JOURNAL OF

MATHEMATICS EDUCATION

AT TEACHERS COLLEGE

A Century of Leadership in Mathematics and Its Teaching

Rethinking Purposes and Best Practices of Mathematics Education

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JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE | FALL 2021 | VOLUME 12, ISSUE 2

Teaching Statistics with an Inquiry-Based Learning Approach

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ABSTRACT Two case studies were conducted to examine whether inquiry-based learning (IBL) can help students in understanding normal distributions, and to determine if IBL methods have any effect on students' conceptual and computational capabilities. There were 16 students in the traditional class and 17 students in the IBL-implemented class who participated in each of two case studies. The experimental group performed significantly better than the control group in terms of conceptual understanding and computational skills. These outcomes suggest that IBL methods such as discussions, asking questions, and guided lectures helped students to develop statistical reasoning.

KEYWORDS *inquiry-based learning, statistics, community college, conceptual thinking, computational skills*

Introduction

Research has indicated that requiring remedial mathematics courses in community colleges is problematic because most students required to take these courses will not finish their degree programs (Belfield & Crosta, 2012; Cooper et al., 2017). As such, many students see the completion of remedial mathematics as a barrier to their future academic and professional careers (Benken et al., 2015). In focusing on remedial courses, students often neglect training in applied mathematics courses such as statistics, which is arguably more broadly applicable than algebra (LaMar & Boaler, 2021). Recognizing this barrier, and also acknowledging the benefits of non-remedial mathematics courses, many Science, Technology, Engineering, and Mathematics (STEM) and non-STEM programs in four-year and two-year colleges have expanded their offerings of statistics courses while reducing remedial requirements for elementary algebra, intermediate algebra, and arithmetic. In this same vein, California passed AB 705, a new law that requires students to complete a transfer-level course in English and mathematics within one year (A.B. 705, 2019). Advocates of this law claim that low-achieving high school students can be placed directly into introductory transfer-level English, statistics, and precalculus courses rather than their remedial prerequisites. Accordingly, many community colleges in California, New York, and other states now offer guided pathways courses, which are courses designed to help students develop academic plans at the start of their degrees by providing more consistent and personalized support (Bailey et al., 2015; Logue et al., 2016).

The push to expand statistics course offerings gives rise to novel challenges. Not all mathematics departments have instructors trained to teach statistics, so the quality of courses varies widely across institutions (Boaler, 2016; Lee, 2011). Additionally, many students entering community colleges struggle in basic mathematics, and statistics could be considered advanced, prompting anxiety and fear of failure among inexperienced students (Boaler, 2016; Lee, 2011; Lee et al., 2013). In particular, many community college students who are low-income or students of color have expressed reluctance to study mathematics on the grounds that traditional teaching methods do not work for them (Ban, 2019; Lee, 2018).

Statistics teaches the fundamental skills of analyzing data drawn from real-life scenarios. In the case of descriptive statistics, teaching statistical thinking involves getting students to infer about the methods of designing research questions, collecting data, and explaining data sets, whereas inferential statistics seeks to extrapolate theories and hypotheses from data sets that can be generalized and applied widely. Courses based solely on using textbooks to solve practice problems could limit students' abilities to explore and develop conceptual ideas (Boesen et al., 2014; Lee, 2018). By contrast, as Golubski (2016) demonstrated, a constructivist pedagogical approach such as inquiry-based learning (IBL) helps students develop conceptual flexibility, discover new ideas, and construct their statistical learning process. Thus, we suggest that teaching structures for a statistics course should be redesigned to increase engagement through collaborative, student-centered, and hands-on learning activities.

This article details an ongoing study, conducted by our team of education researchers, which studied the impact of IBL as applied to teaching statistics at community college. The first case study focused on the statistical concept of normal distribution to study how IBL could be implemented to improve student engagement. The second case study examined the difference between students' computational and conceptual thinking skills in the traditional classroom versus the IBL classroom. The two case studies revealed that IBL methods of teaching statistics potentially increase students' computation and conceptual thinking more significantly than traditional teaching methods.

