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A Century of Leadership in Mathematics and Its Teaching

Rethinking Purposes and Best Practices of Mathematics Education

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NOTES FROM THE FIELD

Evolution in the Field of Mathematics Education: Its People, its Products, and its Directions

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The ways in which a field changes reveal the essence of the field. Understanding the field of mathematics education can benefit from retrospection on major changes in the field. Over the past few decades, those changes have revolved around people, products, and directions–and those changes have been profound in U.S. mathematics education. In this piece, I reflect on the changes over the past few decades that have had the greatest effect on the field of mathematics education.

People and Affinity Groups

Not only have the individuals involved in the field of mathematics education changed as scholars have moved in and out of the community, but the relationships among those individuals and their affiliations have also undergone significant change. Improvements in access to travel and communication over the past few decades, for example, have cultivated growth in the nature and strength of international interrelationships. The quadrennial meetings of the International Congress on Mathematical Education (ICME) which assembles groups of mathematics educators, curriculum developers, mathematicians, researchers in mathematics education, mathematics teachers, mathematics teacher educators, producers of mathematics education resources from throughout the world, and mathematics education researchers from a broad range of nations provides one of a growing number of venues that cultivate those international relationships. The annual meeting of the International Group for the Psychology of Mathematics Education (PME) has emerged as an essential venue, with the original attendance of less than 100 swelling in the last few decades to more than eight times that number. U.S. mathematics educators have a clear presence at each of these venues and both a growing awareness of the contributions of international mathematics education and an increasing appreciation for the strength of international research teams. Reflective of these international relationships, the past few decades have witnessed a series of international handbooks on general and specific issues in mathematics education.

Groups of mathematics educators in U.S. have realigned and emerged as new venues have arisen. With growing interest in special subfields such as undergraduate mathematics education, the education of mathematics teachers, and the use of technology in mathematics education, mathematics educators sought special interest organizations both within and apart from existing mathematics-focused organizations. After several decades of work on developing the perception of mathematics education as an acceptable research field within mathematics, the Special Interest Group on Research on Undergraduate Mathematics Education (SIGMAA on RUME) formed within the Mathematical Association of America (MAA) and expanded from dedicated sessions within the joint AMS-MAA meetings to their own conferences (with their 24th annual meeting scheduled for 2022). Similarly, mathematics teacher educators, after seeking a home that focused on the education of mathematics teachers, inaugurated the Association for Mathematics Teacher Educators (AMTE), which has scheduled its 26th annual conference for 2022. School mathematics teachers have also formed special interest groups and expanded their reach. The Teachers Teaching with Technology (T^3) group began its work in the early 1980s and, along with the more recent T^3IC and teacher-driven organizations that focus on classroom use of specific software such as Desmos or CAS, have provided professional development in technology for both mathematics and science teachers.

Products

As affinity groups arose, new journals developed to highlight their specialty areas. A new journal focused on research on the learning and teaching of mathematics at the post-secondary level, *International Journal for Research on Undergraduate Mathematics,* released its first issue in 2015. A journal focused on research on and practice of the education of mathematics teachers, *Mathematics Teacher Educator,* published its first issue in 2012. And a journal focused on the practice of mathematics education in pre-kindergarten through grade 12, *Mathematics Teacher: PreK–12*, a continuation of National Council of Teachers of Mathematics' (NCTM) *Mathematics Teacher,* was first published in 2020. The ability of special-interest groups to adapt to changing times is exemplified in the evolution of a journal dedicated to a specific piece of software, *International Derive Journal* (1994), to a journal dedicated to a specific genre of software, *International Journal of Computer Algebra in Mathematics Education* (1997), to a journal with an unlimited focus on technology in mathematics education, *International Journal for Technology in Mathematics Education* (2004). The development of new journals and the evolution of existing journals, including journals that are published only online, are but one marker of growth in the field of mathematics education. The handful of mathematics education journals available in 1970 is overshadowed by the more than 50 mathematics education journals being produced today. Not only has the number of places within which mathematics education articles can be published increased, but the range of such venues opens the field to new directions and new areas of inquiry.

Directions

The past few decades in mathematics education have generated not only new pockets of research, new participants, new alliances, and new products of those alliances but also new directions for the field. Based on the 1980 NCTM *Agenda for Action,* the NCTM, in its 1989 Standards document, took the bold and unprecedented step of specifying what should be the general content of school mathematics in grades K through 12. These Standards were embodied in the subsequent decade-long development of NSF-funded school mathematics curricula. The stance resulted in the "Math Wars"—vigorous attacks by groups of mathematicians and parents who preferred what they held to be the traditional curriculum. Although various subsequent attempts (e.g., *Principles and Standards of School Mathematics,* Common Core State Standards) were made to define the nature of needed changes in school mathematics curricula, the Math Wars have yet to be settled to everyone's satisfaction. In the meantime, other directions have drawn the field's attention. Some of these directions are unremitting, presenting a constant and difficult-to-achieve but critical goal. Most prominent among these directions is a universal focus on making mathematics classrooms diverse, equitable, and inclusive. Some other directions arose due to unanticipated immediate events, such as the pandemic-fueled need to develop ways to effectively engage students in online or hybrid mathematics instruction. Still other directions are ephemeral, such as mastering the use of the newest digital applications. As we encounter these new directions for our work in the field, it is important to situate this work within overarching and universal goals.

Parting Thoughts

As I reflect on the changes over the past several decades that seem to have most affected the field of mathematics education, I think about the evolution of movements and the production of resources. An increased capability of connecting with mathematics educators from throughout the world has opened the eyes of mathematics educators to the promise of international and intercultural collaboration. The inauguration of organizations centered on particular mathematics education specialties has led to deeper explorations of those specialties. The establishment of an increasingly larger set of venues for disseminating mathematics education research, ideas, and manuscripts allows more mathematics educators to share their ideas broadly about the teaching and learning of mathematics. Just as there is promise in the changes in the people, products, and directions that have been occurring over the past few decades, there are some cautionary notes.

Change requires being mindful of maintaining balance. As we take advantage of the opportunities to connect internationally, to form new alliances, and to join speciality groups, we need to temper knowledge of our specialty with a broader knowledge of the field. We need to maintain cross-groups allegiances that the field of mathematics education has grown. As the number of venues for our work increases, we need to develop ways to deal with and account for the broad range of ideas that others are sharing. As directions and emphases in the field evolve, we need to place them in the larger context of shared directions and emphases. It is through this balance that we can, together, help the field continue to grow in meaningful and productive ways.