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A Reconceptualized Framework for 'Opportunity to Learn' in School Mathematics

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ABSTRACT We present a reconceptualized framework for opportunity to learn (OTL) in school mathematics that builds on previous conceptualizations of OTL and includes features related to both quantity (i.e., time) and quality. Our framework draws on existing literature and on our own observational research of mathematics teaching practices. Through the presentation of two teaching vignettes, we highlight key features of mathematics instruction that seem to differentiate students' OTL. These features are nested under four key dimensions in our OTL framework: the *teacher's mathematical knowledge for teaching (MKT),* how *time* is utilized in the lesson, the mathematical *tasks* implemented with students, and the nature of the mathematical *talk* in which students engage. We discuss implications for the use of the framework as well as its shortcomings.

KEYWORDS *elementary, mathematics teaching, opportunity to learn*

Educational researchers often use two lenses, achievement and opportunity to learn (OTL), to examine student learning (Floden, 2007). The achievement lens focuses on student performance on assessments; the OTL lens explores the relationship between classroom experiences and student learning. Floden (2007) posits, "it seems as though, in the United States at least, the attention to student opportunity to learn (OTL) is even greater than the attention to achievement results…at least in studies of mathematics and science learning" (p. 231). This focus is likely attributed to research indicating that assessment results need to be interpreted with caution; assessment results are not always a reflection of what is happening inside classrooms due to the nature or format of the assessment, curricula, teacher characteristics, and other contextual factors (Robitaille & Garden, 1989; Scherrer, 2013). For almost fifty years, OTL has been operationalized in a variety of ways (e.g., time, alignment between assessment and instruction); therefore, there have been mixed findings about the relationship between OTL and achievement (Floden, 2007). Many researchers in mathematics education have thus turned their attention to understanding what is happening inside classrooms, typically with an emphasis on standards-based mathematics teaching (Walkowiak, Berry, Meyer, Rimm-Kaufman, & Ottmar, 2014; Berry et al., 2013; Ross, McDougall, Hogaboam-Gray, & LeSage, 2003). Standards-based mathematics teaching refers to mathematics instruction (NCTM, 2014) where conceptual underpinnings of mathematical topics are developed and students have opportunities to engage in processes and practices like problem solving and constructing viable arguments (NCTM, 2000; NGACBP & CCSSO, 2010).

While many researchers have invested time into understanding the whats and hows of standards-based mathematics teaching (e.g., McGee, Wang, & Polly, 2013), others have investigated why certain teaching practices are advantageous over others in terms of the opportunities given to students (e.g., Webb et al., 2014).

While the construct of OTL has been a long-investigated idea (Carroll, 1963; Husen, 1967; Tate, 2005), here we focus on students' learning opportunities that result in deep understandings of mathematics, what we will call *conceptual understanding* (Hiebert, 1986; Skemp, 1976).

A Brief Historical Overview of OTL

The construct of OTL has been conceptualized in several ways and has evolved over time. Carroll (1963) introduced and defined OTL as the amount of allocated time for students to learn a specific concept. Husen (1967) operationalized OTL as the overlap between what is taught to students and what is assessed on achievement tests. Wang (1998), borrowing from the work of Stevens (1993), presented OTL as a four-dimensional construct: content exposure, content coverage, content emphasis, and quality of instructional delivery. The first two dimensions in Wang's framework align with the work of Carroll (1963) and Husen (1967), respectively. Tate (2005) collapsed these first two dimensions in his OTL framework specific to mathematics, but retained content emphasis and quality of instructional delivery as two distinct dimensions. Content emphasis refers to the teacher's choice of what to teach; the teacher decides what content from the curriculum to teach and determines which skills to highlight. Quality of instructional delivery includes the teacher's pedagogical strategies and understanding of the subject matter in order to meet the students' needs.

Recent work exploring OTL in mathematics has focused on quality of instructional delivery by examining how pedagogical and/or curricular features of instruction afford or constrain students' OTL. Wijaya, van den Heuvel-Panhuizen, and Doorman (2015) concluded that a lack of context-based tasks in textbooks limits students' OTL. Other researchers have looked at teachers' implementation. In one study, researchers found links between the set-up of mathematical tasks at the beginning of the lesson and the quality of the closing discussions; students were more likely to be given opportunities to learn significant mathematics when the task's cognitive demand was not reduced during set-up (Jackson, Garrison, Wilson, Gibbons, & Shahan, 2013). Furthermore, Gresalfi, Barnes, & Cross (2012) noted how two teachers' implementations of identical curricular materials resulted in different types of learning opportunities for students engaging in the same mathematical tasks.