Related Literature Review

Inquiry-Based Learning

A constructivist view of education emphasizes the importance of learners constructing knowledge for themselves; having students multiply integers using their own algorithm, for instance, is better than providing a teacher-specified algorithm (Murphy et al., 2018; von Renesse & Ecke, 2017). IBL, a type of constructivist learning, is an approach to teaching and learning in which students are active participants and the teacher plays a decentralized role. This style of pedagogy encourages students to construct knowledge through their own experience and inquiry. That is, students are encouraged to conjecture, solve, explore, discover, collaborate, and communicate without a teacher emphasizing the formula, axioms, theorems, and procedures (Ban, 2019; Capaldi, 2015). Research has shown that IBL can improve student outcomes (Murphy et al., 2018) by developing students' curiosity and urge to explore (von Renesse & Ecke, 2017). In undergraduate mathematics in particular, IBL leads to increased retention, better participation, and stronger conceptual thinking (Kuster et al., 2018). Instructors do not have to be experts in IBL to reap its benefits; for instance, when they teach using group activities, games, or outside technology, they are using IBL (Offenholley, 2012; Poon, 2018; von Renesse & Ecke, 2017), which makes students active learners discovering their own methods through collaboration. When instructors provide conceptual tools and opportunities for exploration in lessons, students generally produce their own ways of reasoning, build on each other's contributions, develop a shared understanding, and connect experience to standard language and notation (Kuster et al., 2018). According to Beswick (2021), the manner of an instructor's questioning style can stimulate a student's engagement with problem solving, or conversely, impede their understanding, depending on its form and content. In these studies, IBL was used in a statistics class to prompt students to explain why each data analysis measure needs a standard deviation, and how the standard deviation relates to normal distribution.

The Impact of IBL on Computational and Conceptual Skills

Conceptual understanding is a key component of students' ability to develop mathematical proficiency. According to Kilpatrick et al. (2001), "Conceptual understanding refers to an integrated and functional grasp of mathematical ideas" (p. 118) in which students are able to make connections between and within these ideas. For students to grasp new topics conceptually, teachers should leverage their related prior knowledge (Boaler, 2016). A variety of strategies can be used with IBL to help students build conceptual understanding, such as open-ended questions, peer group discussions, and whole-class discussions. Student-centered activities such as these not only call upon students' prior knowledge but also increase students' interest and provide context for learning new ideas. Furthermore, when students are able to share and listen to each other's varied perspectives, these activities enhance students' understanding of the mathematical ideas. Furthermore, IBL's emphasis on higher cognitive thinking through analysis, synthesis, and evaluation of mathematical concepts enhances student engagement, exploration, and mathematical communication skills (Smith & Stein, 1998). When applied to mathematics, IBL supports deeper learning and promotes critical communication in the classroom (Uiterwijk-Luijk et al., 2017). The preliminary research from the first case study may indicate positive, recursive effects on student learning outcomes, such as increased engagement and mathematical communication. In the second case study, IBL strategies were used to support the development of students' conceptual and computational skills with respect to normal distributions.

Methodology

Case Study 1: Teaching Normal Distribution Using IBL

Normal distributions can be used to predict the outcomes of many societal and naturally occurring phenomena such as income distribution, standardized test scores, shoe size, weight, and height. However, the concept of normal distribution has been shown to be challenging for many students (Batanero et al., 1999; Libman, 2010). Students may encounter difficulties when the teacher discusses things being "normally distributed" because they lack the understanding of how to decide whether a distribution is normal (Batanero et al., 1999). The nature of many lecture-style classes may make it more difficult for students to understand the usefulness and relevance of what they are learning, which reduces their motivation (Libman, 2010). To help address this issue, the first case study aims to introduce a new IBL teaching strategy for normal distribution. Using a qualitative case study methodology, the research team sought to learn whether IBL strategies were effective for improving learning outcomes. In Case Study 1, we conducted observations and collected data, including student dialogues and feedback, from an introductory statistics course at a community college in New York City. Findings indicated that IBL strategies could have a positive impact, but further research was needed to determine how much and for whom.

The instructor who implemented an IBL lesson on normal distributions developed a short lecture video that students could watch before class to preview the concept of a normal distribution. The goal of the video was to encourage students to bring questions to class to discuss with their peers. During class, students participated in a series of discussions by reading, writing, and solving problems collaboratively. These discussions were centered around active learning worksheets using IBL. The active learning worksheets used real-life examples through which students could apply mathematics. This activity worksheet was referenced in the institution's current textbook and the activity questions were written by the instructor.

A total of 17 students participated in the study. Group discussions and worksheets were analyzed for how students learned normal distribution via IBL. A researcher determined whether a classroom was an IBL-implemented classroom based on the following characteristics: multiple small-group discussions, series of whole-class discussions, open-ended questions, and IBL active learning worksheets.

During this session, students were prompted to discover the characteristics of normal distribution and develop the understanding that z-scores indicate how much a given value deviates from its standard deviation throughout a series of group discussions. To begin the lesson, the instructor provided a scenario about normal distribution in the prompt (Figure 1).

