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A Century of Leadership in Mathematics and Its Teaching

Rethinking Purposes and Best Practices of Mathematics Education

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AIMS AND SCOPE

The *Journal of Mathematics Education at Teachers College (JMETC)* is a recreation of an earlier publication by the Program in Mathematics Education at Teachers College, Columbia University. As a peer-reviewed, semiannual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Although each of the past issues of *JMETC* focused on a theme, the journal accepts articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed.

JMETC readers are educators from pre-kindergarten through twelfth-grade teachers, principals, superintendents, professors of education, and other leaders in education. Articles appearing in the *JMETC* include research reports, commentaries on practice, historical analyses, and responses to issues and recommendations of professional interest.

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PREFACE

This semester witnessed the large-scale return to in-person instruction after two uncertain and often tumultuous school years. Amidst broad efforts to restore classrooms to pre-pandemic normalcy, contributors to the Fall 2021 issue sought instead to reevaluate and innovate on long-standing practices in mathematics education. The four articles in the main section of this issue explore strategies for equipping teachers of varying grade levels to immerse their students in rich and challenging mathematical tasks. Also featured are four "Notes from the Field" by veteran mathematics educators and researchers. These authors share their reflections and insights from decades-long careers in the field. It is our hope that, together, these articles will energize those in the field of mathematics education to continue rethinking ways to engage students and research communities to advance mathematical learning.

As many of us are able to return to the classroom, we can again look at ways to make learning mathematics interesting and meaningful for our students. Our authors in this issue bring to light a number of classroom practices that can help students from all grade levels to problem-solve and form conceptual understanding of mathematical concepts in three practice-based articles. First, in an article by Throop Robinson, we learn about a way to support early elementary students in developing quantity sense through the use of a life-size hundreds chart made out of a shower curtain. Not only does this provide students with an opportunity to be physically engaged with mathematics, but the shower curtain's versatility also enables teacher educators to guide preservice teachers in thinking about and designing a variety of problem posing activities.

Next, Stohlmann outlines two modes of game-based learning for middle school mathematics that are geared toward engaging students and promoting student interest in STEM. Game-based learning, according to Stohlmann, is yet another way for teachers to encourage productive struggle and meaningful discourse.

Thirdly, Lee and Ban propose a way to incorporate inquiry-based learning into statistics courses at community colleges, so as to help students develop conceptual understanding and computational skills when examining problems associated with the normal distribution. Such instructional methods nurture students' development of autonomy in learning while deepening their understanding of mathematics.

The main section concludes with a research-based article that examines strategies for supporting early-career teachers. Recognizing the unique challenges facing secondary mathematics educators within their first few years of teaching, Jakopovic, Weiland, Campitelli, Males, and Amick discuss their design and implementation of an online community of practice aimed at providing beginning mathematics teachers with mentorship and peer collaboration opportunities. The authors discuss challenges with implementation as well as directions for further research.

PREFACE (Continued)

While overcoming the many social, political, and, most recently, health and safety challenges, both current and aspiring researchers have continued to find ways to improve mathematics teaching and learning through research. The Notes from the Field section is dedicated to all those who work in the fields of mathematics and mathematics education, with advice and insight from accomplished educators and educational researchers. Beginning with Dr. Robert Q. Berry, we are reminded how stakeholders can “humanize” mathematics and mathematics education research so as to prioritize the needs of all students. Doing so is imperative, as Dr. M. Kathleen Heid highlights, as the many opportunities, developments, and changes to the field of mathematics education require us to adapt and evolve. Dr. Robert Reys reminds us that, however challenging this path through research may seem, it is always a rewarding one. Dr. Marta Civil concludes this issue by prompting us to make the research we choose to engage in more meaningful, so that studies benefit not only the field of mathematics education but also the participants' communities.

We hope that in looking to contemporary research in conjunction with career-long reflections, our readers will be inspired to adapt lessons from these past two years so as to reimagine and build on past practices—whether in striving for rigorous classroom structures or in reconceiving the purpose of mathematics education as a whole.

Nasriah Morrison
Alyssa MacMahon

Guest Editors

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Jumping on the Shower Curtain: Using the Hundred Chart Kinesthetically to Embody Quantity Sense in Elementary Students

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ABSTRACT In this paper I discuss the possibilities for exploring the hundred chart kinesthetically with elementary preservice teachers using a life-size hundred chart—an innovative tool, homemade from a shower curtain—to build quantity sense. Preservice teachers explored creative approaches to teach mathematics using the life-size hundred chart and planned engaging movement activities for young students. I detail the activity prompts for students, sample lesson ideas from preservice teachers, and reflections from those who implemented the tool in their instruction. I provide additional suggestions for teacher educators to use this tool in building quantity sense with their students.

KEYWORDS *hundred chart, movement, preservice, elementary*

Introduction

Supporting children in building number sense, or the "general understanding of number and operations along with the ability and inclination to use this understanding" (McIntosh et al., 1992, p. 3), features prominently in elementary mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000; NSDEECD, 2019). As Jordan et al. (2007) synthesized in their survey of research on children's number sense development, "Most agree that number sense involves abilities related to counting, number patterns, magnitude comparisons, estimating, and number transformation" (p. 36). Some studies (Gerofsky, 2010; Wagner & Davis, 2010; Zevenbergen, 2004) suggest benefits in opening this concept further to include an appreciation of number, to promote flexibility with number, and to provide opportunities for "feeling quantity" through performance and experiential activity. In this paper, I conceptualize number flexibility and this expressive nature as quantity sense.

Lakoff and Nuñez (2001) argued that mathematical thinking is embodied, meaning that for children, cognition is rooted in their bodily-kinesthetic interactions with the physical world. Kinesthetic approaches to learning mathematics in elementary school range from students' finger counting (Soylu et al., 2018) and skip counting through rhythm and dance (Lindt & Miller, 2017), to indicating similarity and balance through gestures (Thomas & Dueber, 2020). Efforts to make mathematics learning tangible for children with an emphasis on hands-on activities using concrete materials show positive results for developing quantity sense (Swan & Marshall, 2010). Current research also shows increased engagement and conceptual benefits for children who participate in full-body movement and gestural mathematics in playful spaces of learning (Edwards et al., 2014). Tracing graphs in the air with arms and making geometric shapes or angles from students' bodies are good examples of this. Additionally, asking students to walk number pathways on a hundred chart or make jumps to determine a skip counting pattern may show potential for students to feel or embody their developing quantity sense. Learning mathematics playfully sustains engagement and opens students to additional benefits for problem solving including collaboration with peers, opportunities to correct mistakes efficiently, and space for creativity in mathematics to emerge (Featherstone, 2000).

In this paper I demonstrate possibilities for playful learning by embodying quantity sense through a tool that uses a household shower curtain as a hundred chart. The hundred chart is a 10x10 grid showing numbers from 1-100 (or 0-99) that students in Grade 1 often see for the first time as they transition from the focus on numbers 1-20 in kindergarten. In Grades 2 and 3, students continue to develop understanding of numbers to 100 and 200 using a variety of counting strategies. For example, students count forward and backward from different starting points and skip count in different ways to demonstrate problem solving (NSDEECD, 2019). Conventionally, teachers use a paper hundred chart for students to show skip counting patterns, calculate two-digit addition and subtraction problems or play "four in a row"-style games. More recently, online versions of hundred charts provide interaction for students through a touch screen. However, a life-size hundred chart allowing students to develop quantity sense through movement may more fully support their mathematical thinking through embodied cognition arising from interactions with the physical world (Lakoff & Nuñez, 2001).

Participants and Context

I work with elementary school preservice teachers (PSTs) enrolled in a two-year post-degree program at a Faculty of Education located in a small university in rural Eastern Canada. The participants I describe in my classroom were Year 1 students completing their first curriculum and instruction course in mathematics education. This 3-credit course met twice weekly for nine weeks and introduced PSTs to the philosophy and strategies for teaching early elementary mathematics (kindergarten to Grade 2). The course focused on methods for engaging students in developing concrete and pictorial representations of pre- and early number concepts through hands-on activities. By instilling positive beliefs about learning and teaching mathematics, I also aimed to boost content knowledge and confidence levels for PSTs.

Before jumping on the shower curtain, PSTs explored pre-number concepts of sorting, seriating, and patterning. This fostered their understanding of one-to-one correspondence and counting. PSTs used counters, ten frames, Cuisenaire rods, and many partitioning activities including hands-on tasks like Bears in a Cave (using bear counters and an overturned bowl as a cave to play with part-whole concepts) and Making Trains (making a train by connecting rods end-to-end to visualize parts of the whole) to familiarize themselves with number families (e.g., 1 + 9 = 10, 9 + 1 = 10, 10 – 9 = 1, and 10 – 1 = 9) and to consolidate the "make-ten" facts. The central focus was to demonstrate conceptual understanding of joining and separating structures by first manipulating concrete materials and then naming the concepts and articulating the procedures.

To continue this pedagogical focus and to extend the numbers to 100, I presented the PSTs with the hundred chart drawn on a shower curtain. I chose the shower curtain to enlarge the typical 10x10 grid printed on paper to "life-size" so that students could walk, skip, or jump on it. I explained how I made the hundred chart on a plastic shower curtain for minimal cost using a straight edge and a marker after carefully measuring the sides to ensure my lines would make one hundred squares. I also noted that once the template was made, subsequent charts could be drawn with relative ease. I asked the PSTs to observe the new tool and to tell me what they noticed and wondered (Figure 1).

Figure 1

PSTs Observe the Blank Shower Curtain Hundred Chart

Immediately, PSTs responded that there were no numbers on the shower curtain—only empty squares. They wondered how students would make sense of the open chart. This was the beginning of our exploration with the hundred chart and my opportunity to model the tool's versatility and the flexibility of the open design. In the next section, I describe how PSTs used the shower curtain to embody their emerging quantity sense and show their understanding of number to 100.

Jumping on the Shower Curtain

When they made their own shower curtain hundred chart, I asked PSTs to leave the cells blank to promote flexibility with the tool and strengthen students' quantity sense at the same time. I explained to the class that the option to place number cards or objects on the shower curtain as part of activities could highlight specific locations and numbers to support students' learning.

Identifying the Corners

Through careful deliberation, PSTs suggested that given a blank chart, students could determine the numbers and therefore the corner numbers would become quite significant. One corner would have to be "1" and another, "100." I then asked, "What numbers would we put in the other two corners?" Finding answers to this question became our first whole-class activity. Choosing four volunteers, I asked each PST to claim a corner. I watched as the four PSTs stepped up to the shower curtain but did not step on it. I encouraged them to jump right on and to walk across it, not around it, if necessary. My intention was for students to use the tool fully and to move their body on the grid just as they might move a counter on a paper chart. Once I saw PSTs standing on their chosen corners, I turned to one and asked, "What number are you?" The response was quick: "I'm number one!" Turning to the second PST, I asked about her number and, after some thought and support from peers, she said, "I must be ten!" I then asked how she knew she was ten and she pointed at the first square. With her finger, she then counted the spaces between them. I accepted her response but as a follow-up, I asked, "How can you prove you are number ten with your body?" Immediately the PST walked across the top row of the shower curtain and jumped from one space to another until she arrived back at her corner. The proof from her body movement and jumps was all we needed, and we were ready to find out about the other two corners. After much discussion, the class discovered that if someone skipped by tens down the "ten" column, they would arrive at the 100 square (Figure 2).

Figure 2

PSTs "Walk" the Shower Curtain

The PSTs also found that if someone "walked backwards" from 100 they would find 91 in the opposite corner. As PSTs identified each number for the corner squares on the grid, with bodily-kinesthetic proof, I asked others to show me different ways to find the numbers for the corner squares. Volunteers quickly jumped on the shower curtain and skipped, walked, and stretched their bodies to show a variety of number pathways including counting by tens starting on one, skip counting by fives, counting on, and counting back. One PST commented, "This is like the hopscotch activity," referencing a previous class in which we explored a variety of ways for students to count to ten while they hopped on numbers. Once corner squares were identified, we continued to play with the corners by asking volunteers to switch places (and numbers) and new volunteers to take up positions on the corners. This orientation to the corners of the blank chart assured PSTs that students could identify any number they wanted to once the corners were determined.

A new conceptualization of the chart emerged when PSTs started to change the position of 1 and 10. When one PST claimed the square on the lower left side of the shower curtain as 1, many peers wondered if this was correct, or even possible. As PSTs explored further, they realized that not all hundred charts are created the same way, and some could start at the bottom and go up, start on the left and go right, or start on the right and go left. Additionally, PSTs wondered if we could incorporate zero onto the shower curtain and, if we did, how it would change the corners and the patterns found in the chart. Based on PSTs' explorations and questions, I decided to progress the lesson to guide the PSTs to find more meaningful ways to interact with the hundred chart.

Playing on the Shower Curtain

I divided the PSTs into six small groups of six, each with their own shower curtain to play on. Previously we had reviewed the curriculum expectations for Grades 1 and 2 as a class, so I challenged the groups to brainstorm ideas for engaging, interactive tasks with young children that would meet these learning outcomes. All groups initially stood around the edges of the shower curtain, pointing to certain spots, gesturing to rows and columns, and discussing ideas for activities. I circulated among the groups, asking questions to prompt discussion, and offering additional materials that might be considered for activities. These included counters, small stuffed animals, small cards, frames to highlight squares, number cards, and large foam dice (Figure 3). Before long, PSTs started to walk on the shower curtain, many of them manipulating the objects I had distribut-

Figure 3

PSTs Designing Activities on the Shower Curtain

ed. The PSTs placed objects and cards on blank squares to identify the numbers and rolled the dice to generate random numbers to find on the grid. I observed several groups identifying the corners and other groups returning to the outcomes to work on adding and subtracting problems. All groups eventually used additional materials on the shower curtain after successfully creating new activities to share with others. I describe these activities in the following section.

PST Suggestions for Shower Curtain Math

Table 1 lists the shower curtain activities created by PSTs to build students' quantity sense kinesthetically, the learning outcomes (NSDEECD, 2019) for students, directions, and the necessary materials:

In all cases I noticed participants' emphasis on embodying quantities for students—for example, walking the space between numbers, jumping from row to

Table 1
Shower Curtain Activities Designed by PSTs

Learning Outcome	Activity	Directions	Materials
Grade 1			
Students will be expected to say the number sequence by • 1s, forward and backward between any two given numbers, 0 to 100, • 2s to 20, forward starting at 0, • 5s to 100, forward starting at 0, using a hundred chart or a number line, • 10s to 100, forward, starting at 0, using a hundred chart or a number line.	What's Your Number?	Throw an object (e.g., a counter or block) on the shower curtain. Where did it land? Tell the number. Walk the pathway from 1 to find the number of the square where the counter landed. Find a different pathway to your number.	• 100 chart shower curtain • Small objects (e.g., counters, blocks, etc.)
	Help Teddy Find Home	Toss the teddy bear on the shower curtain. Designate another square as "home" with a card and describe how Teddy will walk home. Find another possible route Teddy can take to get home.	• 100 chart shower curtain • Teddy bear • Home" card
	Hop, Skip, Jump	Roll a die or two dice depending on the student's level of understanding. Starting at one, hop, skip, or jump according to the number rolled. Start at a different number other than one. Count backward from 100 or 50.	• 100 chart shower curtain • Two dice
Grade 2			
Students will be expected to demonstrate an understanding of addition (limited to one- and two-digit numerals) with answers to 100 and the corresponding subtraction by • Using personal strategies for adding and subtracting with and without the support of manipulatives • Creating and solving problems that involve addition and subtraction • Explaining and demonstrating that the order in which numbers are added does not affect the sum • Explaining and demonstrating that the order in which numbers are subtracted matters when subtracting.	Shower Curtain Addition	Toss a counter on the shower curtain. Roll two dice. Add the number rolled to the counter and find the sum on the chart. Walk the pathway to find the sum. Walk a different pathway to find the same sum. Modify this to try subtraction sentences.	• 100 chart shower curtain • Two dice
	Arrow Arithmetic	Draw arrows on the blank cards. Down arrow means +10, Up arrow means -10, right arrow means +1, and left arrow means -1. Shuffle the cards and lay four cards down in a row. Toss a counter on the shower curtain to find the starting number. Follow the arrows to determine the solution. Ensure students walk the solution pathway.	• 100 chart shower curtain • Arrow cards
	Close to One Hundred	Using a deck of playing cards, partners each turn over four cards to make two two-digit numbers. Each person walks the solution pathway, using addition or subtraction to get the closest to one hundred.	• 100 chart shower curtain • Playing Cards

row or column to column to represent skip counting, and representing the addend in an addition question by moving their body along a pathway to determine the sum. Participation in these activities involved seeing and doing mathematics concretely through movement. This allowed PSTs to experience and feel quantity by performing number operations with the body. PSTs visualized number relationships through the pathways they walked, at times following directions from peers who commented, for example, "Jump down a row, it means add ten." The activities addressed the development of quantity sense by promoting flexibility as PSTs found different pathways to solutions and represented quantities differently through movement (e.g., to represent 50, start at the top of the "10" column and take 5 steps down, counting by 10's, to land on 50; or, starting on the "1" square, take 10 jumps on the grid, counting by 5's, to show the number sequence that begins: 5, 10, 15, 20… etc.). PSTs also embodied partitioning to show numbers in parts to make them easier to work with, a key concept in developing quantity sense. For example, to show 64 as 6 tens and 4 ones, PSTs hopped by 10's to land on 60 and then jumped four more squares to land on 64. Moreover, they utilized estimation strategies as they made judgments based on visual cues and placement of materials (e.g., "I think it landed about halfway down the column," and "Those objects look farther apart, so about 40 or 50 in between"). Further, PSTs' activities showed options for students to feel the magnitude of numbers as they represented their estimates and operations with movement and gestures.

