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Some Thoughts on Doctoral Preparation in Mathematics Education

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Arguments for significantly improving doctoral programs have long been made, both nationally (Bender, 1997; Jackson, 1996; Golde & Walker, 2006; Walker, et al., 2007) and internationally (Cumming, 2010). The nature and variety of doctoral programs makes it difficult to single out specific changes that would be equally applicable to every discipline-specific doctoral program. Therefore, this commentary will focus on doctoral programs and doctoral preparation in mathematics education.

I have been involved in an evolving doctoral program in mathematics education at a research institution for over one-half of a century. During the last 20 years I have been involved in organizing two national conferences on doctoral programs in mathematics education (Reys & Kilpatrick, 2001; Reys & Dossey, 2008) and doing research focused on doctoral preparation in mathematics education. This experience has helped identify some unanswered questions, often raising dilemmas and challenges associated with doctoral preparation in mathematics education. This commentary provides an opportunity to share some thoughts and hopefully promote a dialogue among those interested in or directly involved with preparing doctorates in mathematics education.

What is the nature of doctoral programs in mathematics education?

Doctoral programs in mathematics education in the United States take many different forms (McIntosh & Crosswhite, 1973; Reys et al., 2001; Reys, et al., 2007) and

in general, remain unspecified to the larger education community. That is, program elements are unique to particular programs and there is little that can be said to be “standard” across different institutions. There is no certifying agency for doctoral programs in mathematics education nor are their minimum national requirements that institutions must meet. Each institution has governance over the pathways toward awarding an earned doctorate in mathematics education.

How are earned doctorates in mathematics education identified nationally over time?

The first doctorate in mathematics education was awarded over a hundred years ago at Teachers College, Columbia University (Donogue, 2001). It was patterned after the PhD in mathematics except the doctoral research was focused on teaching and/or learning mathematics. Since then many institutions have initiated doctoral programs in mathematics education (Reys & Reys, 2016).

Identifying earned doctorates in mathematics education is as challenging as determining the nature of doctoral programs. The *Survey of Earned Doctorates* (SED) began tracking information from institutions in the United States in 1920. Mathematics education as a field of study was first introduced by the SED in 1962 (first collected data specific to mathematics education spans the period July 1, 1961 to June 30, 1962). Every earned doctoral graduate from an institution of higher education in the United States is asked to complete a survey (see <http://www.nsf.gov/statistics/srvydoctorates/#qs>).

One question asks the graduate to, “choose the code that best describes the primary field of your dissertation research.” In education there are more than 20 different codes. The code for Mathematics Education is 874. Other codes include Curriculum & Instruction (800), Elementary Teacher Education (852), and Secondary Teacher Education (856). Consequently, there may be some doctoral graduates from mathematics education programs that select other categories (e.g., “Curriculum & Instructions” or “Secondary Education”) as their field of research. If so, these people are not included as doctorates in “Mathematics Education” in the SED data. Additionally, there may be doctoral graduates in areas such as Educational Psychology (618) and Educational Administration (805) whose dissertation research focuses on teaching/learning mathematics and therefore these graduates may identify Mathematics Education as their field of dissertation research. If so, these graduates are reported as doctorates in mathematics education. Despite its limitations, the SED provides the most reliable data set for doctorates in mathematics education that has been collected over time from all institutions in the United States that award doctoral degrees.

Should there be “core knowledge” for doctoral graduates in mathematics education?

If someone is awarded a doctorate in mathematics, then completion of courses in advanced calculus and analysis is a certainty. Is there any core knowledge (or common coursework) for people completing a doctorate in mathematics education in the USA? Participants of the two earlier mentioned national conferences (Reys & Kilpatrick, 2001; Reys & Dossey, 2008) agreed on two things. First, there was not any specific core knowledge that all doctoral graduates in mathematics education had acquired, and second there was a need for the articulation of at least some core knowledge for doctorates in mathematics education. The Association of Mathematics Teacher Educators (AMTE) appointed a task force to address this issue and produced a report, *Principles to guide the design and implementation of doctoral programs in mathematics education* (AMTE 2003). This report represented an important first step in establishing core knowledge for mathematics educators. Since 2003 there has been increasing attention nationally for more interdisciplinary research collaboration, particularly with regard to STEM fields. This attention together with the fact that over a decade has passed since the initial release of the *Principles*, suggest this *Principles* document warrants a thorough review and updating.