Figure 1

IBL Active Learning Worksheet on Normal Distribution

Prompt 1

Claire and Susan are very close friends with each other. They are going to the same college and have been taking many classes together. They've always been rivals against each other; they always compared each one's performance and evaluated who gets a higher grade (score). One semester, they are taking statistics classes: but different classes with different professors. After the first exam, Claire got 75, and Susan got 77 on their first exam. The average score for Claire's class is 68 with a standard deviation of 8.2. The average score for Susan's class is 70 with a standard deviation of 6.7.

Question 1: Who got the higher grade on the first exam?

The first group discussion was conducted during the first 10-15 minutes of the class. During the discussion, the instructor introduced Prompt 1 and instructed students to use their prior knowledge to determine the characteristics of normal distributions. Students were expected to use their prior knowledge of mean, variance, and standard deviation in identifying these characteristics. After the first group discussion, the whole class discussed who performed better and why according to Prompt 1. In accordance with IBL teaching methods, the instructor did not clearly state whether Claire or

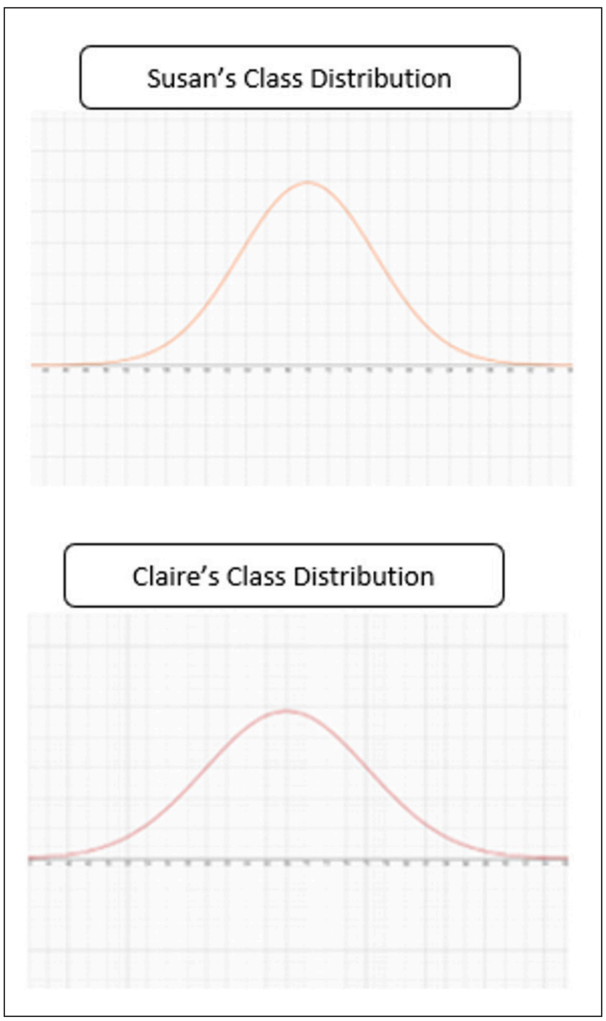
Susan would receive a higher grade in the scenario, nor did they provide a specific procedure for answering the questions; instead, the instructor encouraged students to ask questions and come to their own conclusions. After the first whole-class discussion, the instructor provided the students with graphs that showed the distribution of Claire's and Susan's scores according to the scenario (Figure 2).

In a second group discussion, students were asked to compare two graphs, to identify all similarities and differences between them, and to determine who scored higher. The instructor walked around the groups and told students that both Claire's and Susan's classes had equal variances and were normally distributed. The instructor observed that students compared the graphs while understanding that the two normal curves were equally scaled. During this discussion, which lasted 10-15 minutes, the instructor also asked students to explain why a large standard deviation results in a wider graph and why that is important. Additionally, the instructor asked students to determine who got a higher grade by comparing two graphs as they had not yet figured out how to calculate *z*-scores.

A short lecture followed the second group discussion. The instructor introduced the *z*-score formula and its meaning. During this step, the instructor outlined what *z*-scores represent and how to read *z*-tables. Then, the instructor guided students through the *z*-score formula in the third group discussion while letting them think about the standard deviation of each score and what it represents. The instructor then asked students to analyze Prompt 1 using the formula. Students were expected to explain to their peers what a *z*-score represents and how to read a *z*-table as a third group discussion.

Figure 2

Normal Distribution Curves for Susan's and Clarie's Classes in the Prompt



Case Study 2: Comparing Two Groups' Computational and Conceptual Understandings of Normal Distribution

Case Study 2 is intended to demonstrate how IBL teaching methods affect students' thinking about normal distribution by comparing two different class models for the same-level statistics course: a traditional classroom and an IBL-implemented classroom. In the IBL-implemented classroom, the instructor used worksheets and activities that they designed to help facilitate instruction. The IBL-implemented class consisted of a guided lecture, multiple group discussions, and whole-class discussions. The traditional classroom instructor engaged in direct lectures, individual problem-solving practice, and short class discussions. Both IBL-implemented and traditional classes met twice a week for 90 minutes, at the same institution as in Case Study 1.