Collaborating in Small Groups

Once each group had shared a lesson idea, I distributed a small booklet with contextual problems for PSTs to solve collaboratively. I wanted to focus attention on the learning outcomes for Grade 2 with particular attention to solving a given story problem by modeling it with materials and by showing different ways to arrive at the same solution. I designed questions for adding and subtracting two-digit numbers to model for students solving problems with a variety of unknown numbers, either an initial amount, a change, or a result. Table 2 shows examples of these questions that asked students to show their thinking, model the problem concretely, and demonstrate possible solution pathways kinesthetically on the shower curtain.

I circulated among the groups observing the strategies that each group entertained to problem solve. All groups were completely engaged with their shower curtain as I observed PSTs walking solution pathways to arrive at the appropriate numbers (Figure 4).

I listened to conversations throughout the groups as PSTs discussed finding starting points, selecting materials to place on the shower curtain, and deciding where

Figure 4

PSTs Collaboratively Solve Problems on the Shower Curtain

Table 2

Adding and Subtracting Problems for Small Groups

Learning Outcome	Question	Structure
You have 41 gummi bears. You eat 23 of them.	How many do you have left?	Result unknown
Molly is collecting baseball cards. She has collected 28, but she wants to have a full deck and a full deck is 62 cards.	How many more cards would she have to collect to have a full deck?	Change unknown
You keep some marbles in a bag. Kaveh, your friend, had some doubles so he gave you 15 marbles. When you combined all the marbles you found out that you now have 86 marbles.	How many marbles were in your bag before Kaveh gave you some more?	Initial unknown

they would go, as well as drawing their pathway in the booklet to show the answer pictorially (Figure 5).

In written reflections, many PSTs commented on the "fun" they had working together on problems and using the shower curtain to "prove" their solutions were correct. One student reflected on previous experiences learning mathematics and commented, "The activities were so different from my math class [in which] we were given worksheets to fill out and the teacher wanted us to work alone." Another PST commented on the shower curtain tool saying, "[The shower curtain] was so engaging. I just wanted to hop on it!" Reflections from students also showed how the 100 grid extended their mathematical thinking. One student wrote, "I could see numbers as tens and ones because we were going up and down the shower curtain counting by tens." One student reflected on the significance of the shower curtain activities for her future teaching by writing, "I learned about the importance of creating an engaging learning environment with the use of manipulatives and collaborative group work," while another student commented, "I learned to be more open-minded when it comes to math and math problems. I also learned that math doesn't have to be about worksheets or sitting at your desk and doing work. There is so much more to it!" Hoping to build on the enthusiastic responses from the PSTs, I asked each group to brainstorm follow-up activities for students that could extend the use of the shower curtain. Below, I describe the possibilities that PSTs envisioned for their future classes.

Future Possibilities

One group of PSTs extended the earlier idea to change the numbering on the shower curtain from 0-99, reorientating the numbers to start with 0 in the bottom right corner. This, they reasoned, would afford students more flexibility in visualizing numbers, and would also align with the understanding for students that "Numbers go up so we should go up the shower curtain as numbers get bigger." PSTs also considered doubling the shower curtain dimensions to represent numbers to 200 or renaming the chart from 101 – 200 to expand students' thinking with three-digit numbers. Another group proposed a Hidden Picture activity (Figure 6) in which each card of a set showed a computation question, a pictorial representation of number using base ten materials, or numbers written in words. Students would determine

Figure 5

PSTs Draw Solution Pathways in Problem Solving Booklets

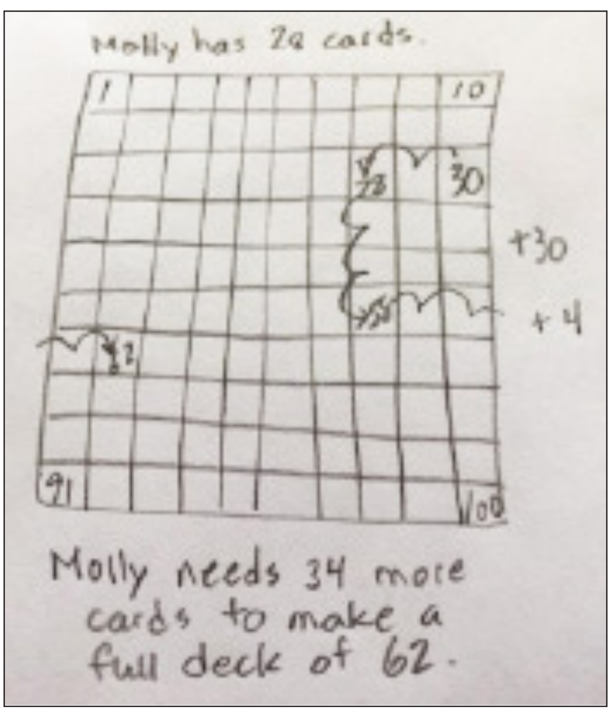

Figure 6

PSTs Design 'Hidden Picture' Activities for Elementary Students

the number on the card, find the location on the hundred chart, and place the card on the appropriate square. Once cards were placed, students could turn each card over to reveal a picture or symbol created by the cards. If all cards were placed appropriately, the image would be clear. This activity would be self-correcting because misplaced cards would alter the final picture. Further, the PSTs considered extending the computation on the cards to include multiplication and division problems for the upper elementary students. In these activities, students would benefit from seeing multiple representations of numbers and from exploring computation concretely to promote flexibility with determining and visualizing number facts.

A third group decided to imagine the shower curtain as a tool for upper elementary students to explore decimals and decided that the entire chart would represent one unit, thereby showing students tenths (each column) or hundredths (each square). This reimagining highlighted the flexibility of the open 100 grid. Along with whole numbers, students could visualize rational numbers for comparing and ordering decimals as well as modeling equivalent decimals. The shower curtain would also serve as a concrete tool for solving problems involving operations with decimals from tenths to hundredths. In these ways, PSTs described experiential activities in which students could use their bodies to show rational number quantities. Such activities support quantity sense development generally as students engage expressively with a wide variety of number relationships found on the grid.

Some groups entertained learning outcomes outside the numbers and operations strands. Activity ideas included exploring measurement by using the squares on the shower curtain to show area and perimeter. One group decided to create giant polygons and irregular shapes cut from cardboard to place over the grid. Students would be encouraged to measure perimeter, first using non-standard units (e.g., lengths of string, dry spaghetti), then using standard units for length and area. Some tasks included extensions for students to find these measurements for compound shapes by deconstructing them in different ways. One group suggested that shower curtain geometry would benefit older students tasked with finding the area of triangles and trapezoids. One activity incorporated spatial reasoning and patterning as PSTs asked students to identify specific numbers with counters to reveal a 2D shape (e.g., place a counter on 25-27, 35-37, and 45-47 to create a square). Tasks involved identifying the shape and looking for patterns in the numbers that make up the shape. PSTs observed, for example, that on one side of the square all the numbers ended in five, on the diagonal the tens and units digit increased or decreased by one, each column represented a pattern of skip counting by ten, and there was an odd-even-odd number pattern in each row. PSTs showed how spatial reasoning could integrate with data analysis when students created concrete graphs on the shower curtain by placing counters, shoes, or other objects in columns on the grid to represent the data using one-to-one correspondence. Students in upper elementary grades could also establish the first quadrant of the Cartesian plane by identifying the bottom of the shower curtain as the x-axis and the left side as the y-axis and create bar graphs representing many-to-one correspondence. With these ideas in mind, PSTs appeared confident to bring the hundred chart shower curtain into their practicum classroom to model interactive and engaging lessons. In contrast to more traditional classroom methods using handouts and seat work, the shower curtain afforded PSTs an opportunity to take advantage of the life-sized grid to support students' developing quantity sense through performance and experiential activity (Gerofsky, 2010; Wagner & Davis, 2010).

Conclusion

The introduction to shower curtain mathematics proved successful for PSTs in allowing them to engage fully in collaborative problem solving through embodied cognition. Although PSTs felt some trepidation at first to walk or jump on the hundred chart, they quickly became engaged and excited about using their bodies to show number pathways, operations, shape and more. Many PSTs reflected on the value of using such a tool to develop children's quantity sense by increasing their interaction, motivation, and enjoyment in mathematics and to appreciate quantities experientially by promoting number flexibility and kinesthetic activity during a lesson. By the end of our introductory class, it was a challenge getting PSTs to step off the shower curtain. This, for me, demonstrated the value in using a tool for kinesthetic learning in mathematics.

In my own practice teaching PSTs at the post-undergraduate level, I recognize value in modeling a new tool and then allowing students the room they need for exploring and imagining possibilities (Iversen et al., 2015). As I circulated the class, I observed how playful movement engaged all learners and stimulated conversation among group participants. Prompts and questions for PSTs (e.g., What are you doing in your activity? What do you see children doing? How can you best support them on the shower curtain?) resulted in richer discussion and more meaningful exchanges (Throop Robinson, 2020) than in previous classes. This example of how to combine simple, everyday materials into an innovative teaching tool may be of interest to mathematics educators seeking to develop quantity sense in their preservice courses for several reasons. Firstly, using a life-size hundred chart appeared to engage students in kinesthetic mathematical activity involving full body movement. As Edwards et al. (2014) suggested, being fully engaged in a task, both mentally and physically, may also yield conceptual benefits for learners as they come to know and feel quantity with their bodies. Secondly, observations suggest that the tool prompted learners to demonstrate flexibility in representing quantity through their embodied cognition (Lakoff & Nuñez, 2001). Finally, the experiential activity of jumping on the shower curtain was perceived to build confidence in mathematical thinking for PSTs who may, in the end, feel more prepared and empowered to teach mathematics to children with playful enjoyment.

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Two Modes of Game-Based Learning for Middle School Mathematics

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ABSTRACT Game-based learning has received more focus as a way to engage students and increase interest in mathematics. Game-based learning should be integrated with effective practices for teaching mathematics, though there is a need for further research on effective implementation models. The purpose of this article is to describe two modes of game-based learning that are aligned with National Council of Teachers of Mathematics effective practices for teaching mathematics including multiple representations and meaningful discourse. Examples will be discussed from the implementation of game-based learning in a Saturday Science, Technology, Engineering and Mathematics (STEM) program for middle school students offered at a large research university.

KEYWORDS *game-based learning, integrated STEM education, middle school*

Middle school is a crucial time for building students' interest in mathematics. In the elementary grades teachers often make use of multiple representations of mathematical concepts using manipulatives and technology, but this occurs less in the middle school grades (Van de Walle et al., 2018). Student interest in mathematics is an important consideration as too often middle school students perceive mathematics to be dull, irrelevant, or too difficult (Grootenboer & Marshman, 2016). In addition, students' interest in mathematics generally declines during the middle school years (Franzel et al., 2010). Making connections to students' interests may be one way to increase interest in mathematics. For instance, students can play sports or video games for hours with few breaks and the time can go by quickly because they are so engaged. A question thus arises: Why can learning in school not foster this kind of engagement?

Game-based learning has been suggested as one way to increase student engagement with mathematics and develop mathematical understanding (Foster & Shah, 2015; Stohlmann, 2019a; Wang et al., 2018). Game-based learning can support mathematical understanding through the incorporation of multiple representations including language, symbolic, pictorial, real-world, and concrete representations. Game-based learning also supports active student-centered learning, including learning through exploration and failure (Devlin, 2011). I adopt Oldfield's definition of *game* (1991): a game is a task that involves a challenge against an opponent, is governed by a set of rules, has a distinct finishing point, and is connected to specific mathematical objectives. When students play games they persevere in problem solving, try new approaches, and continue to develop their strategies when encountering setbacks or failures (Stohlmann et al., 2018). These behaviors are all characteristics of a growth mindset: the belief that one's skills or knowledge can be cultivated through effort (Dweck, 2006), which can lead to increased effort, engagement, and mathematical understanding.

In my prior work I investigated the impact of game-based learning with a class of middle school students that were part of a four-week Saturday Science, Technology, Engineering, and Mathematics (STEM) program. A pre- and post-survey showed that students had statistically significant improvements in aligning their mindsets with a growth mindset. They also improved in the quality of their solutions from the games; by the last week students described more complete strategies, communicated their solutions more clearly, and demonstrated more developed mathematical understanding. Additionally, student engagement was high throughout the program (Stohlmann et al., 2018).

Drawing on this work and my additional implementation of game-based learning with middle school students (Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a; Stohlmann & Kim, 2020), the purpose of this article is to describe two effective modes of game-based learning that middle school teachers can implement: context game-based learning and integrated steM game-based learning. Context game-based learning involves integrating games to pose interesting mathematical questions. In this approach the games are used as a motivating context for the related mathematical task. Integrated steM game-based learning, meanwhile, is the integration of STEM subjects with an explicit focus on mathematics through open-ended technology-based games. In this approach the mathematics is integrated into the gameplay in a substantial way beyond traditional practice problems. Here, students use mathematical thinking and ideas as they play the game. Both modes of game-based learning are aligned with National Council of Teachers of Mathematics (NCTM) effective mathematics teaching practices (NCTM, 2014) including multiple representations and meaningful discourse.

In describing the two modes, I draw on examples of game-based learning that I implemented in the Saturday STEM program. One of the goals of the program was for students to be engaged while completing interesting mathematical tasks. If students' interest and engagement in mathematics can be increased, then it will be more likely that more students will do well in mathematics and possibly be interested in STEM fields (Gandhi-Lee et al., 2015; Lesseig et al., 2016; National Science Board, 2018; Stohlmann, 2019b).

Game-based Learning

Game-based learning has drawn international interest as it has the potential to improve students' motivation and performance in mathematics (Byun & Joung, 2018; Foster & Shah, 2015; Galarza, 2019; Wang et al., 2018). A meta-analysis of studies done with K-16 students found that when compared to nongame comparison groups, technology game-based learning yielded on average a 0.33 standard deviation improvement in learning outcomes across a variety of content areas (Clark et al., 2016). It is possible that the improvement in learning outcomes could be increased with further classroom research on game-based learning that is integrated with best practices for teaching.

The increased prevalence of tablets and computers in schools has created a need to ensure that these technological resources are being wisely used to support students' mathematical understanding (Takeuchi & Vaala, 2014). In a national survey of K-8 teachers in the U.S., 74% reported integrating digital games in instruction, though 80% reported that they would like it to be easier to identify games aligned with standards and objectives (Takeuchi & Vaala, 2014).

Supporting all teachers with game-based learning implementation is important. Denham (2019) conducted case study research with three middle school teachers to investigate how the teachers began to integrate game-based learning. The teachers reported feeling a lack of confidence in how to facilitate the game-based learning effectively. They also noted the importance of thoroughly analyzing the game beforehand to identify how to make the mathematical connections explicit; identify where students may struggle; and note any prerequisite skills students might need to play the game. These findings highlight the importance of having quality examples of game-based learning implementation that can help teachers be confident and successful in implementing game-based learning. Further, the teachers noticed that student engagement was high during the game-based lessons and that the students who usually struggled in mathematics class benefited most from the game-based lessons (Denham, 2019). Their observations highlight the importance of supporting teachers with effective models for game-based learning through teacher-led workshops and/or professional development. This may lead to better game-based learning implementation as many teachers learn about game-based learning from other teachers (Takeuchi & Vaala, 2014). There is a need, then, for guidance on "how, when, for whom, and under what conditions to integrate digital games into formal education" (Van Eck, 2015, p.13).

Implementation and Structure of Game-Based Learning

Guidance and recommendations for effective implementation of game-based learning are important because game-based learning is often not implemented with best practices for teaching mathematics in mind (Byun & Joung, 2018). In 2018, Byun and Joung conducted a meta-analysis of research studies examining the effect of game-based learning on K-12 students' mathematics achievement between 2000 to 2014. Of the 71 authors in the studies reviewed for the meta-analysis, only five of these authors had a background in mathematics education. The effects of not including mathematics education expertise in the majority of the research studies were seen in how the games were structured. Most of the games used in the studies involved drill and practice (Byun & Joung, 2018). In drill and practice-type games, students only receive feedback if answers are correct or incorrect and do not receive support for conceptual understanding. These types of games also emphasize that mathematics is about speed and memorization of facts instead of building conceptual understanding (Bay-Williams & Kling, 2015). Game-based learning for mathematics should move beyond games where drill and practice is the focus so that mathematics is not learned as memorization without understanding. This prior research supports the need for further work in game-based learning in the mathematics education research community so that game-based learning is implemented with effective mathematics teaching practices that support conceptual understanding (NCTM, 2014).

In implementing game-based learning, it is important for teachers not to view games as stand-alone instruction (Galarza, 2019). Instead, teachers should provide students with the opportunity to discuss and reflect on the mathematics within the games. This can involve whole-class discussion and reflection prompts to further develop the mathematical knowledge presented in the games. Foster and Shah (2015) detail one model for this. They propose a four-phase Play Curricular-activity Reflection and Discussion (PCaRD) pedagogical model for game-based learning implementation. In the first phase, play, students are given adequate time to play the game without interruption. During this phase, students are encouraged to discuss the game with other students. The teacher's role is to monitor students while they are working to gather student ideas to share during whole class discussion. The next phase, curricular-activity, is when students complete activities to connect their experience in the game to desired mathematical learning objectives. In the reflection phase, students individually think about and reflect on the connections between the mathematics and the game-play. During this phase the teacher continues to gauge student understanding through listening to or looking at students' ideas and providing assistance as needed. The teacher leads a whole-class discussion in the final phase to address any misconceptions and highlight important student insights.

