneficial learning experience.

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