The researcher developed an assessment designed to measure students' computational and conceptual skills on normal distribution in both the traditional classroom and the IBL classroom from Case Study 1. The researcher designed the assessment to include eight computational problems (see examples in Figure 3) that gauged students' abilities to read the *z*-score and use it to determine probabilities. 16 students from the traditional classroom and 17 students from the IBL classroom agreed to participate in the assessment.

Additionally, the assessment included conceptual problems to test students' understanding of normal graphs based on different standard deviations, as well as their ability to apply these concepts to real-world problems (Figure 4).

Figure 3

Computational Problems on Normal Distribution

Quiz: Standard Normal Distribution

1. Find the indicated area under the standard normal curve.
   a. To the left of z = 1.36
   b. To the right of z = –0.65
   c. Between z = –1.96 and z = 1.96

2. Find the indicated probability using the standard normal distribution.
   a. P (z < 1.45)
   b. P (–2.08 < z < 0)

3. In a recent year, the ACT scores for high school students with a 3.50 to 4.00 grade point average were normally distributed with a mean of 24.2 and a standard deviation of 4.3. A student with a 3.50 to 4.00 grade point average who took the ACT during this time is randomly selected.
   a. Find the probability that the student's ACT score is less than 17.
   b. Find the probability that the student's ACT score is between 20 and 29.
   c. Find the probability that the student's ACT score is more than 32.

Figure 4

Conceptual Problems on Normal Distribution

Using the two given data sets, answer the questions 4 and 5.

Data Set A: Mean = 65, x = 72, s = 7.5

Data Set B: Mean = 70, x = 77, s = 9.12

4. Select the correct statement or statements (there can me more than one).
   a. Data sets A and B have the same mean deviation.
   b. The mean deviation of Data set A is greater than that of Data set B.
   c. The normal curve of Data set A is wider and shorter than that of Data set B.
   d. The normal curve of Data set A is narrow and taller than that of Data set B.

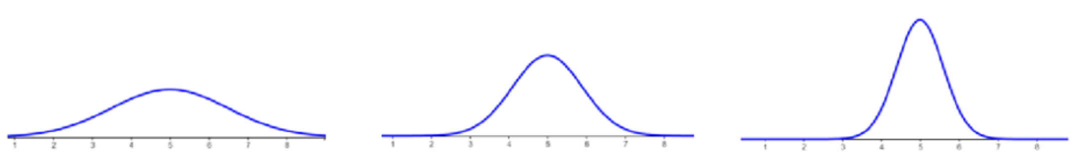
5. Based on the given data, which data set would have greater cumulative area on the standard normal curve?

6. Match each of the following data sets to the most appropriate graph shown below.

   Data set A: Mean is 51 with a standard deviation of 3.2.

   Data set B: Mean is 66 with a standard deviation of 9.1.

   Data set C: Mean is 70 with the standard deviation of 6.



Several instructors who previously taught *Introduction to Statistics* at this institution reviewed and agreed upon this assessment. The researcher used the assessment with the null and experimental groups to measure students' conceptual understanding during the visit. A t-test was used to analyze students' scores on these questions to determine if there were any statistically significant differences between the two classroom models.

Results and Analysis of Case Study 1: Teaching Normal Distribution Based on IBL

Students were given the mean scores and standard deviations of two classes for Group Discussion 1, as well as Claire's and Susan's normal distributions for Group Discussion 2. After the group discussions, the z-score and the table were briefly presented for Group Discussion 3. Without providing additional information, the instructor asked students to determine whether Claire or Susan had the higher grade. At the end of each group discussion the instructor led a brief class-wide discussion. The following dialogue shows how the class discussed the statement after the first group discussion (Dialogue 1).

Dialogue 1

Dialogue After the First Group Discussion

GROUP A: "We think Susan got a higher grade because her score is better than Claire's."

GROUP B: "We cannot say Susan got the higher grade because there are two other values (The class mean and the standard deviation). We think those two values affect the decision."

GROUP C: "If you see the difference between their scores and class averages, the difference is the same. So we can say that they got the same grade."

INSTRUCTOR: *"You mean their mean deviation is the same in two classes?"*

GROUP C: "Yes."

GROUP A: "Oh, I see. That makes sense."

GROUP B: "What about the standard deviation? We really think it affects the decision."

INSTRUCTOR: *"How does the standard deviation affect your decision?"*

GROUP B: "Because…. I don't know. Our group thinks it does something."