In my teaching and research with game-based learning, I found that the discussion phase is better as phase three and the reflection phase as phase four. Having the whole class discussion before individual reflection allows for students to better develop their understanding, clear up any misconceptions, and make the mathematics more explicit. It has been noted that students can have difficulty with reflection on game-based learning (Denham, 2019). Having the whole-class discussion first enables students to make mathematical connections and have the opportunity to determine what they know and what questions they still may have.

The final reflection phase can then be implemented using formative assessment strategies to guide future instruction. Formative assessments used at the end of class often include exit slips or journal prompts. This helps students process what they have learned. For example, students can be asked to write three things they learned that are connected to the mathematical objectives, two things they did well, and one question they still have. Reflection through formative assessments can help students metacognitively monitor their understanding and provide the teacher with information to inform further instruction. Teachers should then use students' responses to identify learning needs or make modifications for future lessons (Black & Wiliam, 1998).

In addition to the implementation of game-based learning, learning how the games are structured is vital. From my research I have developed four general principles for how game-based learning should be structured. First, the mathematical objectives should be connected to grade-level mathematics standards. This is important for any mathematical task or activity so that students learn the required mathematical content (Stohlmann et al., 2011). Second, multiple representations should be integrated. Multiple representations help to develop conceptual understanding through students' use of realistic, language, symbolic, pictorial, and concrete representations (Lesh & Doerr, 2003). There should be support for conceptual understanding of the mathematics in the games and not just problems posed symbolically (Stohlmann, 2019a). Third, discussions and activities make explicit connections to the mathematics in the games (Foster & Shah, 2015; Galarza, 2019). Fourth, students should receive feedback from the game or other students during gameplay. Feedback is an important benefit of technology integration (Schenke et al., 2014; Stohlmann, 2019b) and student-to-student feedback can benefit students' mathematical understanding (Keeley & Tobey, 2011).

Game-based learning implementation is most suc-

Table 1
NCTM Eight Effective Teaching Practices

Effective Teaching Practice	Description
Establish mathematics goals for learning	Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations	Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse	Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions	Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding	Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics	Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking	Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

cessful when it is aligned with effective mathematics teaching practices (Kiili et al., 2015). In this way, game-based learning is not a new teaching technique to be learned, but aligns well with existing teaching practices. Table 1 outlines NCTM's (2014) effective teaching practices that connect well with my principles for game-based learning.

The eight practices align with how game-based learning should be implemented. Clear mathematical goals should be set for game-based learning. Game-based learning should promote reasoning and problem solving as students explore and try new ideas. To be the most effective, game-based learning should incorporate multiple representations to support conceptual understanding. Discourse is included as students discuss their ideas with classmates and in whole class discussion. Teachers should plan purposeful questions to ensure students make connections that are aligned with the mathematical objectives for the lesson. Conceptual understanding can be developed through learning done in relevant contexts supported by multiple representations. Productive struggle is seen as students stay engaged with the work and persevere in problem solving. As teachers monitor students working, they elicit student thinking and use this as the basis for discussion. In this article I focus on four of the effective teaching practices that align especially well with game-based learning: (1) establish mathematical goals for learning, (2) use and connect mathematical representations, (3) facilitate meaningful mathematical discourse, and (4) pose purposeful questions.

Two Modes of Game-Based Learning

I now describe two ways to implement game-based learning that I implemented with middle school students (Stohlmann et al., 2018; Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a): context game-based learning and integrated steM game-based learning. I will then discuss how the four previously mentioned effective teaching practices were integrated into each mode. The students were participants in a Saturday STEM program focused on game-based learning. The students who consented to participate in the study were sixth to eighth grade students, most of whom were in sixth and seventh grade. The implementation was structured as a teaching experiment (English, 2003).

Context Game-based Learning

In context game-based learning, games are used as an interesting context to pose related mathematical problems. Context game-based learning does not need to be technology-based. When students play the games, they are not doing mathematics. The games are played for students to collect data or to become familiar with the game so that mathematics questions can be posed. Students do mathematics after the game when solving these mathematics problems. Games can be taken from popular culture to increase student engagement.

Water Bottle Flipping Activity

The first example I will consider is a water bottle flipping activity. The mathematical objectives for this activity are for students to analyze and solve proportional and linear equations through tables and equations. In this activity students play a game to see how many times in a minute they can flip a partially filled water bottle and get it to land straight up (Stohlmann, 2020b). Students are directed to do five one-minute trials in which they record how many "lands" of the water bottle they can make. The world record for this is 47 lands in one minute. The implementation of this activity was completed in the following sequence: (1) The teacher explains the game to the students and an overview of the accompanying worksheet students will complete (Figure 1). (2) Students play the game. Then, for the mathematical part, students calculate their average number of lands per minute based on the data they collected. Students then fill in a table using this average and answer follow-up question (Figure 1). (3) Students complete the questions on the worksheet. (4) The teacher facilitates a whole-class discussion on students' ideas from the worksheet. (5) Using equations from students, the teacher facilitates additional questions on interpreting graphs in the context of the game. The questions in this game-based learning implementation are planned so that students work with multiple representations—realistic context, table, graph, language, and equations—aligned with the lesson goals.

Class Discussion. Students were able to answer the questions in Figure 1 and some students created their own questions to see how long it would take them to do different numbers of lands, such as 1,000. After students completed the questions in Figure 1, a *Desmos* graphing calculator was used to display one of the students' equations for number of lands: $y = 7x$. The students were asked what the slope of the graph meant. They were able to share that the slope was the number of lands per minute. Another student had an equation of $y = 5.5x$ and so I asked if it was possible to get five and a half lands in a minute. A student replied, "that you cannot have half a land." I followed up with asking, "How did he arrive at 5.5 for the slope?" A student shared that it was from five minutes of flipping bottles. Then I asked, "What was done with the number of lands to get 5.5?" Through further discussion students came to see that in the context of how the data was collected, average number of lands per minute was more precise for the description of the slope.

A couple points on the line $y = 7x$ were then highlighted and students were asked what the points represented. For (4, 28) a student responded that "4 is the number of minutes and 28 is the number of lands." I then asked students how they could use the graph to

Figure 1

Water Bottle Flipping Activity Questions

Fill in the table below using your landing rate for 1 minute. Is the relation between time and lands linear?

Time (minutes)	Lands
0	
1	
2	
3	
4	

Write an equation for number of lands (y) made based on time in minutes (n)

$y =$ ____________________

- Use your equation to determine how many lands would you make if you tossed for 10 minutes?
- Use your equation to determine how long it would take you to make 100 lands?
- If you were given a 10 lands head start, how long would it take you for your total lands to be 145?

solve the questions from Figure 1. After a few minutes of thinking, a student shared, “You can go on the line where the number of minutes is 10 and then see what the number is for the number of lands.” Another student shared, “If you know the number of lands, then you start with that and then see what it matches on the line for the number of minutes.” Students were able to see that a graphical approach can be used in addition to a symbolic approach for solving equations.

Analysis of Learning. The water bottle flipping game implementation connected well with effective teaching practices. There were mathematical goals for the lesson that aligned with questions asked related to the game. Through monitoring students while they were working and the whole class discussion it was evident that students were able to make connections between multiple representations through graphs, tables, equations, and the context of the game. For example, students demonstrated the ability to connect multiple representations when they noted the possibility of extending the graph to predict future outcomes. Meaningful discourse about mathematics occurred as students discussed and shared their ideas in the context of the game. Highlighting this is the discussion on the meaning of slope in the students' equations and what the points on the lines represented. Purposeful questions were posed to have students solve problems in the context of the game and interpret the graph in the context of the game. The questions in Figure 1 and the whole class discussion questions were planned so that students would use graphs, tables, and equations. Students found the planned questions engaging and also the questions asked in the whole class discussion. Doing mathematics in a related problem to the context of water bottle flipping engaged students through movement and healthy competition. The mathematical tasks posed in an interesting relevant context also engaged students in the mathematical work and enabled the mathematics to be discussed in the relevant context of the problem. This engagement was seen as students' conversations were focused on the game and the associated mathematical work with no off-topic discussions.

Integrated steM Game-based Learning

The second mode of game-based learning, integrated steM game-based learning, involves open-ended technology-based games. The integration of the STEM disciplines has the potential to bring together overlapping concepts and mathematical and science practices in meaningful ways. This integration is becoming more important because many jobs rely on STEM knowledge (Fayer et al., 2017). However, there is no widely agreed-upon idea for how this best translates into K-12 education. Schools, districts, and researchers can have different interpretations of STEM education that lead to different implementation models (Johnson, 2013). Further research is needed to investigate effective models for integrated STEM education (Lesseig et al., 2016).

It has been noted that mathematics is often not emphasized in the integration of STEM subjects (English, 2017; Gravemeijer et al., 2017). In response to this, I have proposed that mathematics teachers and researchers focus on integrated steM. Integrated steM is the integration of STEM subjects with an explicit focus on mathematics (Stohlmann, 2018b). It is an effort to combine mathematics with at least one of the three disciplines of science, technology, and engineering, into a class, unit, or lesson that is based on connections between the subjects and that has open-ended problems (Stohlmann, 2018b).

My definition of integrated steM education through game-based learning has several important features that help ensure quality implementation. First, the technology integration should allow for significant task redesign or the creation of new tasks that would not be possible without the technology (Puentedura, 2006). Second, the tasks used should be worthwhile tasks. Hiebert et al. (1997) define worthwhile tasks as tasks without prescribed rules, methods, or perceptions of a specific "correct" solution method. Third, the tasks should be aligned with grade-level standards. Fourth, the tasks should enable students to work with multiple representations. Fifth, the technology should provide students feedback. Finally, the tasks should be open-ended so as to allow for discussion and multiple solutions (Stohlmann, 2019a). When structured well, technology-based mathematics games can engage students in mathematics and help develop their conceptual understanding (Stohlmann, 2019a).

The DiRT Dash Game

The DiRT Dash game from NCTM's Calculation Nation website (NCTM, 2020) is one example of game-based learning through integrated steM education. The mathematical objectives for this game are for the students to use proportional reasoning to solve real-world problems. The implementation of this activity was completed in the following sequence: (1) The teacher explained the directions for the game and did an example game for the whole class. (2) Students played the game for approximately 20 minutes. (3) Students discussed in small groups how to do well in the game. (4) Students provided a written response for how to do well in the game.

(5) Students completed follow-up questions to explore if a straight line is always the best path. (6) The teacher facilitated a whole-class discussion on how to do well in the game and if a straight line is always the best path to take. (7) Further questions were posed using the DiRT dash game end of race information.

Focusing on the relationship between Distance, Rate, and Time, the object of the online game is to drive a virtual car on a variety of terrains to beat one's opponent to the finish line. The race terrain is a patchwork of rectangles that each represent one of three terrains: pavement, dirt, or water (Figure 2). Before the beginning of each race there is the option to select a vehicle. Each vehicle has different speeds for the three terrains. Selecting the right vehicle for each game terrain increases the chances of winning.

Class Discussion. After playing this game several times students were asked to explain how to do well in the game. This question was posed to see if students were considering the relationship between the rate of a vehicle for the different terrains and planning the best path to take. This mathematical reasoning is essential for doing well in the game. Students discussed in small groups how to win the game and then provided a written response. The following are examples of three student explanations from when I implemented this game with middle school students.

"To win the dirt game you would have to pick a car that would match the map. Also you would have to find the fastest way to get to the finish line."

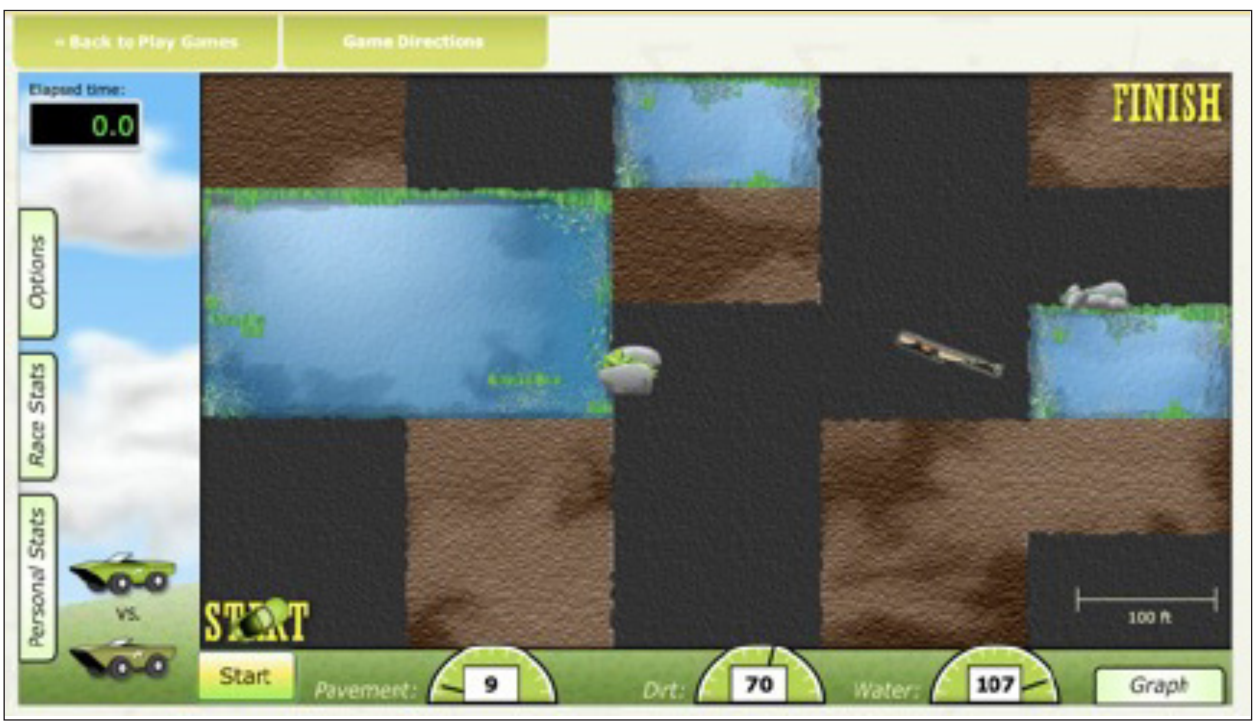
"1. Just cut them off in the start of the race, then just make sure the other car is behind you the whole time.
2. Choose the car that has the most advantages on the area.
3. Try to find the fastest way to the finishing area."

"The best way to win the Dirt Dash game on Calculation Nation is to measure the amount of each element on your path to the finish, and then prioritize certain elements, and choose the car with the high speeds for the element(s) you prioritized."

Though the written examples above do not incorporate specific rate calculations, students used this ratio and rate reasoning in analyzing the terrain of each map including the length of each type of terrain, the best path to take, and the fastest car for this path. Students looked

Figure 2

Example DiRT Dash Game Terrain (NCTM, 2020)



at the terrain and determined the best car to pick, keeping in mind the length of each type of terrain and the available cars speed. This involved determining the car that had the most advantages on the area (terrain type) and then finding the fastest way to the finishing area, keeping in mind the terrain type and the car's speed for each terrain type. Students can realize that a straight line is not always the best path to take in the game in order to win. If there is a vehicle with high speed for a different path in the game, this can lead to a first place finish. Students can use the scale provided in the game to determine the distance that would need to be traveled on the different terrains for different paths to the finish line. As they do this they consider the speed for the different vehicle options. Students did not do the exact calculations but used this mathematical reasoning in the game.

Extension. In order to incorporate further mathematics work with exact calculations by exploring the path taken across different terrains, NCTM has an *Illuminations* activity connected to the DiRT Dash game (Slowbe, 2020). In the DiRT Dash game students did not calculate the exact time it should take them to get to the finish line before playing the game. In this *Illuminations* activity students calculate the exact time for different paths using distance and rate. The *Illuminations* website provides standards-based resources and materials that can be used to implement NCTM's effective teaching practices. The DiRT dash game *Illuminations* activity poses several scenarios to students on the quickest way to get from one place to another by moving through the terrain of sand and/or water. For example, in Figure 3 students are asked to calculate the time it would take to get to their brother when traveling in a straight line towards him. Students can then determine the quickest route to take.

Further, questions can also be posed from the DiRT Dash gameplay. At the end of each race the distance traveled and the time for each terrain are given. A graph is also provided that shows each vehicle's distance traveled and the terrain for the path that was taken (Figure 4). Students can calculate the rate for each terrain and compare this with the ideal rate for each vehicle. The ideal rate may not always be achieved if students have trouble steering or hit an obstacle. Sample graphs can be given as well for students to interpret the graphs in the context of the game for what occurred in the race. For example, I used Figure 4 to ask students questions related to average speed and sections of the graph where there are horizontal lines. Students were asked which vehicle had the greatest average speed between 3 seconds and 12 seconds. Through discussion, students came to realize that they needed to look at the distance traveled for each vehicle in relation to time. I also asked students to explain what was happening when a horizontal line was on the graph. A common misconception was that the vehicle is moving at a constant speed. Since the x-axis is time and the y-axis, distance traveled, students should explain that the vehicle is not moving when there is a horizontal line.