Similar to past researchers, we focus on students' opportunities to develop or build on conceptual understanding, but we also present a framework of key instructional features to maximize OTL. The National Council of Teachers of Mathematics (NCTM) (2014) defines conceptual understanding as "the comprehension and connection of concepts, operations, and relations" (p. 7). In our work, we assume that developing conceptual understanding is foundational to the development of procedural understanding and a fundamental component of students' OTL (e.g., NCTM, 2014; Baroody, Feil, & Johnson, 2007).

Reconceptualizing Opportunity to Learn

The existing conceptualizations of OTL (Carroll, 1963; Husen, 1967; Stevens, 1993; Wang, 1998; Tate, 2005) have provided frameworks to analyze students' experiences in formal schooling; however, the descriptions and examples do not address the *finer-grain* features of mathematics instruction (e.g., how tasks are implemented) that have significant implications for students' learning opportunities. Therefore, we argue that a reconceptualization is needed to define and describe these finer-grain features and to potentially broaden the field's understanding of OTL.

The OTL framework presented herein is grounded in two mixed-methods research studies (Walkowiak, 2010; Pinter, 2013); the qualitative components informed the framework development. The first study (Walkowiak, 2010) examined 27 lessons in 8 third-grade classrooms, and the second study (Pinter, 2013) included 30 lessons in 10 fourth-grade classrooms. The 18 teachers (17 female) in the two studies came from 13 different elementary schools in the same large, suburban school district in the mid-Atlantic region of the United States. For both studies, three or four video-recorded mathematics lessons per teacher were analyzed. Specifically, for each study, the lead researcher watched the lessons, recorded detailed notes throughout the viewing, and wrote a lesson summary for each video. Cross-lesson summaries for each teacher were written after watching the complete set of a teacher's lessons to note patterns in the teacher's instruction. Finally, when examining the data across teachers, themes were identified as similarities and differences among the lessons within each study.

To develop the OTL framework, we combined the similarities and differences from the two studies and considered all 57 video-recorded lessons. When developing, extending, or utilizing conceptual understanding was present within a lesson, we noted distinct features in the mathematics teaching practices that consequently seemed to impact students' learning opportunities. We present two composite teaching vignettes that we utilize to describe the components of our framework by high-

lighting the key differences in students' OTL. Anchored in the themes noted in the cross-study analysis, these two composite vignettes represent the instructional practices of the teachers in our studies.

Vignette #1: Ms. Lawrence

Ms. Lawrence's lesson began with a three-minute review about the names of fraction pieces from circular area models she had displayed. She asked students to name the "one-half" and the "one-third" pieces. Then, Ms. Lawrence said, "our lesson goal for today is to work with these fractional pieces but with quantities that are larger than one."

Next, she spent eight minutes reviewing the task they investigated the previous day about Jack and Jill sharing three brownies. Ms. Lawrence facilitated a brief discussion about some of their strategies. One student gave one brownie each to Jack and Jill and split the third brownie in half such that Jack and Jill each received one whole brownie and one-half of another brownie. Another student split each brownie in half so Jack and Jill each received three halves. Ms. Lawrence acknowledged the two strategies and said "Is one and one-half equal to three-halves? How do you know?" She gave three students the opportunity to explain why the quantities are equal.