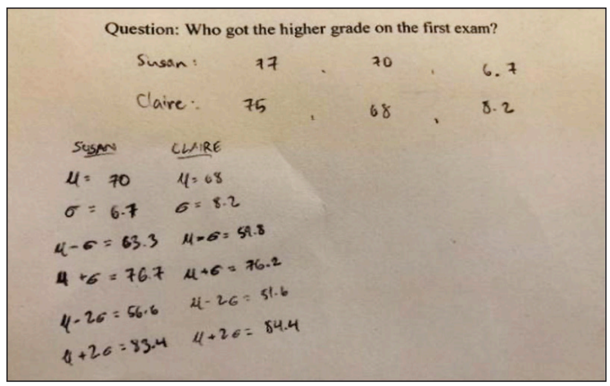
Note: The dialogue was recorded during class and the instructor summarized the class discussion.

Dialogue 1 shows that students in Groups A, B, and C agreed that Susan's and Claire's mean deviations were the same. By asking questions instead of providing direct answers, the instructor guided students to begin to inquire about how standard deviation can affect decision making. Most students knew that standard deviations affect the spread of normal distributions but were unsure how the size of the standard deviation affects its shape. Figure 5 depicts a student's written work on how they determined who received a higher score after the first group discussion.

The student's work in Figure 5 showed that they added and subtracted a standard deviation from the mean. As the students were given the mean and standard deviation, they were able to calculate the distance between the mean and each data point. By using this method students were attempting to construct the concept of a normal distribution.

Figure 5

Student's Written Work Determining Who Received a Higher Score After the First Group Discussion



For the second group discussion, the instructor handed out the normal distribution graphs for Claire's and Susan's classes and asked the students to use them to determine who got a higher grade. After the second discussion, the instructor had a short whole-class discussion (Dialogue 2). All students participated in the group discussions, enabling the instructor to observe their different problem solving approaches.

Observation suggests that most of the groups used the standard deviation to determine who got a higher grade using the graphs. Figure 6 shows two groups' work to determine who got a higher grade using the normal graphs.

Figure 6 illustrates how each group determined the normal graph distribution after the second discussion. One group used the raw data of Susan's and Clarie's scores to find the distance from the average score. Another group used the Empirical Rule, cal-

Dialogue 2

Dialogue After the Second Group Discussion

INSTRUCTOR: *"Okay. Please look at the standard curve of two classes and compare who got a higher grade. Why are you using the standard deviation?"*

GROUP A: "Because the standard deviation is the only value left to do something in this case."

GROUP B: "We need to construct the interval using the standard deviation. We discussed that the standard deviation determines the interval from the mean value."

INSTRUCTOR: *"Okay, that's a good idea. Please show me what you got."*

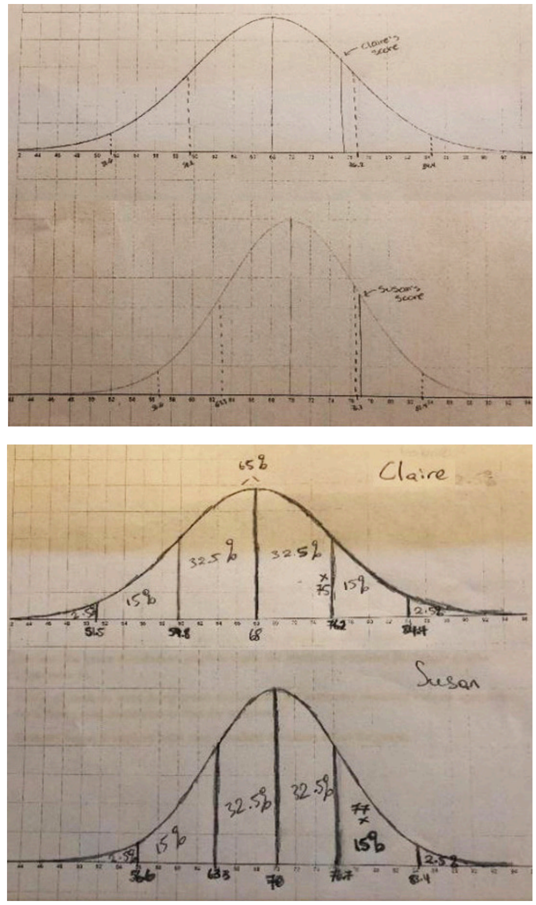
Note: The dialogue was recorded during class and the instructor summarized the class discussion.

culating the percentage to find the distance from the average score. Although those groups used different methods to find the answer, both reached the same conclusion: that Claire's score was below $\mu+\sigma$ and Susan's score was above $\mu+\sigma$. The instructor was curious if students understood how the standard deviation affects the distribution. As a result, the instructor asked the following questions during a whole-class discussion on the topic (Dialogue 3 and Dialogue 4).