How much graduate level mathematics coursework is required for doctorates in mathematics education?

Historically doctorates in mathematics education took graduate courses in mathematics as part of their degree. As mentioned earlier, the initial doctoral program in mathematics education at Teachers College had a strong foundation in mathematics, but their research focused on mathematics learning, teaching, and curriculum rather than pure or applied mathematics (Donogue, 2001). In the 1960s and 1970s the major portion of graduate coursework for doctorates in mathematics education was still mathematics (McIntosh & Crosswhite, 1973). Since then some recommendations for the minimum amount of mathematics coursework for doctoral graduates in mathematics education have been made (Dossey & Lappan, 2001). However, the specific coursework as well as the intensity of graduate level coursework in mathematics varies greatly and has decreased significantly for most doctoral programs in mathematics education, with some doctorates in mathematics education having completed no graduate level courses in mathematics (Reys, et al., 2001).

It is worth noting that if doctorates in mathematics education want to be employed in mathematics departments in institutions of higher education, then they need to have a substantial amount of graduate level mathematics, typically at least the equivalent of a master’s degree in mathematics. Therefore, doctoral graduates in mathematics education with an extensive amount of graduate level mathematics coursework have greater job opportunities because they are employable in either a mathematics department of college/school of education.

Should doctoral programs in mathematics education focus on preparing researchers or teacher educators or both?

All doctoral graduates engage in research that leads to their doctoral dissertation. However, once their degree is completed the career path may not lead them to engage in new research. About 15 percent of doctoral graduates assume educational positions related to PreK-12 education, in school districts, or as regional/state mathematics supervisors (Glasgow, 2000). Responsibilities of these positions include keeping up with research findings and then filtering, using, and disseminating relevant findings. They typically do not include spearheading new research studies. About 40 percent of the doctoral graduates in mathematics education are employed in

community colleges and four-year institutions that are characterized by heavy teaching loads (Glasgow, 2000). While some of these faculty members may choose to do research, their primary responsibility is teaching and/or teacher education.

About 40 percent of the doctoral graduates in mathematics in the U.S. are employed in doctoral granting institutions (Shih, Reys, & Engledowl, 2016). Faculty members in these institutions are expected to establish and maintain a strong research agenda in order to be promoted and tenured. While these faculty members will also be teaching when tenure and promotion decisions are made, a solid record of research productivity typically trumps an outstanding record of teaching.

Since less than one-half of doctoral graduates in mathematics education are employed in research institutions, it raises questions about how research should be weighted in a doctoral program. Achieving a proper foundational balance between research and teaching has been a constant challenge for doctoral programs in all disciplines (Golde & Walker, 2006; Walker et al., 2007). James (1903) discussed this dilemma, and argued that too much doctoral preparation was devoted to preparing researchers, and too little attention was given to preparing teachers even though for many doctoral graduates they would have major commitments to teaching. One solution to this dilemma is for institutions with doctoral programs to identify a focus, (i.e., research, college teaching of mathematics, mathematics curriculum) and design their doctoral experiences to reflect that focus (Hiebert et al., 2008). This would allow doctoral students interested in a particular focus to choose institutions aligned with their interest and to acquire in-depth knowledge and engage in research related to specific areas of expertise. Some institutions, such as the University of Northern Colorado, have chosen to identify a specific niche (in their case, preparing collegiate teachers of mathematics) in the mathematics education doctoral program. Other institutions might gain national acclaim by carving out specific foci for their doctoral programs in mathematics education and making their foci clear to potential doctoral students.