> *Simon:* I'm starting with the three halves. Three halves can be thought of as one-half plus one-half plus one-half. Two of those halves are equal to 1, and there is one half left over.
>
> *Ms. Lawrence:* Can someone show this symbolically?
>
> *Laura:* (writes on the board) $\frac{3}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ so $\frac{3}{2} = 1 + \frac{1}{2}$
>
> *Ms. Lawrence:* Laura, tell me more about why you wrote $\frac{3}{2}$ equals 1 plus $\frac{1}{2}$.
>
> *Laura:* I knew $\frac{1}{2}$ plus $\frac{1}{2}$ equals 1, just like Simon said. And, then there's the extra $\frac{1}{2}$.
>
> *Ms. Lawrence:* Can someone else explain why $1\frac{1}{2}$ is equal to $\frac{3}{2}$?
>
> *Catherine:* I broke up the one and one-half instead. I thought of the 1 as one-half and one-half plus there's the extra one-half, and I knew that was three-halves total.
>
> *Ms. Lawrence:* So, Simon decomposed the three-halves, and Catherine decomposed the one and one-half, but they both showed that $1\frac{1}{2}$ is equal to $\frac{3}{2}$. Now, we are going to use our understanding of unit fractions and our fraction language to name models of quantities greater than one.

Next, Ms. Lawrence spent 10 minutes asking students to name models of mixed numbers. For example, one of her models had four rectangles cut into thirds (Figure 1). Ms. Lawrence asked how they would describe the quantity if one of the rectangles broken into thirds represented one whole. A student suggested combining the individual thirds to make another whole (Figure 2). Then, Ms. Lawrence asked if students could represent the quantity symbolically in more than one way. A student wrote $5\frac{1}{3}$ on the Smartboard and explained how she saw five wholes and an extra one-third piece in the drawing. Another student counted thirds to show 16 thirds are in the model. Ms. Lawrence was explicit in pointing out that $\frac{16}{3}$ and $5\frac{1}{3}$ are the same quantity. Ms. Lawrence completed two additional examples to round out this 10-minute segment of class.
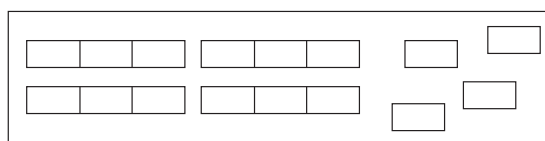

Figure 1. Ms. Lawrence's model of four rectangles (divided into thirds) and four more parts of a rectangle.
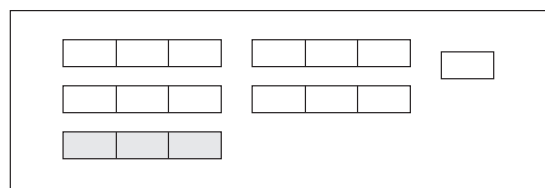

Figure 2. Ms. Lawrence's model after parts were combined to make another whole rectangle (in gray).

During the remainder of class (40 minutes), Ms. Lawrence spent five minutes explaining the task students were about to complete. Then, students spent 28 minutes with a partner matching cards that had either: a numerical form written as mixed number, numerical form written as an improper fraction, word form, or picture representation. Each match included four cards. The pictures included a variety of shapes (e.g., hearts, circles, rectangles). Partners were required to take turns explaining any matches they found. Finally, Ms. Lawrence facilitated a five-minute discussion. She asked, "how did you figure out how to represent a picture in numerical form?" and "tell me more about what you did to translate from numerical form to the picture form." Two students responded to each question, and Ms. Lawrence

probed their thinking for further explanation (e.g., "Why did you use that approach?"). Before recess, students completed one problem individually, similar to the problem in Figure 1, on a slip of paper (i.e., "exit pass"); most students finished in two minutes or less.

Vignette #2: Ms. Davis

Ms. Davis's lesson began with eight minutes of students practicing addition of two-digit numbers using the traditional U.S. algorithm, an already learned procedure for the third-graders. Some students finished quickly and were instructed to read silently while they waited. After reviewing the answers as a class, Ms. Davis read a book to the students entitled, *Full House: An Invitation to Fractions* (Dodds, 2007), for ten minutes. At the book's conclusion, Ms. Davis said, "before we move on, boys and girls, remember how we define fractions. Fractions are formed when you take something like a pizza or cake and divide it up into parts."