In Dialogue 3, students demonstrated that z-scores describe the relationship between a value and the mean of a group of values. Students illustrated that the z-score

Figure 6

Students' Work After the Second Group Discussion



Dialogue 3

Dialogue After the Third Group Discussion

INSTRUCTOR: *"What happened here? How could Claire's score be below $\mu+\sigma$, and Susan's score be above $\mu+\sigma$?"*

GROUP B: "As you see our graph, Claire's class $\mu+\sigma$ is 76.2 because we added the mean value 68 and the standard deviation value 8.2. We did the same thing for Susan: $\mu+\sigma = 76.7$. And, we compared their scores and the $\mu+\sigma$ value. As a result, Claire's score is below 76.2: she got 75, and Susan's score is above 76.7: her score is 77. Therefore, we concluded that Susan got a higher grade."

GROUP A: "We agreed with it. We found the z-score of each one's data."

INSTRUCTOR: *"How could you get the z-score even though we did not discuss it yet?"*

GROUP A: "As you suggested, we watched the lecture video and found that out."

INSTRUCTOR: *"How did you know the z-formula is related to today's activity?*

GROUP A: "Because there are three different values: students' scores, class mean values, and standard deviations. We compared this activity with the z-formula and realized that the activity is related to standard deviation."

INSTRUCTOR: *"Great! Could you explain how you applied it?"*

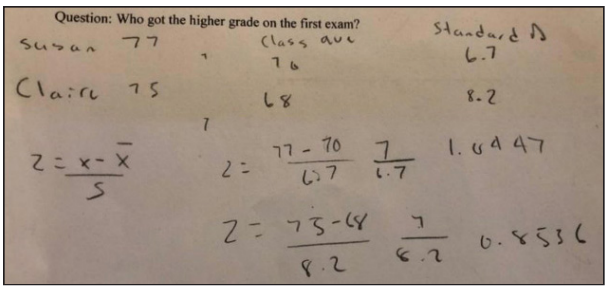
GROUP A: "We used the z-formula: $z = \frac{x-\mu}{\sigma}$, and found Claire's z-score is 0.8536, and Susan's z-score is 1.04447. As you see Susan's z-score is higher than Claire's, so we concluded that Susan performed better."

Note: The dialogue was recorded during class and the instructor summarized the class discussion is based on standard deviations from the mean, and a z-score of 0 indicates that the data point's score is the same as the mean. In addition, by asking questions, the instructor helped students review what they have learned, allowing them to see how z-scores are formed by standard deviation.

In Dialogue 4, students noticed that z-scores of $\mu+\sigma$ indicate values that are one standard deviation away from the mean. The z-scores may be positive or negative, with a positive value indicating a higher score than the mean and a negative score indicating a lower score than the mean. Figure 7 illustrates how students use a z-score formula to calculate standard deviations.

Figure 7

Student Work to Determine Who Got a Higher Grade Using the Normal Graphs After the Third Discussion



Dialogue 4

Continued Dialogue After the Third Group Discussion

INSTRUCTOR: *"Would you compare the two graphs carefully?"*

GROUP A: "We found out that Susan's class graph is taller and narrower."

INSTRUCTOR: *"Do you understand what "Group A" just mentioned?"*

GROUP B: "It makes sense. The narrower and taller graphs generate a bigger z-score."

INSTRUCTOR: *"When is the case that the normal graph is narrower and taller?"*

GROUP B: "When the standard deviation is smaller."

INSTRUCTOR: *"Why?"*

GROUP A: "When we divided the mean deviation by the standard deviation, we got a bigger z-score on Susan's class, and Susan's class standard deviation was smaller."

GROUP C: "That's right. Because we divide it by a smaller number, the value is bigger."

Note: The dialogue was recorded during class and the instructor summarized the class discussion.

After a brief lecture and whole-class conversation about z-scores, students were able to use z-scores to compare different scales of distribution and engage in an extended discussion to find out who had the higher score. By the end of this activity, with the understanding of standard deviation that students had developed through dialogues, students recognized that graphs can be narrower and taller, or wider and shorter, depending on their z-score. During the analysis of the series of discussions, the researcher found that students could better engage when they are asked to write down their learning, discuss it with peers, and participate in the classroom discussion. The instructor consistently guided students through the inquiry by asking questions. Based on our observations, asking questions stimulated students' interest, guided them towards clear ideas, and aided them in developing their own ideas. Case Study 1 shows how, through a series of group discussions and whole-class discussions, students are able to make connections between various aspects of mathematics (Gillies, 2007). In addition, peer-to-peer discussions promote higher-order thinking and collaboration among students, which has the benefit of helping students gain confidence in talking with their peers, understanding the topic, and sharing their opinions with one another (Gillies, 2007; Johnson et al., 1998). The findings from Case Study 1 suggest that open-questions, discussions, and collaboration can help students develop an understanding of the concept of normal distribution and its properties.