Should there be national certification/ accreditation of doctoral programs in mathematics education?

I believe the answer is a definite YES, and here is why. Institutions differ greatly in the number of mathematics educators on their faculty, the number of full-time doctoral students, and the institutional resources made

available to support a doctoral program in mathematics education (Reys, et al., 2007). As a result, some institutions have shaped their doctoral programs to reflect some of the *Principles to guide the design and implementation of doctoral programs in mathematics education*; however, many institutions remain unaware of the existence of the report or have been unable to make progress toward alignment to the recommendations.

According to the SED data from 2000-2014 there were 180 different institutions in the United States that graduated at least one doctorate in mathematics education (NSF, 2016). Further examination revealed that over one-half of these institutions awarded a total of five or less doctorates in mathematics education during this 15-year period, and nearly 40% of the institutions awarded two or less doctorates in mathematics education (Reys & Reys, 2016). Levine's 2007 comment is particularly appropriate to this situation. He said, "our country has too many under-resourced doctoral programs for the preparation of education scholars" (p. 60).

In the spirit of full disclosure, I have served as an external reviewer for a number of doctoral programs in mathematics education. In every case, these institutions were seeking information about the health and direction of their doctoral program, and they allocated sufficient budget to provide travel expenses and honorarium for the review team members. Such a regular external program review should be a standard for all doctoral programs in mathematics education.

External review site visits are typically preceded by self-assessments of the program by faculty members and feedback from current doctoral students and doctoral graduates in mathematics education. This self-assessment, while ostensibly prepared for the review team, becomes particularly valuable to the faculty members of the institution as they reflect on ways to strengthen their doctoral program. During the site visit, the review team members engage in conversations with faculty members and current doctoral students as well as deans and provosts. Each visit culminates in a debriefing with program leaders and administrators, and follow-up written summaries identifying strengths and weaknesses of the doctoral program. Although standard criteria could have facilitated these reviews, they didn't exist; as a result, the content and format of the reviews varies. Nevertheless, in every instance I have received feedback from the institution about the worth of the external review and how the review process led to improving their doctoral program in mathematics education. The self-examination process provided valuable information to the faculty members, and that information together with the entire

external review process helped mathematics education become more visible to the institutional administrators. Strong programs were strengthened, and marginal programs used the external review to acquire more resources (sometimes more faculty members) to better support the doctoral program in mathematics education.

How might certification/accreditation of doctoral programs get started?

Naysayers argue that it can't be done or it will harm the uniqueness of doctoral programs. I argue that it will help insure some core knowledge for doctoral graduates in mathematics education and raise the overall quality of doctoral programs in mathematics education. It has been done by other disciplines, such as music education (See <http://nasm.arts-accredit.org/index.jsp?page=Standards-Handbook>.) If certification/accreditation of doctoral programs in mathematics education is to happen, then details about specific criteria to be used in the certification process for doctoral programs in mathematics education will need to be established. Care will need to be taken to insure that the best interests of a wide range of diverse programs are represented. Faculty members actively involved in a range of doctoral programs in mathematics will need to shape the certification process, and a professional organization such as AMTE would be a natural leader for the effort. If not AMTE, then perhaps a new organization—Association for Certification of Doctoral Programs in Mathematics Education (ACDPME) will be established. While it will take time to get a national certification program organized and operating, in my judgment the result will have long term benefits for doctoral programs and ultimately the doctoral graduates from these programs. I believe that national certification/accreditation of doctoral programs in mathematics education will raise the overall quality of doctoral programs in mathematics education.

Closing

A goal of our doctoral preparation should be to develop future stewards of our discipline of mathematics education. Hopefully, this Commentary will stimulate discussion of doctoral preparation in the U.S. and throughout the international mathematics education community. The doctoral graduates in mathematics education today represent the future of mathematics education, so whatever can be done to strengthen their doctoral program preparation in every institution should be a high priority.

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