Ms. Davis then spent approximately three minutes reviewing the word "equivalent" and discussed "equivalent fractions." She sketched two rectangles on the board, divided one rectangle into eight parts and the second one into five parts. She emphasized that the rectangles are equal and told her students to imagine they are bread. She said, "if I eat two pieces of bread from that one (shades in two of five pieces) and I eat three pieces of bread from this one (shades in three of eight pieces), are they still equal?" There was a mix of responses in the room, both yes and no. Ms. Davis said, "They are still equal, aren't they? Because there is still this much left." She put her hands around the unshaded ⅗ and ⅝, and a student says, "I thought you meant the pieces that are gone." Ms. Davis said, "No, but the fractions are still equal, aren't they?" A few children responded yes, and Ms. Davis moved on.

The lesson segued into a whole-group activity lasting approximately 16 minutes. Students made a fraction strip (fourths on one side, sixths on the other side) and compared fractions. The students drew lines by approximating to divide the strips into fourths and sixths. Ms. Davis gave directions like "Divide your strip into six parts. Five lines, but six parts. Spread the lines out evenly." As she gave the directions, she sketched her own fraction strips as rectangles on the board. Then, she asked a series of questions related to comparing the fractions, referencing her own strips on the board. Ms. Davis said to pretend that the strips of paper were "pizza bread." She first represented ⅓ on her drawing and then asked students to draw a fraction equivalent to ⅓ using their fraction strip that was cut into sixths.

Ms. Davis: How many would you want to shade if you want this fraction (the sixths) to be equivalent to one-third? If we fill in three, would that be equal to what's shaded (points to the one part shaded on the thirds)?

Student #1: No.

Ms. Davis: How many are equal to one-third?

Student #2: Two pieces.

Ms. Davis: Okay, everyone shade in two pieces on your strip. What fraction did you just make?

Student #3: Two-sixths.

Ms. Davis: Give me a thumbs-up if you think one-third is equal to two-sixths and a thumbs-down if you think they are not equal. (Most students give her a thumbs-up, and 5 students give her a thumbs-down.) Yes, they are equal. Okay, let's do one more. Which one [of the three fractions ⅓, ²⁄₆, or ¼], say if you had pizza bread, would you want to have [assuming you want the most pizza bread]? One-third of the bread? Would you want to have two out of six pieces? Or, one-fourth of the bread? Would you want to have any of them?

Student #4: Two out of six.

Ms. Davis: Why?

Student #4: Because two is more than one

Ms. Davis: But it is the same loaf of bread, right? Aren't the fractions all the same size?

Many students: No

Ms. Davis: Yes they are, they are all the same size.

There was a two-minute transition before students worked independently on two worksheets on equivalent fractions for the remaining 18 minutes of class.

Comparing and Contrasting the Two Lessons

Ms. Lawrence and Ms. Davis offered different types of learning opportunities during their lessons on fractions. We first describe the differences between the two teachers' lessons. These differences are nested under four key teaching dimensions: mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), time utilization, mathematical tasks, and mathematical talk.

Mathematical Knowledge for Teaching.

Decisions in the lesson indicated some differences in each teacher's mathematical knowledge for teaching (MKT). MKT refers to the special type of knowledge that

teachers need in order to engage in the mathematical work of teaching such as analyzing students' strategies, probing student thinking, and utilizing appropriate and accurate mathematical representations (Ball, Thames, and Phelps, 2008). Ms. Lawrence was mathematically accurate throughout her lesson. In contrast, Ms. Davis made mathematical errors with her sketched fraction strips on the board. When the students were approximating the subdivision of their own strips (also problematic), her own sketched strips were divided into unequal parts. Consequently, it appeared one-fourth and one-third were equivalent, hence the reason she said $\frac{1}{3}$, $\frac{2}{6}$, and $\frac{1}{4}$ were the same amount of pizza bread.

Unlike Ms. Lawrence, Ms. Davis also promoted misconceptions multiple times in her lesson. When she said that "fractions are formed when you take something like a pizza or a cake and divide it up into parts," she is limiting students' conceptions of fractions to the part-whole interpretation (Lamon, 2012). For example, her definition does not consider that fractions are also formed by division (e.g., 2 cookies shared evenly among 3 people). More importantly, she disregarded that the whole can be a set of discrete objects (part-of-set model), not just a composite unit like a pizza.. Interestingly, the story book included part-of-whole and part-of-set models within it, but she did not bring attention to it in her limited "definition." Additionally, when students made their own fraction strips, Ms. Davis told them to make "five lines, but six parts" which caused students to create unequal parts, resulting in inaccurate representations and promoting the misconception that unit fractions can be unequal in size relative to the whole (e.g., one half can be bigger than the other half).