Analysis of Learning. The DiRT Dash game implementation connected well with effective teaching practices. There were mathematical goals for the lesson, for students to use ratio and rate reasoning, that aligned with the mathematics students used during the game and questions asked related to the game. Students were given the opportunity to make connections between multiple representations through discussions about the game with other students, the real-world context of the game, and the symbolic mathematics. Meaningful discourse occurred as students discussed and shared their ideas in the whole-class discussion and as I monitored their small group discussion. Students considered the relationship between distance and the rate of speed of

Figure 3

Example DiRT Dash Game Illuminations Activity Question (Slowbe, 2020)

Your brother notices a starfish on the ocean floor and calls for you to see it. You want to get there as fast as possible, but you are starting in the sand. Refer to the picture below.

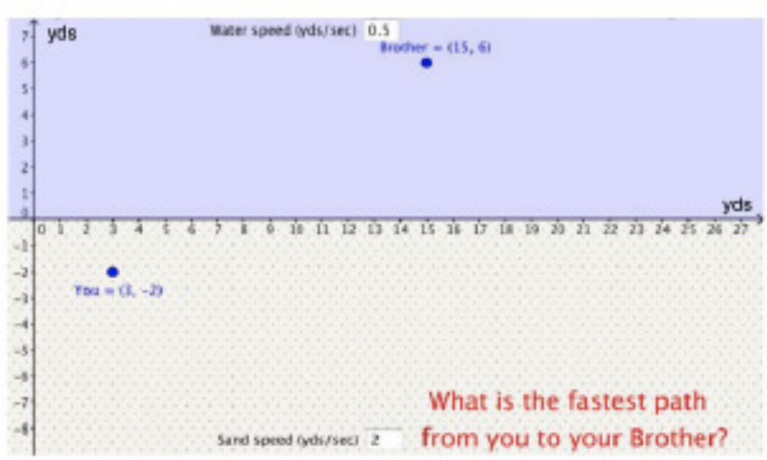
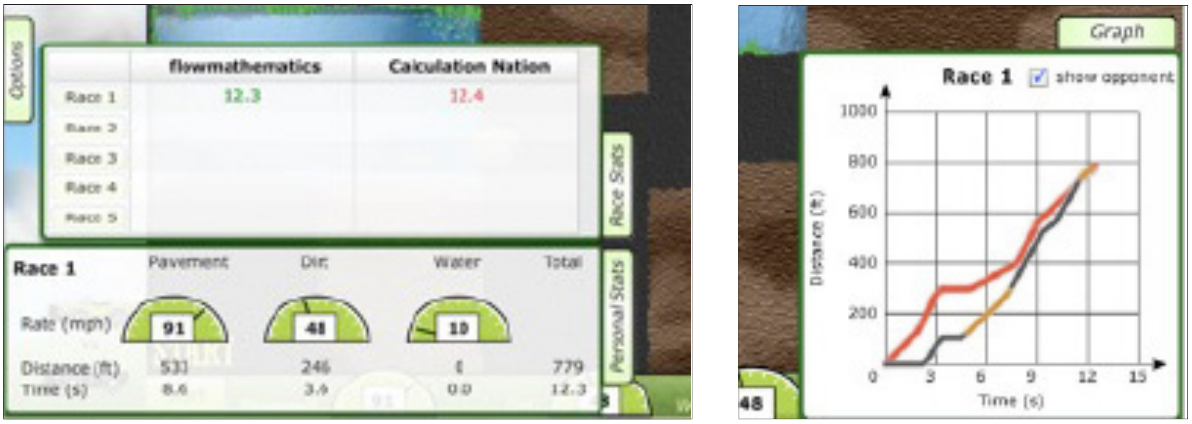


Figure 4

Example DiRT Dash Game End of Race Information (NCTM, 2020)



the vehicles to get to the finish line in the quickest time. For example, a class discussion involved students solving $d = rt$ for t and then discussing how to minimize the time. In the context of the game students discussed generally wanting the distance to be smaller and the rate larger for $t = \frac{d}{r}$. Purposeful questions were posed to have students solve problems in the context of the game and interpret the graph in the context of the game. For example, the questions posed in the *Illuminations* activity had students determine the best path to take to arrive in the quickest time. Students were also asked to describe an end of race graph for someone else's race to see if they could explain what happened in the race. The students were engaged with the game and were able to describe strategies for doing well in the game through the written responses and whole class discussion.

Additional Game-based Learning Activities

A few more examples of types of games that fit within the two modes of game-based learning are robotics and escape rooms. Both of these types of games have engaged students and enabled students to demonstrate mathematical understanding through multiple representations (Stohlmann, 2020a; Stohlmann & Kim, 2020). Students can use programming and robotics, a type of integrated steM game-based learning, to investigate the speed, time, and distance traveled of a robotic ball and then use this knowledge to play several games. For example, this can be done through programming a robotic ball with different speeds to travel in a straight line for three seconds and measuring the distance traveled. Students can then record this information in a table and work to come up with a method to predict the distance traveled given a speed. Along with this task, various games can be incorporated with the robotic balls in which students can use their knowledge from the previous task. Mini golf holes can be drawn on paper or setup with blocks and students can program the balls to end up in the hole. Along with proportional reasoning and measurement, students also apply knowledge of angles to get their ball to go on a path to the hole. Bowling pins can be setup as another game to be incorporated with the programming. A track can also be setup and students can program the balls to race each other around the track (Stohlmann & Kim, 2020). In addition, Dunbar & Rich (2020) describe how a robotic ball may enable students to use proportional thinking to program a ball to go through a maze. This is similar to the idea of mini golf holes as students must program the ball to move in a route from the start to the end of the maze.

Escape rooms are another type of context game-based learning that engages students and encourages mathematical communication. Escape rooms are a "game during which teams solve multiple puzzles using clues, hints, and strategy to determine how to escape from a locked room" (Stohlmann, 2020a, p. 383). Mathematical escape rooms can be designed with a unifying theme, a brief backstory, structures to help students persevere in problem solving, the inclusion of hints if needed, and a compelling twist (Stohlmann, 2020a). I have developed and classroom-tested escape rooms based on the topics of lines (Stohlmann, 2020a) and ratios and proportions (Stohlmann & Kim, 2020). Students displayed high levels of engagement while completing the escape rooms. Moreover, they demonstrated mathematical understanding through modeling the content using symbolic, verbal, and pictorial representations (Stohlmann, 2020a; Stohlmann & Kim, 2020). As in the other examples of game-based learning, design and implementation of escape rooms should align with the NCTM (2014) effective teaching practices to ensure its success.

Conclusion

Difficulty with mathematics can prevent students from degree completion and from pursuing certain careers (Lee, 2012). When students see mathematics as interesting and relevant, this can help students do well in the subject (Kim et al., 2015). The ability to persevere in problem solving is a vital part of mathematics, and game-based learning helps students to do this more effectively (Stohlmann et al., 2018). When structured well, game-based learning has the potential to engage students and support the development of mathematical understanding. Context game-based learning and integrated steM game-based learning are two promising modes of game-based learning that deserve further research work. Yet, many teachers report challenges in implementing game-based learning effectively (Denham, 2019) and there is a need for more guidance on how to integrate games to help develop conceptual mathematical understanding (Van Eck, 2015). Moreover, teachers have reported that they would like it to be easier to identify games aligned with standards and objectives (Takeuchi & Vaala, 2014).

In this article I described two modes of game-based learning that I found to be effective with middle school mathematics students: context game-based learning and integrated steM game-based learning (Stohlmann, 2019a; Stohlmann & Kim, 2020). Both modes incorporate multiple representations, student feedback, and an explicit focus on mathematics. The main difference between the two modes is that integrated steM game-based learning includes open-ended tasks and always includes technology integration.

Both modes enable teachers to implement effective teaching practices including posing purposeful questions, having students learn through multiple representations, generating meaningful discourse, and having clear mathematical goals (NCTM, 2014). These effective teaching practices are aligned with my general principles for game-based learning implementation. Clear mathematical goals help teachers focus on grade-level mathematics standards. Students make use of multiple representations in demonstrating their understanding. Mathematical discussion occurs in the context of the game and questions are posed to make the mathematics explicit. Students also receive feedback from the technology or from other students. Both modes of game-based learning allow for student-centered learning and students to socially construct their knowledge. In the examples that I described in this article and my prior work with game-based learning (Stohlmann, 2018a; Stohlmann, 2019a; Stohlmann, 2020a; Stohlmann & Kim, 2020) students have been engaged with the games and the mathematical work that went along with the games. This is important as students' interest in mathematics generally declines in the middle grades (Franzel et al., 2010). Future research with game-based learning can focus on supporting teachers for effective implementation of game-based learning, further development of games that are well-structured and support conceptual understanding, and investigating the impact of game-based learning on low achieving mathematics students. Game-based learning is a promising method that deserves further focus.

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Teaching Statistics with an Inquiry-Based Learning Approach

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ABSTRACT Two case studies were conducted to examine whether inquiry-based learning (IBL) can help students in understanding normal distributions, and to determine if IBL methods have any effect on students' conceptual and computational capabilities. There were 16 students in the traditional class and 17 students in the IBL-implemented class who participated in each of two case studies. The experimental group performed significantly better than the control group in terms of conceptual understanding and computational skills. These outcomes suggest that IBL methods such as discussions, asking questions, and guided lectures helped students to develop statistical reasoning.

KEYWORDS *inquiry-based learning, statistics, community college, conceptual thinking, computational skills*

Introduction

Research has indicated that requiring remedial mathematics courses in community colleges is problematic because most students required to take these courses will not finish their degree programs (Belfield & Crosta, 2012; Cooper et al., 2017). As such, many students see the completion of remedial mathematics as a barrier to their future academic and professional careers (Benken et al., 2015). In focusing on remedial courses, students often neglect training in applied mathematics courses such as statistics, which is arguably more broadly applicable than algebra (LaMar & Boaler, 2021). Recognizing this barrier, and also acknowledging the benefits of non-remedial mathematics courses, many Science, Technology, Engineering, and Mathematics (STEM) and non-STEM programs in four-year and two-year colleges have expanded their offerings of statistics courses while reducing remedial requirements for elementary algebra, intermediate algebra, and arithmetic. In this same vein, California passed AB 705, a new law that requires students to complete a transfer-level course in English and mathematics within one year (A.B. 705, 2019). Advocates of this law claim that low-achieving high school students can be placed directly into introductory transfer-level English, statistics, and precalculus courses rather than their remedial prerequisites. Accordingly, many community colleges in California, New York, and other states now offer guided pathways courses, which are courses designed to help students develop academic plans at the start of their degrees by providing more consistent and personalized support (Bailey et al., 2015; Logue et al., 2016).

The push to expand statistics course offerings gives rise to novel challenges. Not all mathematics departments have instructors trained to teach statistics, so the quality of courses varies widely across institutions (Boaler, 2016; Lee, 2011). Additionally, many students entering community colleges struggle in basic mathematics, and statistics could be considered advanced, prompting anxiety and fear of failure among inexperienced students (Boaler, 2016; Lee, 2011; Lee et al., 2013). In particular, many community college students who are low-income or students of color have expressed reluctance to study mathematics on the grounds that traditional teaching methods do not work for them (Ban, 2019; Lee, 2018).

Statistics teaches the fundamental skills of analyzing data drawn from real-life scenarios. In the case of descriptive statistics, teaching statistical thinking involves getting students to infer about the methods of designing research questions, collecting data, and explaining data sets, whereas inferential statistics seeks to extrapolate theories and hypotheses from data sets that can be generalized and applied widely. Courses based solely on using textbooks to solve practice problems could limit students' abilities to explore and develop conceptual ideas (Boesen et al., 2014; Lee, 2018). By contrast, as Golubski (2016) demonstrated, a constructivist pedagogical approach such as inquiry-based learning (IBL) helps students develop conceptual flexibility, discover new ideas, and construct their statistical learning process. Thus, we suggest that teaching structures for a statistics course should be redesigned to increase engagement through collaborative, student-centered, and hands-on learning activities.

This article details an ongoing study, conducted by our team of education researchers, which studied the impact of IBL as applied to teaching statistics at community college. The first case study focused on the statistical concept of normal distribution to study how IBL could be implemented to improve student engagement. The second case study examined the difference between students' computational and conceptual thinking skills in the traditional classroom versus the IBL classroom. The two case studies revealed that IBL methods of teaching statistics potentially increase students' computation and conceptual thinking more significantly than traditional teaching methods.

Related Literature Review

Inquiry-Based Learning

A constructivist view of education emphasizes the importance of learners constructing knowledge for themselves; having students multiply integers using their own algorithm, for instance, is better than providing a teacher-specified algorithm (Murphy et al., 2018; von Renesse & Ecke, 2017). IBL, a type of constructivist learning, is an approach to teaching and learning in which students are active participants and the teacher plays a decentralized role. This style of pedagogy encourages students to construct knowledge through their own experience and inquiry. That is, students are encouraged to conjecture, solve, explore, discover, collaborate, and communicate without a teacher emphasizing the formula, axioms, theorems, and procedures (Ban, 2019; Capaldi, 2015). Research has shown that IBL can improve student outcomes (Murphy et al., 2018) by developing students' curiosity and urge to explore (von Renesse & Ecke, 2017). In undergraduate mathematics in particular, IBL leads to increased retention, better participation, and stronger conceptual thinking (Kuster et al., 2018). Instructors do not have to be experts in IBL to reap its benefits; for instance, when they teach using group activities, games, or outside technology, they are using IBL (Offenholley, 2012; Poon, 2018; von Renesse & Ecke, 2017), which makes students active learners discovering their own methods through collaboration. When instructors provide conceptual tools and opportunities for exploration in lessons, students generally produce their own ways of reasoning, build on each other's contributions, develop a shared understanding, and connect experience to standard language and notation (Kuster et al., 2018). According to Beswick (2021), the manner of an instructor's questioning style can stimulate a student's engagement with problem solving, or conversely, impede their understanding, depending on its form and content. In these studies, IBL was used in a statistics class to prompt students to explain why each data analysis measure needs a standard deviation, and how the standard deviation relates to normal distribution.

The Impact of IBL on Computational and Conceptual Skills

Conceptual understanding is a key component of students' ability to develop mathematical proficiency. According to Kilpatrick et al. (2001), "Conceptual understanding refers to an integrated and functional grasp of mathematical ideas" (p. 118) in which students are able to make connections between and within these ideas. For students to grasp new topics conceptually, teachers should leverage their related prior knowledge (Boaler, 2016). A variety of strategies can be used with IBL to help students build conceptual understanding, such as open-ended questions, peer group discussions, and whole-class discussions. Student-centered activities such as these not only call upon students' prior knowledge but also increase students' interest and provide context for learning new ideas. Furthermore, when students are able to share and listen to each other's varied perspectives, these activities enhance students' understanding of the mathematical ideas. Furthermore, IBL's emphasis on higher cognitive thinking through analysis, synthesis, and evaluation of mathematical concepts enhances student engagement, exploration, and mathematical communication skills (Smith & Stein, 1998). When applied to mathematics, IBL supports deeper learning and promotes critical communication in the classroom (Uiterwijk-Luijk et al., 2017). The preliminary research from the first case study may indicate positive, recursive effects on student learning outcomes, such as increased engagement and mathematical communication. In the second case study, IBL strategies were used to support the development of students' conceptual and computational skills with respect to normal distributions.

Methodology

Case Study 1: Teaching Normal Distribution Using IBL

Normal distributions can be used to predict the outcomes of many societal and naturally occurring phenomena such as income distribution, standardized test scores, shoe size, weight, and height. However, the concept of normal distribution has been shown to be challenging for many students (Batanero et al., 1999; Libman, 2010). Students may encounter difficulties when the teacher discusses things being "normally distributed" because they lack the understanding of how to decide whether a distribution is normal (Batanero et al., 1999). The nature of many lecture-style classes may make it more difficult for students to understand the usefulness and relevance of what they are learning, which reduces their motivation (Libman, 2010). To help address this issue, the first case study aims to introduce a new IBL teaching strategy for normal distribution. Using a qualitative case study methodology, the research team sought to learn whether IBL strategies were effective for improving learning outcomes. In Case Study 1, we conducted observations and collected data, including student dialogues and feedback, from an introductory statistics course at a community college in New York City. Findings indicated that IBL strategies could have a positive impact, but further research was needed to determine how much and for whom.

The instructor who implemented an IBL lesson on normal distributions developed a short lecture video that students could watch before class to preview the concept of a normal distribution. The goal of the video was to encourage students to bring questions to class to discuss with their peers. During class, students participated in a series of discussions by reading, writing, and solving problems collaboratively. These discussions were centered around active learning worksheets using IBL. The active learning worksheets used real-life examples through which students could apply mathematics. This activity worksheet was referenced in the institution's current textbook and the activity questions were written by the instructor.

A total of 17 students participated in the study. Group discussions and worksheets were analyzed for how students learned normal distribution via IBL. A researcher determined whether a classroom was an IBL-implemented classroom based on the following characteristics: multiple small-group discussions, series of whole-class discussions, open-ended questions, and IBL active learning worksheets.

During this session, students were prompted to discover the characteristics of normal distribution and develop the understanding that z-scores indicate how much a given value deviates from its standard deviation throughout a series of group discussions. To begin the lesson, the instructor provided a scenario about normal distribution in the prompt (Figure 1).

Figure 1

IBL Active Learning Worksheet on Normal Distribution

Prompt 1

Claire and Susan are very close friends with each other. They are going to the same college and have been taking many classes together. They've always been rivals against each other; they always compared each one's performance and evaluated who gets a higher grade (score). One semester, they are taking statistics classes: but different classes with different professors. After the first exam, Claire got 75, and Susan got 77 on their first exam. The average score for Claire's class is 68 with a standard deviation of 8.2. The average score for Susan's class is 70 with a standard deviation of 6.7.

Question 1: Who got the higher grade on the first exam?