Results and Analysis of Case Study 2: The Impact of IBL on Conceptual and Computational Skills

Figure 8 compares the proficiency for computational thinking between the two groups with the x-axis serving as the question number, and y-axis serving as the percentage of responses that were correct.

Overall, both groups performed well on computational problems when reading the z-scores and calculating their probabilities. Most students in both classrooms had correct answers to computation questions, which implies that algebraic procedures of problem-solving were successfully taught by both teaching methods. According to the t-test (Table 1), however, the IBL classroom performed slightly better on the computational problems than the traditional classroom. Table 1 shows the results of the t-test between

Figure 8

Comparison of Scores on Computational Problems Between Traditional and IBL-Implemented Classrooms

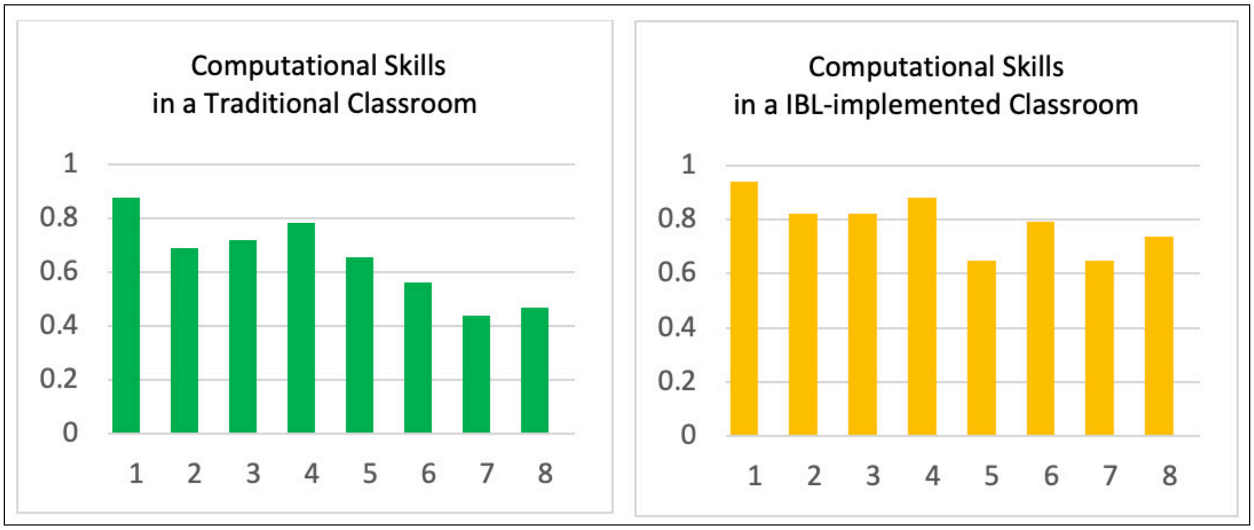


Table 1

t-test of Scores on Computational Problems Between the Two Groups

t-test: Two-Sample Assuming Equal Variances		
	Experiment	*Null*
Mean	0.701680672	0.446428571
Variance	0.022738507	0.07473386
Observations	14	14
Pooled Variance	0.048736184	
Hypothesized Mean Difference	0	
df	26	
t Stat	3.059092341	
P (T < = t) one-tail	0.002548234	
t Critical one-tail	1.70561792	
P (T < = t) two-tail	0.005096467	
t Critical two-tail	2.055529439	

the two groups on computational problem solving.