Time Utilization.
Ms. Lawrence and Ms. Davis utilized their time differently, partly because their goals varied in specificity and narrowness. Ms. Lawrence explicitly told students the learning goal after a brief three-minute review on naming fraction parts relative to whole. The remaining 58 minutes were used to focus on representing quantities larger than one with mixed numbers, improper fractions, and concrete models. In contrast, while 47 of 57 minutes of Ms. Davis's lesson were focused on fractions, the lesson goal within the topic of fractions was not clear. They read a book that used stories to represent fractions, but there was no discussion about the book. Additionally, the fraction strip activities seemed to be focused on equivalency and comparing, but the worksheets only focused on equivalency. The lesson lacked a clear mathematical goal and perhaps had multiple goals.

Unlike Ms. Davis, Ms. Lawrence utilized lesson components that built on each other through the closure when students synthesized their learning. In Ms. Lawrence's lesson, there was a clear progression in the activities: she built on background knowledge from previous lessons, chose simple tasks to build foundational knowledge, facilitated whole-group discussion and practice, provided collaborative application practice with peers, and concluded with a discussion to synthesize learning. Ms. Davis's lesson moved through activities, but it was difficult to see a thread because the goal was unclear.

Mathematical Tasks.
The tasks in both lessons could be considered more procedural in nature, especially if taught in a teacher-centered manner. Ms. Lawrence structured her task implementation by focusing on student-constructed knowledge. The initial task in the lesson was to think about sharing three brownies fairly between two people. Ms. Lawrence intentionally asked students probing questions, giving them space to explore the task, and structuring discussion so that students were building their conjectures rather than being told a specific procedure to follow. In contrast, when Ms. Davis posed questions about fraction equivalencies and comparisons, she directed every step and answered each question herself. Students were not given the opportunity to explore the task in a way that allowed their own knowledge construction.

During the tasks of the lessons, the teachers utilized mathematical representations differently. Ms. Lawrence used concrete and symbolic models in her lesson for mixed numbers and improper fractions; students translated among the representations when they matched cards and discussed the brownie problem (e.g., Laura wrote equations on board to represent Simon's thinking). Although Ms. Davis attempted to use multiple representations as part of her implemented tasks, the inaccuracies in her representations prevented students from using them for understanding and from translating among them.

Mathematical Talk.
Ms. Lawrence and Ms. Davis also utilized talk in contrasting ways in their lessons. In Ms. Lawrence's lesson, there were frequent opportunities for students to explain their thinking to the teacher or each other (e.g., Ms. Lawrence's facilitation of discussion about the brownie problem and students' explanation of card matches to partners). Ms. Davis utilized an IRE (initiate, respond, evaluate) discourse pattern with her students (Mehan,

1979) as seen in the fraction strips task. She asked direct questions that typically required one-word answers from her students.

The presence of probing questions appears to be a distinguishing feature in lessons with optimal learning opportunities. Ms. Davis missed opportunities to use probing questions. When she asked students to indicate (using their hands) whether or not they thought one-third and two-sixths were equivalent, students gave conflicting responses. Rather than following up with probing questions or asking students to share their thinking, Ms. Davis simply stated they were equal and moved forward. Ms. Lawrence, however, probes Laura about her symbolic representation of Simon's thinking, and she probes students about *why* they used their selected strategies for matching cards after they explained how they matched picture and symbolic forms.