The first group discussion was conducted during the first 10-15 minutes of the class. During the discussion, the instructor introduced Prompt 1 and instructed students to use their prior knowledge to determine the characteristics of normal distributions. Students were expected to use their prior knowledge of mean, variance, and standard deviation in identifying these characteristics. After the first group discussion, the whole class discussed who performed better and why according to Prompt 1. In accordance with IBL teaching methods, the instructor did not clearly state whether Claire or

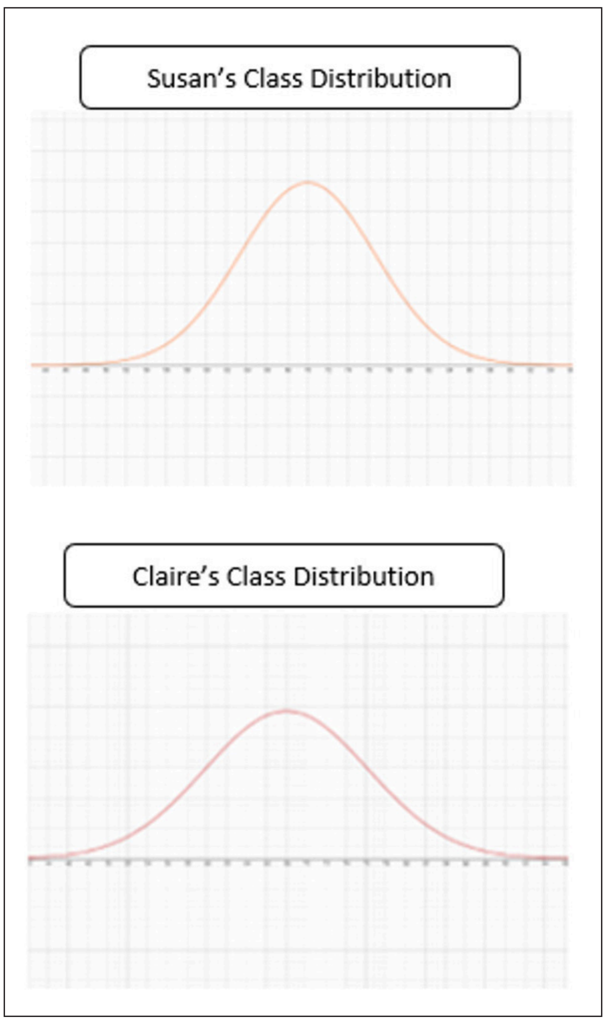
Susan would receive a higher grade in the scenario, nor did they provide a specific procedure for answering the questions; instead, the instructor encouraged students to ask questions and come to their own conclusions. After the first whole-class discussion, the instructor provided the students with graphs that showed the distribution of Claire's and Susan's scores according to the scenario (Figure 2).

In a second group discussion, students were asked to compare two graphs, to identify all similarities and differences between them, and to determine who scored higher. The instructor walked around the groups and told students that both Claire's and Susan's classes had equal variances and were normally distributed. The instructor observed that students compared the graphs while understanding that the two normal curves were equally scaled. During this discussion, which lasted 10-15 minutes, the instructor also asked students to explain why a large standard deviation results in a wider graph and why that is important. Additionally, the instructor asked students to determine who got a higher grade by comparing two graphs as they had not yet figured out how to calculate *z*-scores.

A short lecture followed the second group discussion. The instructor introduced the *z*-score formula and its meaning. During this step, the instructor outlined what *z*-scores represent and how to read *z*-tables. Then, the instructor guided students through the *z*-score formula in the third group discussion while letting them think about the standard deviation of each score and what it represents. The instructor then asked students to analyze Prompt 1 using the formula. Students were expected to explain to their peers what a *z*-score represents and how to read a *z*-table as a third group discussion.

Figure 2

Normal Distribution Curves for Susan's and Clarie's Classes in the Prompt



Case Study 2: Comparing Two Groups' Computational and Conceptual Understandings of Normal Distribution

Case Study 2 is intended to demonstrate how IBL teaching methods affect students' thinking about normal distribution by comparing two different class models for the same-level statistics course: a traditional classroom and an IBL-implemented classroom. In the IBL-implemented classroom, the instructor used worksheets and activities that they designed to help facilitate instruction. The IBL-implemented class consisted of a guided lecture, multiple group discussions, and whole-class discussions. The traditional classroom instructor engaged in direct lectures, individual problem-solving practice, and short class discussions. Both IBL-implemented and traditional classes met twice a week for 90 minutes, at the same institution as in Case Study 1.

The researcher developed an assessment designed to measure students' computational and conceptual skills on normal distribution in both the traditional classroom and the IBL classroom from Case Study 1. The researcher designed the assessment to include eight computational problems (see examples in Figure 3) that gauged students' abilities to read the *z*-score and use it to determine probabilities. 16 students from the traditional classroom and 17 students from the IBL classroom agreed to participate in the assessment.

Additionally, the assessment included conceptual problems to test students' understanding of normal graphs based on different standard deviations, as well as their ability to apply these concepts to real-world problems (Figure 4).

Figure 3

Computational Problems on Normal Distribution

Quiz: Standard Normal Distribution

1. Find the indicated area under the standard normal curve.
   a. To the left of $z = 1.36$
   b. To the right of $z = -0.65$
   c. Between $z = -1.96$ and $z = 1.96$

2. Find the indicated probability using the standard normal distribution.
   a. $P(z < 1.45)$
   b. $P(-2.08 < z < 0)$

3. In a recent year, the ACT scores for high school students with a 3.50 to 4.00 grade point average were normally distributed with a mean of 24.2 and a standard deviation of 4.3. A student with a 3.50 to 4.00 grade point average who took the ACT during this time is randomly selected.
   a. Find the probability that the student's ACT score is less than 17.
   b. Find the probability that the student's ACT score is between 20 and 29.
   c. Find the probability that the student's ACT score is more than 32.

Figure 4

Conceptual Problems on Normal Distribution

Using the two given data sets, answer the questions 4 and 5.

Data Set A: Mean = 65, x = 72, s = 7.5

Data Set B: Mean = 70, x = 77, s = 9.12

4. Select the correct statement or statements (there can me more than one).
   a. Data sets A and B have the same mean deviation.
   b. The mean deviation of Data set A is greater than that of Data set B.
   c. The normal curve of Data set A is wider and shorter than that of Data set B.
   d. The normal curve of Data set A is narrow and taller than that of Data set B.

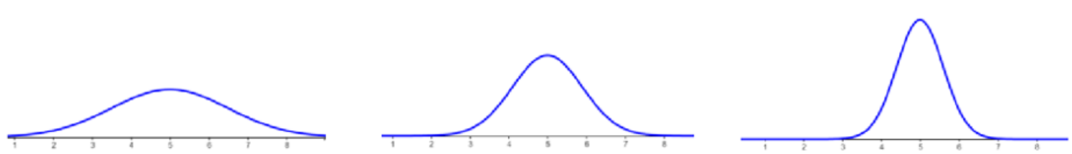
5. Based on the given data, which data set would have greater cumulative area on the standard normal curve?

6. Match each of the following data sets to the most appropriate graph shown below.

   Data set A: Mean is 51 with a standard deviation of 3.2.

   Data set B: Mean is 66 with a standard deviation of 9.1.

   Data set C: Mean is 70 with the standard deviation of 6.



Several instructors who previously taught *Introduction to Statistics* at this institution reviewed and agreed upon this assessment. The researcher used the assessment with the null and experimental groups to measure students' conceptual understanding during the visit. A t-test was used to analyze students' scores on these questions to determine if there were any statistically significant differences between the two classroom models.

Results and Analysis of Case Study 1: Teaching Normal Distribution Based on IBL

Students were given the mean scores and standard deviations of two classes for Group Discussion 1, as well as Claire's and Susan's normal distributions for Group Discussion 2. After the group discussions, the z-score and the table were briefly presented for Group Discussion 3. Without providing additional information, the instructor asked students to determine whether Claire or Susan had the higher grade. At the end of each group discussion the instructor led a brief class-wide discussion. The following dialogue shows how the class discussed the statement after the first group discussion (Dialogue 1).

Dialogue 1

Dialogue After the First Group Discussion

GROUP A: "We think Susan got a higher grade because her score is better than Claire's."

GROUP B: "We cannot say Susan got the higher grade because there are two other values (The class mean and the standard deviation). We think those two values affect the decision."

GROUP C: "If you see the difference between their scores and class averages, the difference is the same. So we can say that they got the same grade."

INSTRUCTOR: *"You mean their mean deviation is the same in two classes?"*

GROUP C: "Yes."

GROUP A: "Oh, I see. That makes sense."

GROUP B: "What about the standard deviation? We really think it affects the decision."

INSTRUCTOR: *"How does the standard deviation affect your decision?"*

GROUP B: "Because…. I don't know. Our group thinks it does something."

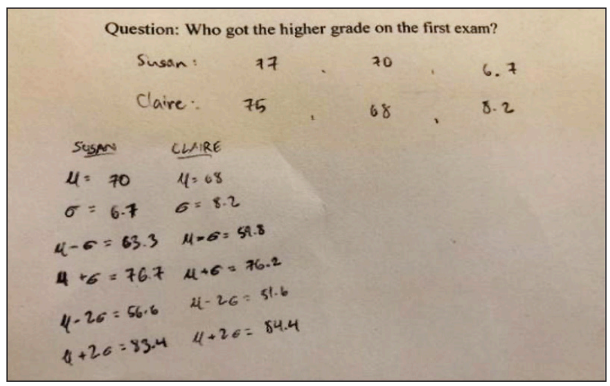
Note: The dialogue was recorded during class and the instructor summarized the class discussion.

Dialogue 1 shows that students in Groups A, B, and C agreed that Susan's and Claire's mean deviations were the same. By asking questions instead of providing direct answers, the instructor guided students to begin to inquire about how standard deviation can affect decision making. Most students knew that standard deviations affect the spread of normal distributions but were unsure how the size of the standard deviation affects its shape. Figure 5 depicts a student's written work on how they determined who received a higher score after the first group discussion.

The student's work in Figure 5 showed that they added and subtracted a standard deviation from the mean. As the students were given the mean and standard deviation, they were able to calculate the distance between the mean and each data point. By using this method students were attempting to construct the concept of a normal distribution.

Figure 5

Student's Written Work Determining Who Received a Higher Score After the First Group Discussion



For the second group discussion, the instructor handed out the normal distribution graphs for Claire's and Susan's classes and asked the students to use them to determine who got a higher grade. After the second discussion, the instructor had a short whole-class discussion (Dialogue 2). All students participated in the group discussions, enabling the instructor to observe their different problem solving approaches.

Observation suggests that most of the groups used the standard deviation to determine who got a higher grade using the graphs. Figure 6 shows two groups' work to determine who got a higher grade using the normal graphs.

Figure 6 illustrates how each group determined the normal graph distribution after the second discussion. One group used the raw data of Susan's and Clarie's scores to find the distance from the average score. Another group used the Empirical Rule, cal-

Dialogue 2

Dialogue After the Second Group Discussion

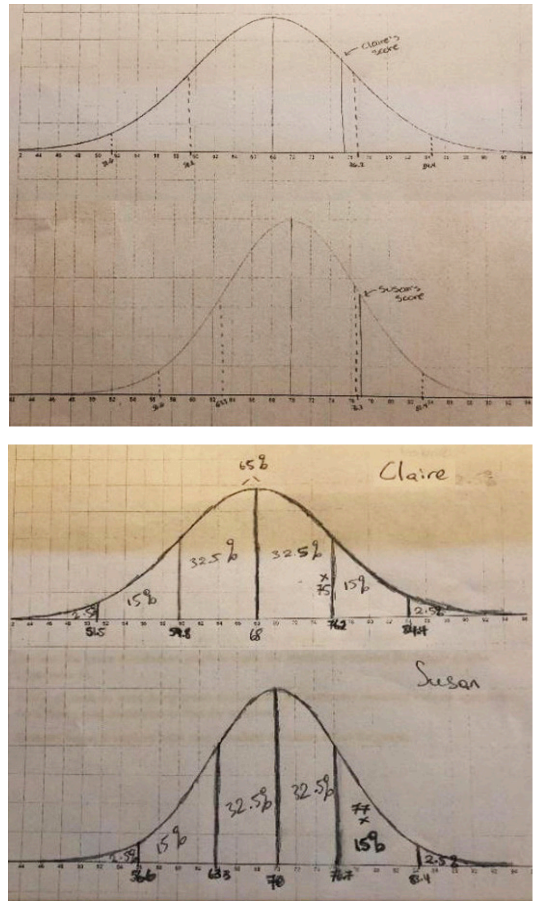
> ***INSTRUCTOR:*** *"Okay. Please look at the standard curve of two classes and compare who got a higher grade. Why are you using the standard deviation?"*
>
> **GROUP A:** "Because the standard deviation is the only value left to do something in this case."
>
> **GROUP B:** "We need to construct the interval using the standard deviation. We discussed that the standard deviation determines the interval from the mean value."
>
> ***INSTRUCTOR:*** *"Okay, that's a good idea. Please show me what you got."*
>
> **Note:** The dialogue was recorded during class and the instructor summarized the class discussion.

culating the percentage to find the distance from the average score. Although those groups used different methods to find the answer, both reached the same conclusion: that Claire's score was below $\mu+\sigma$ and Susan's score was above $\mu+\sigma$. The instructor was curious if students understood how the standard deviation affects the distribution. As a result, the instructor asked the following questions during a whole-class discussion on the topic (Dialogue 3 and Dialogue 4).

In Dialogue 3, students demonstrated that *z*-scores describe the relationship between a value and the mean of a group of values. Students illustrated that the *z*-score

Figure 6

Students' Work After the Second Group Discussion



Dialogue 3

Dialogue After the Third Group Discussion

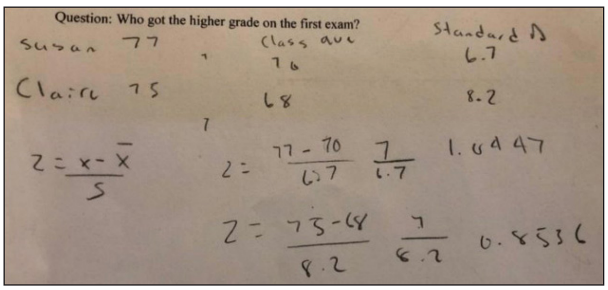
> ***INSTRUCTOR:*** *"What happened here? How could Claire's score be below $\mu+\sigma$, and Susan's score be above $\mu+\sigma$?"*
>
> **GROUP B:** "As you see our graph, Claire's class $\mu+\sigma$ is 76.2 because we added the mean value 68 and the standard deviation value 8.2. We did the same thing for Susan: $\mu+\sigma = 76.7$. And, we compared their scores and the $\mu+\sigma$ value. As a result, Claire's score is below 76.2: she got 75, and Susan's score is above 76.7: her score is 77. Therefore, we concluded that Susan got a higher grade."
>
> **GROUP A:** "We agreed with it. We found the *z*-score of each one's data."
>
> ***INSTRUCTOR:*** *"How could you get the z-score even though we did not discuss it yet?"*
>
> **GROUP A:** "As you suggested, we watched the lecture video and found that out."
>
> ***INSTRUCTOR:*** *"How did you know the z-formula is related to today's activity?*
>
> **GROUP A:** "Because there are three different values: students' scores, class mean values, and standard deviations. We compared this activity with the *z*-formula and realized that the activity is related to standard deviation."
>
> ***INSTRUCTOR:*** *"Great! Could you explain how you applied it?"*
>
> **GROUP A:** "We used the *z*-formula: $z = \frac{x-\mu}{\sigma}$, and found Claire's *z*-score is 0.8536, and Susan's *z*-score is 1.04447. As you see Susan's *z*-score is higher than Claire's, so we concluded that Susan performed better."
>
> **Note:** The dialogue was recorded during class and the instructor summarized the class discussion

is based on standard deviations from the mean, and a z-score of 0 indicates that the data point's score is the same as the mean. In addition, by asking questions, the instructor helped students review what they have learned, allowing them to see how z-scores are formed by standard deviation.

In Dialogue 4, students noticed that z-scores of $\mu+\sigma$ indicate values that are one standard deviation away from the mean. The z-scores may be positive or negative, with a positive value indicating a higher score than the mean and a negative score indicating a lower score than the mean. Figure 7 illustrates how students use a z-score formula to calculate standard deviations.

Figure 7

Student Work to Determine Who Got a Higher Grade Using the Normal Graphs After the Third Discussion



After a brief lecture and whole-class conversation about z-scores, students were able to use z-scores to compare different scales of distribution and engage in an extended discussion to find out who had the higher score. By the end of this activity, with the understanding of standard deviation that students had developed through dialogues, students recognized that graphs can be narrower and taller, or wider and shorter, depending on their z-score. During the analysis of the series of discussions, the researcher found that students could better engage when they are asked to write down their learning, discuss it with peers, and participate in the classroom discussion. The instructor consistently guided students through the inquiry by asking questions. Based on our observations, asking questions stimulated students' interest, guided them towards clear ideas, and aided them in developing their own ideas. Case Study 1 shows how, through a series of group discussions and whole-class discussions, students are able to make connections between various aspects of mathematics (Gillies, 2007). In addition, peer-to-peer discussions promote higher-order thinking and collaboration among students, which has the benefit of helping students gain confidence in talking with their peers, understanding the topic, and sharing their opinions with one another (Gillies, 2007; Johnson et al., 1998). The findings from Case Study 1 suggest that open-questions, discussions, and collaboration can help students develop an understanding of the concept of normal distribution and its properties.