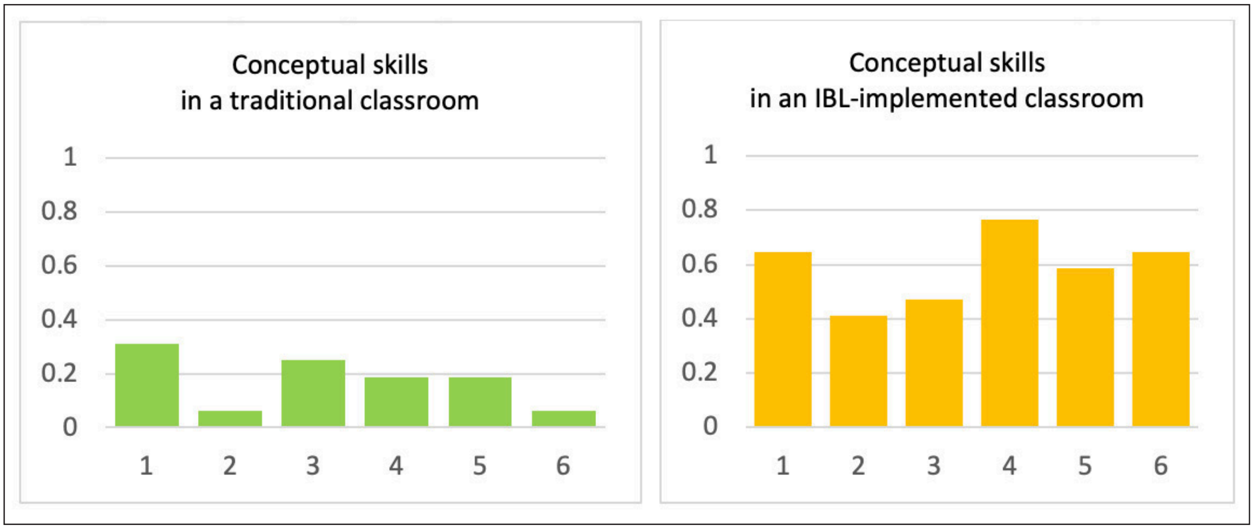
In Table 1, the *p* values (both with one tail and two tails) indicated that there was a significant difference in the performances of the two groups on computational problems. The results may suggest that students are more likely to succeed as a result of peer interactions and activities. In addition, the results suggest that engaging and collaborative learning experiences may lead to deeper students' understanding of the subject.

In addition, Case Study 2 tested students' conceptual understanding of normal distribution. Figure 9 shows the students' performance on conceptual problems with the *x*-axis serving as the question number and y-axis serving as the percentage of correct responses.

The IBL classroom's conceptual skills, as shown in the second graph in Figure 9, are generally much higher than that of the null group. For all of the conceptual problems, the students in a traditional classroom rated

Figure 9

Comparison of Scores on Conceptual Problems Between Traditional and IBL-Implemented Classrooms



below 0.4, while the students in the IBL-implemented classroom rated above 0.5 for most questions. There is a clear and considerable difference between the two groups when it comes to conceptual understanding. These findings suggest that IBL simulated development of students' conceptual understanding of course topics.

We also conducted a *t*-test between the two groups on conceptual understanding (Table 2).

These *p* values (both one tail and two tails) further suggest that IBL strategies, such as allowing students to share their perspectives on problem solving, can improve students' conceptual skills.

Table 2

t-test of Scores on Conceptual Problems Between the Two Groups

t-test: Two-Sample Assuming Equal Variances		
	Experiment	*Null*
Mean	0.588235294	0.177083333
Variance	0.016608997	0.010026042
Observations	6	6
Pooled Variance	0.013317519	
Hypothesized Mean Difference	0	
df	10	
t Stat	6.170940073	
P (T < = t) one-tail	5.26933E-05	
t Critical one-tail	1.812461123	
P (T < = t) two-tail	0.000105387	
t Critical two-tail	2.228138852	

Conclusion

In IBL, students are encouraged to become active learners through open-ended questions and peer- and whole-class discussions that guide inquiry. Through this approach, students can increase their proficiency in mathematics, improve their understanding, and stimulate their interests (Riegle-Crumb et al., 2019). In particular, through IBL in statistics courses, students can gain social skills such as listening, problem solving, leadership, and teamwork. The results of this study suggest that IBL provided students with the opportunity to discover important concepts related to normal distribution on their own. Results also suggest that IBL can improve students' communication skills, written work, and statistical analysis, which leads to improvements in their computational skills and conceptual understanding. Moreover, this study indicates that group discussions, open-ended questions, whole-class discussions, and activity worksheets are important elements of statistics courses as they help students develop their own ideas and computational skills. The results are consistent with the findings from a study showing that students were able to talk and engage with their peers to practice thinking about course content through IBL (Dorier & Maass, 2020), which enhanced their statistical thinking abilities. That is, the findings suggest that IBL can be a potentially effective method for developing statistical reasoning through the development of conceptual understanding of normal distribution and the improvement of computational skills in z-score computation. Case Study 2 provides an illustration of how IBL methods in Case Study 1 can be utilized to benefit students' computational and conceptual abilities. It implies that students can gain a deeper conceptual understanding of normal distribution in an IBL setting than in a traditional teacher-focused lecture.

Suggestions for Further Study

This study compared student learning on only one statistical concept: normal distribution. Future research should test the effectiveness of IBL for teaching other statistical topics as well. Furthermore, future studies should examine overall student grades, pass rates, and student satisfaction with their learning experience in order to gain greater insight into the potential effectiveness of IBL in statistics courses.

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