Our OTL Framework

After considering the key differences between lessons like those of Ms. Lawrence and Ms. Davis, we built our re-conceptualized framework for OTL. Our conceptualization builds on the aforementioned OTL frameworks and includes both quantity (i.e., time) and quality. Figure 3 displays a graphic organizer of the framework. We see the teacher's mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) as a critical variable (Hill et al., 2008; Walkowiak, 2010) that is related to how a teacher utilizes time (Wenglinsky, 2004), selects and implements tasks (Stein, Smith, Henningsen, & Silver, 2009), and facilitates mathematical talk (Hill et al., 2008). Furthermore, the connecting arcs represent interactions between the dimensions because as a teacher makes decisions about time, tasks, and talk, his or her MKT could be affected. For example, a teacher can potentially deepen his/her own understanding of mathematical concepts through task selection and implementation (Wilhelm, 2014). Each of the dimensions in the reconceptualized OTL framework includes finer-grain features (Table 1) that are worded to represent the optimal enactment of the given feature. While not exhaustive, we argue that these are the *essential* features of mathematics instruction and represent starting points for analyzing instruction.
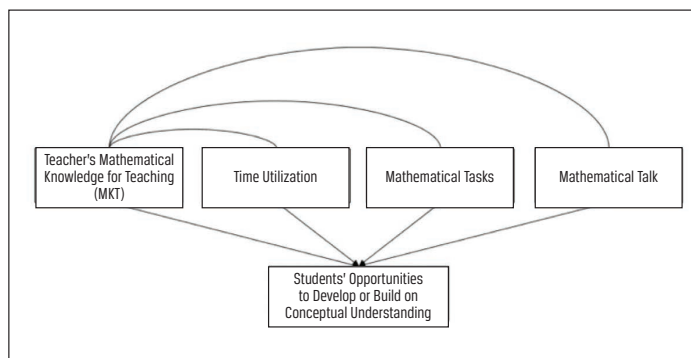
Figure 3. A reconceptualized OTL framework.

Table 1

Reconceptualized OTL Framework: Dimensions and Their Respective Features

Opportunity-to-Learn Dimension	Dimension's Features
Teacher's MKT	1. The lesson content is mathematically accurate. 2. The lesson promotes accurate conceptions among students.
Time	1. The majority of time in the lesson is used to reach the mathematical goal. 2. The time is structured so that the lesson components build on each other with explicit attention to the mathematical goal.
Tasks	1. The implementation of tasks is student-focused, allowing students to make sense of the mathematics. 2. Tasks involve the use of and translation among two or more representations.
Talk	1. Students have opportunities to explain their mathematical thinking. 2. Talk is utilized to move students toward a deeper understanding of the mathematical goal.

Teacher's Mathematical Knowledge for Teaching

The first dimension in our OTL framework is the teacher's mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008). When a teacher has a deep understanding of mathematics, they are more likely to be accurate in their mathematical language, explanations, and use of representations (Hill, Rowan, & Ball, 2005). Further, they are less likely to promote misconceptions (e.g., "multiplication makes numbers bigger") because they are aware of mathematical "rules" that are not always true (Faulkner, 2013; Karp, Bush, & Dougherty, 2014). This dimension has two features.

The lesson content is mathematically accurate.
Mathematical accuracy has implications for students' understandings (Ma, 1999). In contrast to Ms. Lawrence's lesson, Ms. Davis's lesson included mathematical errors. These mathematical errors impact students' OTL because it is likely that many students walked away from the lesson with unclear or inaccurate notions about fractions and their relative size.

The lesson promotes accurate student conceptions.
Beyond mathematical accuracy, teachers have opportunities to promote accurate conceptions or misconceptions. When a teacher promotes misconceptions, students' OTL is negatively affected; sometimes, misconceptions become roadblocks in future learning experiences, particularly when those misconceptions are not given attention (Lee & Ginsburg, 2009). Ms. Davis promoted misconceptions throughout her lesson. One example was the generation of fraction strips by the students when Ms. Davis ignored attention to precision.

Time

Consistent with Carroll's (1963) initial conceptualization of OTL, researchers have found a positive correlation between the number of minutes of content exposure and students' outcomes on assessments (Ottmar, Decker, Cameron, Curby, & Rimm-Kaufman, 2014), but time utilization varies highly among classrooms (Smith, 2000; Elmore, 2006). Beyond the number of minutes allocated to mathematics instruction, there are two important features relative to time that can maximize, or diminish, students' OTL.

The majority of time in the lesson is used to reach the mathematical goal.
In order to maximize the time in a lesson, it is important to first identify a narrow, well-defined learning goal (NCTM, 2014). Once the goal is set, a majority of the time

should be spent working toward that goal. Ms. Lawrence utilized the majority of time in her lesson on the mathematical goal. In contrast, the breadth of topics in Ms. Davis's lesson creates a surface-level treatment of the mathematics, rather than an in-depth focus on one goal. In our own observational research (Walkowiak, 2010; Pinter, 2013), lessons sometimes moved through activities with no mathematical relation (e.g., multiplication to geometric shapes); in these cases, multiple goals do not allow for sufficient time for conceptual development.