Dialogue 4

Continued Dialogue After the Third Group Discussion

INSTRUCTOR: *"Would you compare the two graphs carefully?"*

GROUP A: "We found out that Susan's class graph is taller and narrower."

INSTRUCTOR: *"Do you understand what "Group A" just mentioned?"*

GROUP B: "It makes sense. The narrower and taller graphs generate a bigger z-score."

INSTRUCTOR: *"When is the case that the normal graph is narrower and taller?"*

GROUP B: "When the standard deviation is smaller."

INSTRUCTOR: *"Why?"*

GROUP A: "When we divided the mean deviation by the standard deviation, we got a bigger z-score on Susan's class, and Susan's class standard deviation was smaller."

GROUP C: "That's right. Because we divide it by a smaller number, the value is bigger."

Note: The dialogue was recorded during class and the instructor summarized the class discussion.

Results and Analysis of Case Study 2: The Impact of IBL on Conceptual and Computational Skills

Figure 8 compares the proficiency for computational thinking between the two groups with the x-axis serving as the question number, and y-axis serving as the percentage of responses that were correct.

Overall, both groups performed well on computational problems when reading the z-scores and calculating their probabilities. Most students in both classrooms had correct answers to computation questions, which implies that algebraic procedures of problem-solving were successfully taught by both teaching methods. According to the t-test (Table 1), however, the IBL classroom performed slightly better on the computational problems than the traditional classroom. Table 1 shows the results of the t-test between

Figure 8

Comparison of Scores on Computational Problems Between Traditional and IBL-Implemented Classrooms

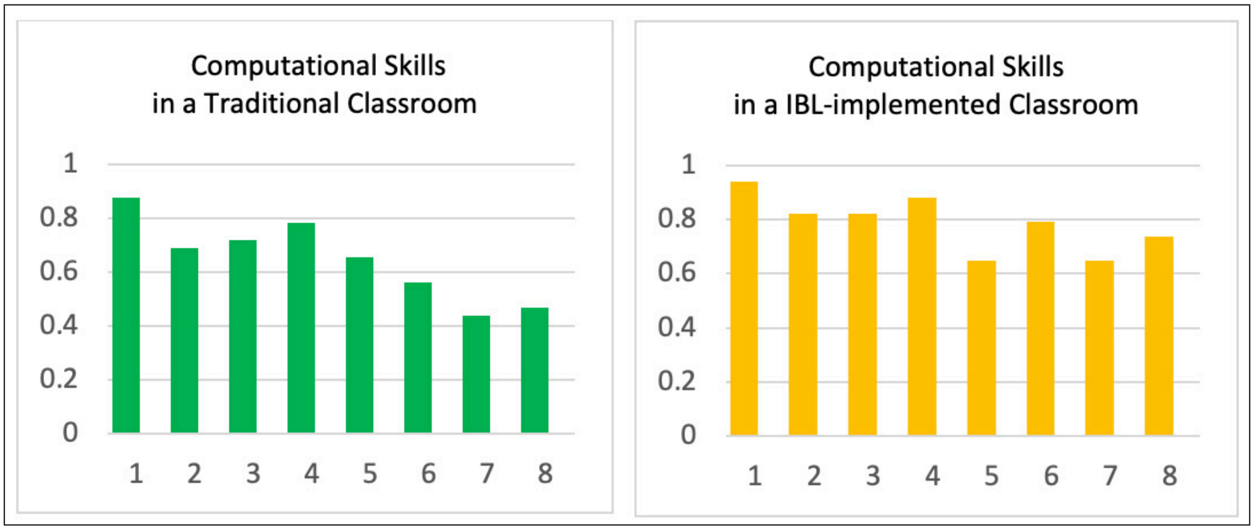


Table 1

t-test of Scores on Computational Problems Between the Two Groups

t-test: Two-Sample Assuming Equal Variances		
	Experiment	*Null*
Mean	0.701680672	0.446428571
Variance	0.022738507	0.07473386
Observations	14	14
Pooled Variance	0.048736184	
Hypothesized Mean Difference	0	
df	26	
t Stat	3.059092341	
P (T < = t) one-tail	0.002548234	
t Critical one-tail	1.70561792	
P (T < = t) two-tail	0.005096467	
t Critical two-tail	2.055529439	

the two groups on computational problem solving.

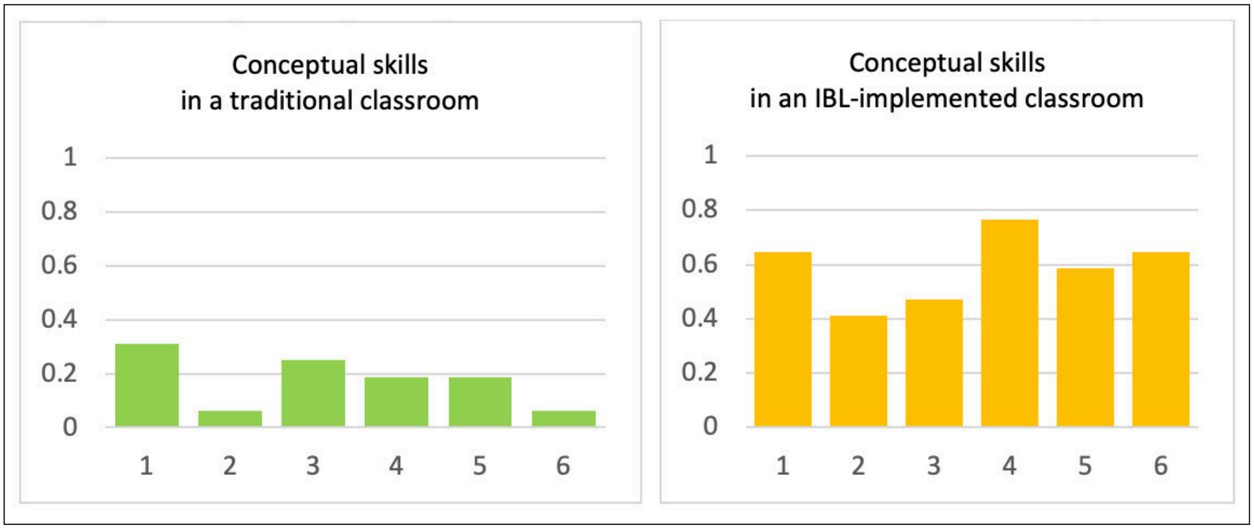
In Table 1, the p values (both with one tail and two tails) indicated that there was a significant difference in the performances of the two groups on computational problems. The results may suggest that students are more likely to succeed as a result of peer interactions and activities. In addition, the results suggest that engaging and collaborative learning experiences may lead to deeper students' understanding of the subject.

In addition, Case Study 2 tested students' conceptual understanding of normal distribution. Figure 9 shows the students' performance on conceptual problems with the x-axis serving as the question number and y-axis serving as the percentage of correct responses.

The IBL classroom's conceptual skills, as shown in the second graph in Figure 9, are generally much higher than that of the null group. For all of the conceptual problems, the students in a traditional classroom rated

Figure 9

Comparison of Scores on Conceptual Problems Between Traditional and IBL-Implemented Classrooms



below 0.4, while the students in the IBL-implemented classroom rated above 0.5 for most questions. There is a clear and considerable difference between the two groups when it comes to conceptual understanding. These findings suggest that IBL simulated development of students' conceptual understanding of course topics.

We also conducted a *t*-test between the two groups on conceptual understanding (Table 2).

These *p* values (both one tail and two tails) further suggest that IBL strategies, such as allowing students to share their perspectives on problem solving, can improve students' conceptual skills.

Table 2

t-test of Scores on Conceptual Problems Between the Two Groups

t-test: Two-Sample Assuming Equal Variances		
	Experiment	*Null*
Mean	0.588235294	0.177083333
Variance	0.016608997	0.010026042
Observations	6	6
Pooled Variance	0.013317519	
Hypothesized Mean Difference	0	
df	10	
t Stat	6.170940073	
P (T < = t) one-tail	5.26933E-05	
t Critical one-tail	1.812461123	
P (T < = t) two-tail	0.000105387	
t Critical two-tail	2.228138852	

Conclusion

In IBL, students are encouraged to become active learners through open-ended questions and peer- and whole-class discussions that guide inquiry. Through this approach, students can increase their proficiency in mathematics, improve their understanding, and stimulate their interests (Riegle-Crumb et al., 2019). In particular, through IBL in statistics courses, students can gain social skills such as listening, problem solving, leadership, and teamwork. The results of this study suggest that IBL provided students with the opportunity to discover important concepts related to normal distribution on their own. Results also suggest that IBL can improve students' communication skills, written work, and statistical analysis, which leads to improvements in their computational skills and conceptual understanding. Moreover, this study indicates that group discussions, open-ended questions, whole-class discussions, and activity worksheets are important elements of statistics courses as they help students develop their own ideas and computational skills. The results are consistent with the findings from a study showing that students were able to talk and engage with their peers to practice thinking about course content through IBL (Dorier & Maass, 2020), which enhanced their statistical thinking abilities. That is, the findings suggest that IBL can be a potentially effective method for developing statistical reasoning through the development of conceptual understanding of normal distribution and the improvement of computational skills in z-score computation. Case Study 2 provides an illustration of how IBL methods in Case Study 1 can be utilized to benefit students' computational and conceptual abilities. It implies that students can gain a deeper conceptual understanding of normal distribution in an IBL setting than in a traditional teacher-focused lecture.

Suggestions for Further Study

This study compared student learning on only one statistical concept: normal distribution. Future research should test the effectiveness of IBL for teaching other statistical topics as well. Furthermore, future studies should examine overall student grades, pass rates, and student satisfaction with their learning experience in order to gain greater insight into the potential effectiveness of IBL in statistics courses.

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Supporting and Retaining Early Career Mathematics Teachers Using an Online Community of Practice

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ABSTRACT This study reports on efforts over several years to design and implement a yearlong intervention intended to support secondary mathematics teachers in their early years of teaching. The intervention is designed to retain early career secondary mathematics teachers in the profession by engaging them in the development of meaningful professional relationships with school-based mentors, and by creating an online community of practice for support with other professionals. The intervention itself consists of early career teachers and their mentors participating in monthly professional learning activities such as online meetings, videoconferencing panels with experts, and collaborative reading and discussions of timely, purposeful, and relevant content. The intervention is designed not to overburden participants and to be feasible for national implementation with little to no funding. This article presents the purpose, design, and implementation of the intervention, as well as a discussion of challenges faced and potential next steps and future directions for similar work.

KEYWORDS *teacher education, in-service professional development, secondary mathematics*

Purpose of Study

The most extensive survey of teacher mobility is the Teacher Follow-up Survey, conducted by the National Center of Educational Statistics and last administered in 2012-2013. In their analysis of this survey data, Carver-Thomas and Darling-Hammond (2019) reported that approximately 8% of all teachers are considered "movers'' and move to a different teaching job each year, while another 8% are considered "leavers" and leave the profession altogether, putting overall annual teacher turnover at about 16%. Research has found that the highest rate of teacher attrition occurs in high-needs schools (Fantilli & McDougall, 2009; Goldring et al., 2014). Most recently, Carver-Thomas and Darling-Hammond (2019) reported that turnover rates of mathematics and science teachers are almost 70% greater in Title 1 schools than non-Title 1 schools. As a result, the United States spends upwards of $7.3 billion annually to address teacher attrition (Learning Policy Institute, 2017; National Math + Science Initiative, 2014).

According to Carver-Thomas (2018), 40% of new mathematics and science teachers are underprepared, and are most likely to teach in high-needs schools with a majority of historically marginalized populations of students. Research has shown that when teachers leave the profession, this results in an increase in the proportion of teachers lacking experience or certification (Carver-Thomas & Darling-Hammond, 2019; Sutcher et al., 2019). This issue is further exacerbated in schools that predominantly serve students of color or low-income families, where teachers are much more likely to be prepared and recruited through alternative certification pathways (Carver-Thomas & Darling-Hammond, 2019). This attrition rate, in turn, is believed to "signal lower quality of education for students" (Sorensen & Ladd, 2020, p. 13). In addition, enrollment in teacher preparation programs has also declined nationally by more than one third since 2010 (Partelow, 2019). The culmination of these factors has led to classrooms staffed with underprepared or unqualified teachers, which profoundly affects the mathematical preparation of students in high school, college, and beyond (Kini & Podolsky, 2016).

Researchers agree that challenges with mathematics teacher recruitment and retention need to be addressed to support the development of a stable and qualified teacher workforce (Carver-Thomas & Darling-Hammond, 2019; Fuller & Pendula, 2019). This entails building more cohesive systems of teacher preparation, development, and support (Espinoza et al., 2018; Mehta et al., 2015), as well as prioritizing recruitment and retention. In their synthesis of research on Science, Technology, Engineering, and Mathematics (STEM) teacher preparation and retention, Fuller and Pendola (2019) found several factors related to teacher attrition and turnover, including personal characteristics, school characteristics, school leadership, and the quality of preservice preparation. Teachers with certain personal characteristics, such as being younger, less experienced, close to retirement, identifying as a person of color, or having a graduate degree are more likely to leave a school or the profession (Carver-Thomas & Darling-Hammond, 2019). Fuller and Pendola (2019) also consistently found that school districts paying lower salaries tended to have higher attrition rates. Other studies found high attrition rates for school characteristics such as working conditions (unclean/unsafe facilities, poor administrative support, large class sizes, insufficient resources for students) as well as a school's Title I status (Scafidi et al., 2007; Simon & Johnson, 2015). Schools and districts need to put systems in place to support teachers, particularly those who are less experienced and who work in underserved communities, in order to retain them over time.

Many of the factors described above are systemic and cannot easily be addressed by an intervention. There is evidence, however, that other factors can be leveraged by teacher preparation programs and school districts. Research shows that teachers feel supported when given the opportunities to be involved and connected with colleagues, and that supports such as induction programs can have a positive impact on teacher retention (Ingersoll, 2012). These programs are designed to be comprehensive supports for novice teachers as they develop their practice and can include both tools and resources for teaching as well as professional development and mentoring partnerships (Kaufmann, 2007). A national survey of teachers found that a significant number of teachers identified involvement in professional learning communities, coaching, or mentoring as important support mechanisms for professional growth (Goe et al., 2017). Additionally, the more involved teachers were in professional development, the more likely they were to express a desire to teach in the long term (Coldwell, 2017). Berry and Berry (2017) state, "For teachers, a key aspect of the [National Council of Teachers of Mathematics (NCTM)] Professionalism Principle is recognizing that their own learning is never finished and that they must build a culture of professional collaboration that is driven by a sense of interdependence and collective responsibility" (p. 155). This suggests that teachers should be collaborative and prioritize professional learning early in their careers. Working from this literature and a survey conducted by our research team (Amick et al., 2020), we chose to focus on addressing the issue of teacher retention through an intervention meant to promote effective mentoring and provide professional development for novice teachers.

We report on the design and implementation of a cost-effective, easily replicable intervention for early career secondary mathematics teachers with the goal of positively impacting teacher retention. Thus far we have had mixed results in terms of participation, both in terms of the number of total participants as well as how actively participants have engaged in the interventions. Therefore, we also share the challenges faced and present lessons learned over two years of enacting the interventions to provide suggestions for future research.

Background

The authors of this paper are members of the Secondary Teacher Recruitment and Induction in Diverse Educational Settings (STRIDES) research action cluster (RAC) and work at institutions of higher education and state organizations across the United States. The work of this RAC addresses the Association of Mathematics Teacher Educators' (AMTE, 2017) *Standards for Preparing Teachers of Mathematics* Standard P.5—Recruitment and Retention of Teacher Candidates. Standard P.5 specifically delineates teacher recruitment and retention as one of the "five important standards that are essential aspects of effective mathematics teacher preparation programs" (p. 43). The STRIDES RAC is one of five RACs that were formed within Mathematics Teacher Education Partnership (MTE-P) that utilizes a Network Improvement Community (NIC) model, an alternative to locally structured problem solving groups, where professionals from different geographical and institutional settings bring individual strengths and perspectives to bear on educational issues. The NIC model is supported by researchers such as Bryk et al. (2011) who state, "In an arena such as education, where market mechanisms are weak and where hierarchical command and control is not possible, networks provide a plausible alternative for productively organizing the diverse expertise needed to solve complex educational problems" (p. 5).

Our research team came together through the MTE-P with a shared interest in early career teacher support and retention. MTE-P as a whole is composed of over 40 teams across 31 states that include over 100 universities, university systems, community colleges, K-12 schools, and school districts. MTE-P participants work collaboratively to redesign secondary mathematics teacher preparation programs through coordinated research, development, and implementation efforts. The partnership takes a comprehensive approach to tackling this challenge, convening community colleges, universities, and university systems, as well as K-12 schools, state departments of education, and other education-focused organizations (for more information about the partnership, see Martin et al., 2020). The partnership serves as a clearinghouse for model programs and practices, and also advocates for change at university, state, and national levels.