The time is structured so that the lesson components build on each other with explicit attention to the mathematical goal.
Making learning goals explicit to students has been deemed a critical part of lesson implementation (Hiebert, Morris, Berk, & Jansen, 2007). The components of Ms. Lawrence's lesson built on each other, but Ms. Davis's lesson lacked coherence across the lesson. The connections from the story to the fraction strips to the individual practice were absent; therefore, students were unlikely to assimilate the lesson components into a working schema that would move them forward in their conceptual understanding of, in this case, fractions.

Tasks

The planned and implemented tasks in a mathematics lesson matter. Research has shown that higher level thinking during a mathematics lesson increases students' engagement with mathematical ideas (Boaler & Staples, 2008; Stein & Lane, 1996; Tarr et al., 2008). This higher level thinking occurs through the use of cognitively demanding tasks. High-demand tasks involve connecting procedures to their underlying concepts or completing complex, non-algorithmic tasks; low-demand tasks involve memorization or completing procedures without connecting to the underlying concept (Stein, Smith, Henningsen, & Silver, 2009). OTL is optimal when two task features are present.

The implementation of tasks is student-focused, allowing students to make sense of the mathematics.
When students have time to make sense of the mathematical concepts, rather than simply being told what to do, their learning opportunities are increased. Ms. Lawrence gave opportunities for students to develop their conceptual understanding of mixed numbers and improper fractions. On the other hand, Ms. Davis tended to answer her own questions and limit her students' opportunities to make sense of fractions.

Tasks involve the use of and translation among two or more representations.

Opportunities to utilize more than one representation (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003) and to translate among representations (Duval, 2006) have been deemed important task components in building students' conceptual understanding (NCTM, 2000). We argue that even in lessons where the goal is to increase utility with one representation, students' experiences and utility with that representation will likely be improved if they translate to oral or written language, at the very minimum. In her lesson, Ms. Lawrence and her students used multiple representations to develop the mathematical concepts. While Ms. Davis also attempted to use concrete representations, the errors in the representations did not allow for their effective use.

Mathematical Talk

A growing body of literature in the field of mathematics education underscores not only the importance, but also the complexities, of mathematical talk, the fourth dimension in our framework (Ryve, 2011; Schleppenbach, Perry, Miller, Sims, & Fang, 2007). While mathematical talk falls under the broader construct of discourse (NCTM, 2000) that includes talking, listening, and writing, we focus on talk as a key first step in a more complicated process of fostering mathematical discourse where the responsibility rests upon the teacher to maximize its use. The presence of talk alone in mathematics lessons is certainly not enough to promote understanding (Franke, Kazemi, & Battey, 2007; Lampert & Cobb, 2003), but its presence has been proven to positively impact student learning (Walshaw & Anthony, 2008). Researchers have suggested moves that teachers can make to promote student talk (e.g., turn and talk, revoice, press for reasoning) (Chapin, O'Connor, & Anderson, 2009; Smith & Stein, 2011), but leveraging that talk to push students along in their mathematical understanding is perhaps a more critical part of orchestrating discourse (Hufferd-Ackles, Fuson, & Sherin, 2004). Overall, optimal instruction has two talk features.

Students have opportunities to explain their mathematical thinking.

Opportunities to talk should include asking students to explain their mathematical strategies and thinking (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). We know that explanations can vary, from a simple explanation of a procedure to a more elaborate explanation to reveal understanding (or not) of a concept. However, giving students opportunities to explain their thinking is the first essential discourse feature. In our own observational research, some teachers' lessons tended to look like Ms. Lawrence's lesson where students had many opportunities to explain thinking. Other lessons looked like Ms. Davis's lesson where students' explanations of mathematical ideas were few. If students had been given the opportunity to share their thinking in Ms. Davis's lesson, the dialogue may have uncovered their misconceptions as well as errors in the fraction strip representations.

Talk is utilized to move students toward a deeper understanding of the mathematical goal.

This second feature extends the first one: it is important for teachers to utilize student talk to probe students' thinking for further justification (Franke, Webb, Chan, Ing, Freund, Battey, 2009) with prompts like "why did you do that?" or "tell me more about…." The lessons of Ms. Lawrence and Ms. Davis demonstrate contrasting examples. Ms. Lawrence asked probing questions like "how do you know?" and "why did you use that approach?" to push students in their own mathematical understandings. Probing questions were not present in Ms. Davis's lesson.

Discussion

We have presented a reconceptualized OTL framework for mathematics that includes four dimensions: the teacher's mathematical knowledge for teaching, time, tasks, and mathematical talk. The framework provides a lens through which teachers, teacher educators, and scholars can use in discussion, planning, implementation, and analysis of mathematics lessons. Based upon both existing research and our own observational research, we posit the framework's dimensions and their defining features are central, and we argue essential, to students' opportunities to engage in processes that would deepen their understanding of mathematics. Impact becomes exponentially larger, particularly if instruction across a school year tends to look more like Ms. Lawrence versus Ms. Davis.

While we believe this framework has potential as a practical tool for teachers and teacher educators and as a theoretical tool for researchers, we acknowledge that the framework has shortcomings. First, while the framework provides essential features for maximizing students' opportunities to learn, the features may not capture all nuances of instruction. Based upon existing research in instructional dimensions such as discourse or representations, there may be other features to con-

sider. For example, we acknowledge that researchers have found other features of talk (Hufferd-Ackles, Fuson, & Sherin, 2004) (e.g., scaffolding of students' thinking) that potentially impact students' understanding. However, our goal was to draw from our observational research to develop a framework of dimensions and features that appear to be essential in mathematics lessons.

Second, the framework does not include connections between mathematics lessons and students' lives because these connections did not emerge as essential in differentiating lessons where OTL was maximized. We recognize that research has indicated the importance of culturally relevant pedagogy and lessons that capitalize on what students bring to the classroom (e.g., Ladson-Billings, 1995; Tate, 1995; Morrison, Robbins, & Rose, 2008). Further, making connections to their lives outside of school and giving them opportunities to apply mathematical ideas to real-world scenarios is important. When students perceive mathematics to be relevant and valuable, they are likely to be more engaged and to develop more positive attitudes toward the discipline. While our data did not indicate this feature as essential to OTL perhaps due to the sampling protocol (3-4 lessons per teacher), we recommend attention to students' backgrounds within units of instruction.

Third, the socioemotional climate of the classroom is not included as a component of the framework. We recognize the importance of providing a classroom culture with positive reinforcement and emotional support when the aim is to have productive discourse occurring (Reyes, Brackett, Rivers, White, & Salovey, 2012). Researchers have pointed to the importance of setting sociomathematical norms that create an environment for mathematical sense making, problem solving, and reasoning within the classroom (Cobb, Stephan, McClain, & Gravemeijer, 2001). In our own data analyses, we were focused on the mathematical features of instruction and were limited to video recorded observations, likely limiting our ability to be attentive to socioemotional aspects of the classroom.

Finally, the framework does not give explicit attention to assessment. While assessment is implicit within the nested features (e.g., utilizing students' responses to questions to move students toward a deeper understanding), there is not explicit acknowledgement of the role of assessment in students' opportunities to learn.

Despite the fact that the framework is not exhaustive, it does provide a lens for defining "opportunity to learn" in the teaching and learning of mathematics, conceptu-

alized to include both the quantity (Carroll, 1963) and quality (Wang, 1998; Tate, 2005) of learning opportunities for students. Much of the conversation in the mathematics education community about the impact of mathematics instruction focuses on what is happening during mathematics lessons, the very focus of this framework. This framework has practical implications for teacher educators and professional development facilitators who are designing and implementing professional learning experiences with prospective and/or practicing teachers. We currently use the components of this framework with these audiences, coupled with rubrics on an observational measure (Berry et al., 2013), to develop understanding of standards-based mathematics teaching practices. Furthermore, this OTL framework seeks to provide a theoretical framework for the research of mathematics teaching practices. Although our work was situated in upper elementary classrooms, we propose that this OTL framework can be applied to mathematics instruction in grades K-16.

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