Theoretical Framework

Novice teachers often feel isolated, and those feelings of isolation are often associated with teachers leaving the field (Carroll & Fulton, 2004; Schlichte et al., 2005). Our STRIDES RAC promotes the perspective that teacher retention would improve with the development of communities of practice to provide a support network to draw upon, including online communities (Wenger, 2011). Communities of practice are "groups of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (Wenger, 2011, p.1). Wenger (2011) further shares three features that characterize communities of practice: a domain of interest, common members who participate in joint activities and discussions, and shared practice. For our work, the domain of interest is middle or high school mathematics teachings in the early years of a teacher's career. In this context, the community of practice consists of early career teachers who are being inducted into the field, mid-career mentor teachers, and university mathematics teacher education faculty. We recognize that the work of retaining teachers requires, in part, a focus on developing relationships within the educational community and promoting connectedness within the larger community (Minarik et al., 2003). Recently, our RAC has focused on creating an online community of practice with early career teachers, mentors, and university faculty to develop relationships across multiple states and teacher preparations programs to try to support and retain early career secondary mathematics teachers. It is our hope that by creating and fostering these communities of practice, early career secondary mathematics teachers will feel a sense of support and that any feelings of teacher isolation that may cause them to leave would be mitigated.

Past Work: Driving the Design

In 2014 our RAC created a survey as an initial step to study the current support systems of early career secondary mathematics teachers. The main goal of the survey was to better understand the degree to which early career mathematics teachers perceived various learning opportunities as influential to their interest in teaching mathematics. One research question guiding this work was: What is the perceived scope, nature, and impact of professional support for early career mathematics teachers? Members of our team created the survey through an iterative design and vetting process that extended from the fall of 2014 throughout early 2016. The survey consisted of 25 questions including multiple-choice, Likert scale, and open-ended items, asking respondents to report on their current support systems, job satisfaction, projected longevity in the field, and other related topics. We administered the survey in November of 2016 and gathered 144 responses from teachers across the United States. Results from this study were presented in Amick et al. (2020). By better understanding current support systems, we hoped to develop interventions that would strengthen and replicate systems proven to work, and attempt to improve broken ones. We present some of the main results that guided the design of our interventions in Table 1.

As shown in Table 1, the vast majority of novice teachers (134 out of 144) reported participating in mentoring or coaching. Of these 134 teachers, 88.8% found the experience to be moderately or very influential on their enthusiasm for teaching mathematics. This finding is consistent with other research on induction programs (Ingersoll & Strong, 2011; Youngs et al., 2019). In their review, Ingersoll and Strong (2011) found that induction programs and especially teacher mentoring programs positively influenced novice teachers' satisfaction, commitment, and/or retention. Further, the rates at which early career teachers left the field decreased significantly when they were paired with a mentor who taught the same subject area and shared a common planning

Table 1

Results from 2016 National Survey of Early-Career Mathematics Teachers

Learning Opportunity	Number of Participants	Influence of Experiences on Enthusiasm for Teaching Mathematics	
		Not Influential (% of Participants)	Moderately/Very Influential (% of Participants)
Mentoring/Coaching	134*	11.2	88.8
School/Department meetings	128*	34.4	65.6
Professional Coursework	60*	20	80
Professional Conference	60*	11.7	88.3
Online Community of Practice	50*	34	66
Online Resources	51*	35.3	64.7
* Out of 144 total respondents			

time (Ingersoll, 2012). He also found that having multiple induction supports such as meeting regularly with building leadership, engaging in formal mentoring programs, or being provided common planning time with colleagues, had a strong positive effect on retention (Ingersoll, 2012). With this past research and our survey results in mind, we sought to develop systems that could effectively support and retain early career mathematics teachers. We argue this work arises from the need to further explore, strengthen, and replicate these systems nationwide in an effort to increase early career mathematics teachers' longevity in the field.

Methods

In this section we report on the methods for recruiting participants; the design of two iterations of the implementation of a yearlong series of intervention activities situated within an online community of practice; and the sources of our data. Our overall methodology for this work is a design experiment approach (Cobb et al., 2003), focusing on a problem in practice and pragmatically designing an intervention to impact that problem with multiple iterations of implementation and (re)design.

Participants

To recruit early career secondary mathematics teachers for this study, we extended email invitations to recent graduates of teacher preparation programs at our home institutions at the start of the academic year. For our first year of implementation during the 2018-2019 academic year we only invited teachers in their first year of teaching, though in the second year of implementation in 2019-2020, we opened the participant pool to both first- and second-year secondary mathematics teachers in hopes of increasing the number of participants. We asked participants to commit to participating one hour per month in the study in exchange for a yearlong support intervention and a community of practice consisting of teachers in the same places in their careers. The goal of the interventions was to provide timely resources and information to participants that could subsequently foster opportunities to collaborate and discuss with mentors as well as the online community. We requested that participants recruit a mentor teacher if they did not have one assigned to them already, and we encouraged participants to seek a mentor who also taught mathematics, as research states having a mentor that teaches in the same field is most impactful (Ingersoll, 2012).

The participants for both years of implementation included a diverse group of early career secondary mathematics teachers located in Kentucky, California, Texas, New York, and Ohio, who taught a variety of mathematics courses and grade levels (6-12) in settings that ranged from large urban districts to small rural schools. During year one, seven teachers volunteered to participate. At the time of their registration (late August 2018), four of those teachers did not know who their school site mentors would be. During year two we expanded our recruitment of early career teachers and 15 teachers initially registered, 10 of whom already had school site mentors assigned.

Design

Due to both current research in the field and our RAC's survey results pointing toward mentoring as an extremely impactful induction experience, our group focused on the mentoring relationship as the basis for the first iteration of our intervention. We enacted the first-year intervention throughout the 2018-2019 academic year. We designed the intervention to provide targeted support to first-year teachers by (1) strengthening the mentor/mentee relationship through monthly communications; (2) suggesting targeted discussion topics between the mentor/mentee teachers; and (3) providing synchronous online meetings to build a community of practice.

We designed the intervention to include approximately one hour of active participation each month to minimize the strain on the participating early career teachers. Furthermore, we designed the intervention to engage early career teachers with their mentors and to foster their engagement in an online community. For example, in one monthly intervention we sent out an article about effective mathematics teaching practices for participants and their mentors to read together and discuss. Activities such as this were intended to provide opportunities for the pairs to take part in learning together to provide a supported space for mentor teachers and first-year teachers to meet and build positive relationships. As our STRIDES RAC did not have funding to incentivize participation, the goal was to provide a structure to facilitate the development of a functioning community of practice by helping participants build relationships with their mentors, connect with other new teachers, and have access to timely, easy-to-implement interventions.

These activities were based both on the literature on novice teacher needs, as well as on the experiences of our research team. Several of our team currently support or have previously supported first- and second-year teachers through state-level induction programs. We relied heavily on those experiences in tandem with the current research to create a list of monthly engagement activities that would serve as the year one intervention. Current early career teachers who were graduates of the authors' teacher preparation programs also contributed to listing out monthly topics/activities that they felt would benefit them in these early and critical years. The activities were designed to be timely (for example, discussing high stakes testing prep in the spring). In addition to identifying topics related to general new teacher supports, we selected several topics aligned to the Common Core State Standards of Mathematical Practice (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and NCTM's (2014) *Effective Mathematics Teaching Practices.* For example, in October 2018 we organized a videoconference panel of teachers with three to five years of experience— early enough in their careers to be relatable to the participants. The goal of this videoconference was to offer participants a sounding board with which to vent frustrations, ask questions, and seek advice. According to the National Institute for Excellence in Teaching (2020), novice teachers often need support not only in learning the curriculum, but also in developing their ability to create and manage an engaging learning environment to meet the needs of diverse learners. Prevalent topics in our virtual discussion therefore included eliciting student thinking, student engagement and participation, and selecting and/or modifying curriculum. In November, we introduced participants to the *First Year Teacher Curve* (Moir, 1990), which describes the stages of anticipation, survival, disillusionment, rejuvenation, reflection, and anticipation that many first-year teachers experience. We encouraged participants to reflect on the curve, determine their current location on the curve, and talk to their mentors about their needs for support. We specifically chose to do this activity at this time of year because the First Year Teacher Curve suggests that the lowest point in a teacher's first year occurs during late October through December.

Before the winter break, we asked participants to engage in professional goal setting that would enable them to start fresh in the upcoming semester. Some of their goals included refocusing on classroom management by restating expectations and following through consistently with consequences; effectively using formative assessments to gauge procedural and conceptual understanding; and promoting student engagement through teacher questioning. After winter break, we asked the participants for midyear feedback based on their experiences thus far and the goals they set for the second semester. Our aim in soliciting this feedback was to determine if we needed to make modifications in the spring. One desire participants reported was to begin the second semester by establishing more collaborative and student-centered teaching practices. Based on this feedback, our research team distributed a number of mathematical tasks that focused on student-to-student discourse and supporting students in productive struggle, both proven methods to increase student engagement (NCTM, 2014). We followed up with a second videoconferencing session.

In an effort to improve the intervention, and in keeping with a design experiment approach, we modified the intervention over the course of the year based on continual analysis. To do so, we solicited monthly feedback from participants via email, as well as through a midyear survey administered in December 2018 and an end-of-year survey in May 2019. The research team utilized this information to make ongoing adjustments to the intervention model in an attempt to better support participants and keep them engaged for the entire intervention year. After year one of implementation the team went through a data-driven (re)design process during the summer of 2019 to prepare for the second year of implementation in 2019-2020.

During the second intervention year, we made several changes to the program after examining the collective data from year one. For example, we added activities around standardized testing in mathematics and issues of equity as spring interventions based on participant feedback. In an effort to foster a stronger community during the second iteration of the intervention, we created a Facebook group for participants as another way to connect in the virtual environment between monthly sessions. We then posted resources to the page periodically to attempt to create an online space for dialogue among participants. We also sent an email in early September including several self-care resources and asked participants to peruse and discuss these resources with their mentor teachers. Making a personal commitment to self-care early on each school year was one common desire of the year-one participants.

Data Sources

The main data sources of this study consisted of feedback from the participants over the course of the year in response to prompts provided in monthly intervention emails, and also in midyear and end-of-year surveys completed by both the teachers and their mentors each intervention year. The mid- and end-of-year surveys collected information on how useful the new teachers found each of the monthly interventions, what supports they had, and what additional supports they wanted. The surveys also asked the new teachers whether the support they received through our program had an impact on whether or not they intended to continue in the profession in the future. We surveyed the mentor teachers at a later date on which aspects of the intervention seemed useful to the early career teachers, and to gather feedback on what we might improve for future implementations.

Results

In the first year of intervention, participants identified that they would like suggestions to address challenges they faced in their classrooms, including broad topics like classroom management as well as topics dealing specifically with implementing effective mathematics teaching practices. This led us to implement a revised set of intervention topics in the second year, as well as to include additional structures intended to foster community among participants. In spring 2020, as we completed our second year of implementation, we were still met with mixed results as to the usefulness of the intervention. We identified several specific challenges that may have impacted the success of the intervention in developing a well-functioning online community of practice: a lack of common times to engage synchronously, low engagement by mentors, and a sense of being overwhelmed by the participants by midyear.

First, for those who remained engaged with the group and attended sessions consistently both years, we received overwhelmingly positive feedback. This feedback was received via email and survey feedback and spoke to the usefulness of the videoconferencing teacher panels, both during the panel as well as after the panels. The early career teacher participants reported the panels as being helpful to connect and talk to others that have a shared experience. They also greatly appreciated the practical advice from the RAC members that was grounded in research. One participant expressed her gratitude for the work of the RAC in stating, "I am so very appreciative of all the work that you are doing for us and our kids. I always find the content you provide so insightful, and it helps me reflect." Unfortunately, the attendance for the videoconferencing teacher panels was low, with only two to five early career teachers participating in sessions each time they were offered across the two-year cycle of interventions. We attempted to address this in year two by staggering the panel meeting times, hosting a weekday evening session in the fall and a Saturday afternoon session in the spring. Novice teacher participants continued to report several challenges that impeded attending the panels including limited access to videoconferencing while on their school district's network, as well as afterschool and weekend coaching obligations during the proposed panel times. In the 2019-2020 year of implementation, even after staggering the meeting times, we had only four total participants in the teacher panels.

Second, although we encouraged the engagement of mentor teachers both years, we had no mentors join any of our virtual panels in either year of the interventions. In fact, in the data we collected across both cycles of intervention, many of our participating teachers reported that they had very little interaction with their assigned mentor teachers. For example, during year two of the intervention, a total of five participants completed the midyear survey and, of those, three reported having "little to no interaction with [their] mentor." In the survey, one participant described her relationship with her mentor teacher as "non-existent. [It is] based on appearances and there is no real support. I lean on my ELA teacher team for support." Another participant shared a similar situation, stating that her mentor teacher was "distant." Even when participants reported having positive relationships with their mentors, the mentors did not engage with our interventions or teacher panels. One of the main intervention goals was to help foster relationships between new teachers and their mentors to provide a system of support, but instead, members of the research team and invited panelists were the only ones who engaged in the community of practice we attempted to develop. As a number of participants reported the lack of strong mentoring support being provided on site, our team sought to identify other ways to help support them through our interventions.

Finally, the novice teachers we received feedback from often reported feeling overwhelmed. Novice teachers were most overwhelmed with planning and managing student engagement. For example, in the midyear survey one participant shared, "Planning is my biggest issue this year. I feel as though I am just above water with planning. I don't want to spend all of my free time on school; it's not fair to my fiancé or myself!" Although we had hoped that our Facebook community could be a supportive environment for our early career teachers, this also required them to use their limited free time to engage. In addition, Facebook did not seem to be an engaging environment for some. One novice teacher explained their rationale for this as well and offered a potential suggestion: "I am not a Facebook person, so that is probably why I did not engage in many of those conversations. I saw them too late. I think it would be beneficial to have a Google Classroom setup instead. This would be an easy way to organize any materials you want to share and set up 'assignments' so we receive automatic reminders." Another participant shared that they tended to use Twitter as a preferred social media platform, while others reported face-to-face interactions as their favored method of networking with other teachers. The lack of a consistently used platform by participants made it difficult to foster relationships among participants via social media.

Overall, we struggled to keep novice teachers involved and engaged in the intervention beyond the first few months. This low participation aligned with their self-ratings on the midyear survey question "On a scale of 1 to 10, how overwhelmed do you feel in your job?" Novice teachers reported an average of 6.2 on their self-rankings of being overwhelmed (the highest teacher ranking themself a 10). In addition, four participants shared being either in "survival" or "disillusionment" on the new teacher curve when asked on Facebook, "Where are you on the new teacher curve?"

Discussion

The overall lack of participation across the two years, coupled with the decline in participation at the midway point of year two, led us to think critically about our approach in early spring 2020. In March 2020, with the onset of COVID-19 and the transition to remote learning by many schools around the country, the research team ended the yearlong intervention early so as not to overburden teachers further in a time of uncertainty. Even so, the team learned lessons that could benefit others who support early career teachers.

Our findings echo those of Youngs and colleagues (2019), whose synthesis of research on teacher induction programs found that interventions with first-year teachers seem to have little effect on the retention of STEM teachers. They attributed this to early career teachers likely being overwhelmed (Youngs et al., 2019). Although our sample size was small in years one and two of our intervention, the data we gathered and analyzed thus far support this finding. We found that keeping early career teachers engaged in a yearlong intervention was a major obstacle and, though the challenges were often not specific to mathematics, the potential resulting attrition rate of participation raises the question of what can be done to improve future efforts. We are now considering a focus on teachers in their second and third years of teaching and recommend the need for future research in this area. One potentially influencing factor was the inability of our research team to incentivize participation beyond the perceived benefits of participation itself.

The ways in which our participating teachers described the relationships with their mentors contradicts the results from our national survey of early career teachers (Amick et al., 2020). One possible reason for this disconnect might be the result of our intervention design. We were only able to encourage teachers to participate, but if it was a requirement, it may have resulted in better participation. We considered whether this lack of participation by mentors may have potentially influenced sustained teacher participation across the yearlong commitment. Fantilli and McDougall (2009) identified the need for induction programs to refine the mentor selection process in order to ensure novice teachers have adequate support. Therefore, another idea we are examining is to center on mentors or teacher leaders for future iterations of the intervention, and to consider ways to help them support groups of mentees. This would be a significant shift in focus from our current intervention and participant recruitment methods, but it might help to develop strong mentorship teams. We have also considered taking a school team approach and involving an administrator, as past research has shown that perceived administrator support has an impact on early career STEM teachers (Youngs et al., 2019). Another subgroup of our RAC is exploring fostering new teachers' relationships with their administrators. For future work, we are considering combining efforts to strengthen support for early career mathematics teachers. We propose one approach: to build community within school-based teams of early career teachers, one to two mentor teachers, and an administrator. We hope that a collaborative approach might increase participation and thus help novice teachers succeed in their early years of teaching. We also look to expand the University faculty involved in this work to other MTEP partnerships as we are just a small subset of the larger NIC. Being a partner of MTEP allows for rapid dissemination of findings and also scaling of innovations, which we plan on taking advantage of in future iterations of this work.

Additionally, there is a need to consider the potential challenges that come with seeking to create a community of practice via a virtual environment extending across the country. Part of the goal of our intervention was to provide structures to support local mentor-mentee relationships, as well as to connect novice teachers to near peers with whom they could share ideas, challenges, and exchange support. Though we had difficulty keeping early career teachers engaged in our current intervention, we hypothesize that the "new normal" of virtual interactions established during the pandemic might lead to this type of intervention being more successful in the future. For example, limited teacher access to web conferencing software, such as Zoom or Teams, is likely no longer an issue as teaching moved online for many at some point during the pandemic. Furthermore, the pandemic has forced many teachers to move from making connections with their colleagues and students in person to doing so in virtual settings. The ease of participating in virtual workshops, training opportunities, and networking events has eliminated the barriers of distance and travel to engage with a broader community of practice. Such experiences may make early career teachers more likely to engage and remain engaged in online communities. Online support communities may also continue to be useful as the pandemic, at the time we are writing this, is still ongoing. Regardless of the situation, teachers still need support—particularly early career teachers. We see the creation and investigation of online communities of practice being even more important now because of the conditions created by the pandemic. We hope that others take up this work and can build upon the lessons we have learned to create meaningful and supportive online communities of practice that support and retain early career mathematics teachers.

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NOTES FROM THE FIELD

Reflections on the Field of Mathematics Education

By sharing their perspectives, recommendations, and lessons learned with the broader mathematics education community, leaders in the field can support both current and future mathematics educators and researchers in bettering their practice. In the following section, *Reflections on the Field of Mathematics Education*, four people with long, storied careers in mathematics education—Robert Q Berry, III, M. Kathleen Heid, Robert Reys, and Marta Civil—provide reflections on the field of mathematics education. Drawing from their own experiences, they each highlight different aspects of the field that they deem especially pertinent today.

Robert Q. Berry, III reflects on the influential people he met throughout his mathematics career—teachers and professors who not only helped him become a better mathematics educator but also inspired him to investigate the ways in which we can humanize mathematics, particularly for underrepresented students in STEM. Looking backwards and forwards, M. Kathleen Heid reflects on the changes she has seen in the field of mathematics education, focusing on growth in the people, groups, and institutions. Robert Reys provides perspective on and encouragement in pursuing the many activities, projects, and opportunities that those of us in the field of mathematics education will encounter throughout our careers. Finally, reflecting on her qualitative research interests, Marta Civil reminds us that research should benefit both the research and the researched.

Among the common themes from these authors is the need to think deeply about the motivations for mathematics education research, aiming to ensure that such research benefits the community as a whole—the researchers, the participants, the readers, and potential stakeholders. Moreover, we are reminded about the richness that globalism has brought to the field. Colleagues from a variety of backgrounds and perspectives can help us all to identify the needs of the field of mathematics education, whether with respect to classroom practices, mathematical standards, or socio-political policies. We hope that reflecting on these themes and insights will provide our readers with a path forward as they strive to make meaningful contributions of their own to the field of mathematics education.

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NOTES FROM THE FIELD

Humanizing Mathematics to Broaden the Space of Participation

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As I reflect on mathematics education, I am reminded of those who have contributed to my development in mathematics education. I was influenced by a second-grade teacher who told me I was smart and capable; a sixth-grade teacher who was a warm-demander and expected excellence; a high school mathematics teacher who called me out on my laziness because he knew I was not performing up to my potential; a mathematics professor who made me sit in her office for hours until I could prove that I understood it; and a doctoral advisor who saw my potential and nurtured me as a mathematics educator. These people and many students have influenced my work. The stories and evidence are compelling that humanizing mathematics teaching and learning positively impacts persistence and engagement with the field. Within the last 30 years, there has been significant growth in research and work to understand people's mathematical experiences and how it contributes to their mathematical identity, agency, and their worldview of mathematics. The underlining questions appear to be, "what is the purpose for teaching mathematics?"

There are many ways in which researchers, teachers, and leaders investigate this question. Some take on an objective approach that is socially and politically neutral. And, others acknowledge that mathematics is neither objective nor politically and socially neutral. The field is questioning how mathematics is often positioned as objective, politically and socially neutral. As a field, we must engage the questions such as:

- How do we humanize/rehumanize mathematics education? What role do curricula play in humanizing mathematics, and how does it intersect with teaching and learning?
- How does the field attune to contexts and conditions in mathematics teaching and learning? Specifically, how does the field support learners to use mathematics to critique their contexts, understand their conditions, and read the world?
- In what ways do racial-equity, gender-equity, and different forms of representations push the field towards being more equitable in research, teaching, and learning?

For many mathematics educators, I believe that their love for mathematics, students, and communities is the common thread that binds us together as a field. It explains our willingness to engage in activities supportive of building a community around mathematics. It explains the willingness to engage in professional networks and critical conversations about mathematics teaching and learning. The field of mathematics education must engage in critical conversations about humanizing and rehumanizing mathematics to broaden the space of participation.

Too often, humanizing, context, and conditions are ignored in the field. For example, a common theme among policy and reform documents is a call for increased participation of historically excluded learners in mathematics. This call usually references increased and new demands of the U.S. economy, the drive to exceed international competitors, and a need to secure the U.S. from international security threats. Rarely are there references focused on the circumstances of historically excluded people and communities. Positioning historically excluded people increased participation in mathematics to meet the interests of international competition, national security threats, and the economy to the exclusion of the communities of historically excluded people is a form of racial commodification (Basile & Lopez, 2015).

These policies and reform do not provide clarity on how historically excluded people and their communities will benefit from increased participation in mathematics. The field must consider how policies and reforms maintain the interests of those who have power. When we consider power issues in the field, we broaden our understanding of how those with power are often centered and those from historically excluded communities continue to be marginalized.

A growing body of research is engaged in understanding contexts, conditions, and race in mathematics education. This growing body of research challenges the dominant discourse and pushes mathematics education to consider sociological, anthropological, and critical theories. A significant contribution to the field would be developing how this body of work informs practice and research methodologies.

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NOTES FROM THE FIELD

Evolution in the Field of Mathematics Education: Its People, its Products, and its Directions

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The ways in which a field changes reveal the essence of the field. Understanding the field of mathematics education can benefit from retrospection on major changes in the field. Over the past few decades, those changes have revolved around people, products, and directions–and those changes have been profound in U.S. mathematics education. In this piece, I reflect on the changes over the past few decades that have had the greatest effect on the field of mathematics education.

People and Affinity Groups

Not only have the individuals involved in the field of mathematics education changed as scholars have moved in and out of the community, but the relationships among those individuals and their affiliations have also undergone significant change. Improvements in access to travel and communication over the past few decades, for example, have cultivated growth in the nature and strength of international interrelationships. The quadrennial meetings of the International Congress on Mathematical Education (ICME) which assembles groups of mathematics educators, curriculum developers, mathematicians, researchers in mathematics education, mathematics teachers, mathematics teacher educators, producers of mathematics education resources from throughout the world, and mathematics education researchers from a broad range of nations provides one of a growing number of venues that cultivate those international relationships. The annual meeting of the International Group for the Psychology of Mathematics Education (PME) has emerged as an essential venue, with the original attendance of less than 100 swelling in the last few decades to more than eight times that number. U.S. mathematics educators have a clear presence at each of these venues and both a growing awareness of the contributions of international mathematics education and an increasing appreciation for the strength of international research teams. Reflective of these international relationships, the past few decades have witnessed a series of international handbooks on general and specific issues in mathematics education.

Groups of mathematics educators in U.S. have realigned and emerged as new venues have arisen. With growing interest in special subfields such as undergraduate mathematics education, the education of mathematics teachers, and the use of technology in mathematics education, mathematics educators sought special interest organizations both within and apart from existing mathematics-focused organizations. After several decades of work on developing the perception of mathematics education as an acceptable research field within mathematics, the Special Interest Group on Research on Undergraduate Mathematics Education (SIGMAA on RUME) formed within the Mathematical Association of America (MAA) and expanded from dedicated sessions within the joint AMS-MAA meetings to their own conferences (with their 24th annual meeting scheduled for 2022). Similarly, mathematics teacher educators, after seeking a home that focused on the education of mathematics teachers, inaugurated the Association for Mathematics Teacher Educators (AMTE), which has scheduled its 26th annual conference for 2022. School mathematics teachers have also formed special interest groups and expanded their reach. The Teachers Teaching with Technology (T^3) group began its work in the early 1980s and, along with the more recent T^3IC and teacher-driven organizations that focus on classroom use of specific software such as Desmos or CAS, have provided professional development in technology for both mathematics and science teachers.

Products

As affinity groups arose, new journals developed to highlight their specialty areas. A new journal focused on research on the learning and teaching of mathematics at the post-secondary level, *International Journal for Research on Undergraduate Mathematics,* released its first issue in 2015. A journal focused on research on and practice of the education of mathematics teachers, *Mathematics Teacher Educator,* published its first issue in 2012. And a journal focused on the practice of mathematics education in pre-kindergarten through grade 12, *Mathematics Teacher: PreK–12*, a continuation of National Council of Teachers of Mathematics' (NCTM) *Mathematics Teacher,* was first published in 2020. The ability of special-interest groups to adapt to changing times is exemplified in the evolution of a journal dedicated to a specific piece of software, *International Derive Journal* (1994), to a journal dedicated to a specific genre of software, *International Journal of Computer Algebra in Mathematics Education* (1997), to a journal with an unlimited focus on technology in mathematics education, *International Journal for Technology in Mathematics Education* (2004). The development of new journals and the evolution of existing journals, including journals that are published only online, are but one marker of growth in the field of mathematics education. The handful of mathematics education journals available in 1970 is overshadowed by the more than 50 mathematics education journals being produced today. Not only has the number of places within which mathematics education articles can be published increased, but the range of such venues opens the field to new directions and new areas of inquiry.

Directions

The past few decades in mathematics education have generated not only new pockets of research, new participants, new alliances, and new products of those alliances but also new directions for the field. Based on the 1980 NCTM *Agenda for Action,* the NCTM, in its 1989 Standards document, took the bold and unprecedented step of specifying what should be the general content of school mathematics in grades K through 12. These Standards were embodied in the subsequent decade-long development of NSF-funded school mathematics curricula. The stance resulted in the "Math Wars"—vigorous attacks by groups of mathematicians and parents who preferred what they held to be the traditional curriculum. Although various subsequent attempts (e.g., *Principles and Standards of School Mathematics,* Common Core State Standards) were made to define the nature of needed changes in school mathematics curricula, the Math Wars have yet to be settled to everyone's satisfaction. In the meantime, other directions have drawn the field's attention. Some of these directions are unremitting, presenting a constant and difficult-to-achieve but critical goal. Most prominent among these directions is a universal focus on making mathematics classrooms diverse, equitable, and inclusive. Some other directions arose due to unanticipated immediate events, such as the pandemic-fueled need to develop ways to effectively engage students in online or hybrid mathematics instruction. Still other directions are ephemeral, such as mastering the use of the newest digital applications. As we encounter these new directions for our work in the field, it is important to situate this work within overarching and universal goals.

Parting Thoughts

As I reflect on the changes over the past several decades that seem to have most affected the field of mathematics education, I think about the evolution of movements and the production of resources. An increased capability of connecting with mathematics educators from throughout the world has opened the eyes of mathematics educators to the promise of international and intercultural collaboration. The inauguration of organizations centered on particular mathematics education specialties has led to deeper explorations of those specialties. The establishment of an increasingly larger set of venues for disseminating mathematics education research, ideas, and manuscripts allows more mathematics educators to share their ideas broadly about the teaching and learning of mathematics. Just as there is promise in the changes in the people, products, and directions that have been occurring over the past few decades, there are some cautionary notes.

Change requires being mindful of maintaining balance. As we take advantage of the opportunities to connect internationally, to form new alliances, and to join speciality groups, we need to temper knowledge of our specialty with a broader knowledge of the field. We need to maintain cross-groups allegiances that the field of mathematics education has grown. As the number of venues for our work increases, we need to develop ways to deal with and account for the broad range of ideas that others are sharing. As directions and emphases in the field evolve, we need to place them in the larger context of shared directions and emphases. It is through this balance that we can, together, help the field continue to grow in meaningful and productive ways.

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NOTES FROM THE FIELD

Some Career Reflections on Research and Scholarship in Mathematics Education

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Context for Reflections

I received my doctorate in mathematics education from the University of Missouri (MU) in 1966; since that time, I have edited or co-edited ten books and authored or co-authored over 20 books and over 240 articles in refereed journals. During my over 50 years as a faculty member at MU I served as an undergraduate advisor to more than 500 undergraduates preparing to become secondary mathematics teachers and 100 master's and doctoral students in mathematics education. These students kept me grounded, challenged my thinking, provided direction for much of my research, and overall contributed greatly to my career growth and professional development.

Since beginning my career in mathematics education, I met and worked with mathematics educators nationally and internationally. I also witnessed many changes that impacted the mathematics education community. It is said that life is made up of choices. In my own career, often casual or even random events resulted in uncharted paths for my career which often led to new and exciting directions. In this article I share some lessons learned through these experiences that might be useful to future generations of mathematics educators pursuing a career in higher education.

Mathematics Education is an Exciting Career Field

No one can predict the opportunities that arise or the many career paths that might be taken. When I earned my doctorate at the University of Missouri, I never anticipated that I would spend my entire academic career there. Once your doctorate is completed, your first decision is where to work. If you choose higher education, your initial job choice is very important because most mathematics educators stay at the institution that initially hired them throughout their career. Additionally you must ask yourself, do you want to work in a teaching institution or a research-oriented institution? Each of these choices can be rewarding but provide very different career paths.

Sustained Research

While sustained research is important, your research focus may change and be directed by unseen forces or opportunities. In my case, my dissertation provided a start, and reporting on the assessment of mathematical knowledge of entering kindergarteners provided some initial visibility. My experience as a member of two interpretative teams of National Council of Teachers of Mathematics for the first two mathematics assessments of the National Assessment of Educational progress lasted over a decade and provided knowledge and valuable experience. Our research on calculator use in elementary classrooms led to work with estimation, mental computation, and number sense for nearly 20 years, and these topics were of interest to mathematics educators nationally and internationally. The last 20 years of my career included work in helping to implement innovative mathematics curricula, along with a continual focus on doctoral programs in mathematics education. Fresh ideas and innovation are always needed to push thinking and research to address the constantly changing world of mathematics education and sustained research is a powerful vehicle.

Report Research Results in Peer-reviewed Journals

Respected journals provide a mechanism to share results and introduce your work to people on your campus, in your state, in the United States, and around the world. Your writing may inform and stimulate interest in a topic that is new for a reader. It may alert others who want to learn more about your work. You never know when or where an article you have written stimulates interest among the readers and what kind of collaboration may result.

Do Not Give up on a Good Idea

While I was successful in getting multiple proposals funded by external agencies, many of them were funded after second or third reviews. Each time, we used feedback from the reviewers to strengthen the proposal and communicate more clearly its objectives and detail how the project would reach the stated goals. Similarly, many articles that were published in peer-reviewed journals were submitted and initially rejected. Sometimes the feedback from the reviews was useful in rewriting the papers for resubmission to the same journal. In other cases it became clear that the paper should be submitted to a different journal better-aligned with the content of the paper. The moral of this reflection is to believe in yourself and trust that with patience and continued work, your idea will be well received.

Focus on Long-Term Projects

My first funded project was for one year—too short to have any lasting impact. Research, and service efforts to improve teaching, curriculum, and learning in schools, need to be long-term (i.e., multiyear) if any significant results are to be forthcoming. While planning multiyear projects is challenging and competition for long-term funding is keen, the potential rewards are great.

Mathematics Education Programs Can Grow

MU is a public and state-supported institution, and member of the Association of American Universities (AAU). For the last 60 years there have been peaks and valleys in state support for higher education. However, every year department chairs, deans, and provosts made decisions that either enhance or reduce support for specific programs. At MU, mathematics education became a priority area because its faculty members had an established track record of success in scholarly publications, securing external funds, and growing the number of full-time master's and doctoral students. These elements, along with strong and sustained support from the administration, brought national visibility to the mathematics education program and allowed our faculty to grow from two to ten tenure track positions over 20 years.

Mathematics Education Community

In higher education, the mathematics education community is quite small and provides opportunities to develop close personal relationships. It is essential to get to know, learn from, and work with people. I have been fortunate to meet and work with undergraduate and graduate students from MU, graduate students from other institutions, classroom teachers from many different schools and districts, and faculty at a range of national and international institutions of higher education. These interactions helped stimulate my thinking, learn new knowledge, and grow professionally.

Opportunities Arise in Many Places

Opportunities arise from many, often unexpected places. The opportunity might be an internal announcement for research funds at your institution; it might be a call for proposals from your state; it might be a new initiative from funding agencies, such as the National Science Foundation (NSF) or the Department of Education; it might be an invitation of where to spend your sabbatical; it might be encouragement to enter the Fulbright Program; or it might be a letter from a colleague or mathematics educator you do not know that leads to some exciting collaboration. The moral here is that if opportunity knocks at the door, answer and investigate it!

Enjoy Every Day and Try to Learn Something

Enjoy every day and try to learn something new from reading or from someone. The days, semesters, and years go by rapidly. One thing I wish I had done throughout my career was to keep a regular journal, capturing memorable happenings. It might have been something that happened in a class, perhaps how a student's response to a question was different than what was anticipated. It might have been an encounter with a colleague, a presentation that went well or one that was a disaster. It might have been an acceptance or rejection letter for a submitted manuscript or proposal. Such a log will keep you grounded to reality and may lead to exciting places. At the least, the journal provides a record of personal milestones and memories.

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NOTES FROM THE FIELD

Learning With and From the Community

Marta Civil
University of Arizona

As I thought about what I wanted to communicate in this piece, I realized that for me, doing research has always been about following my passion. As a graduate student, I remember some of my peers in education assuming that I would probably do a quantitative study for my dissertation because of my mathematics background. Yet, the research methods course that spoke to me the most was one on qualitative interviewing. I realized that my passion was listening to students as they explained mathematics, and later on listening to parents (mostly mothers of Mexican origin) as they told me about their and their children's experiences with mathematics. Much of what I see as my responsibility in doing research is listening to the students, the teachers, the parents with whom I am working, and then conveying their stories as accurately as I can in my writing. I do not claim to "give them a voice," because they can and should speak for themselves, but the reality is that often they do not have access to having their stories heard. The most powerful conference presentations for me are those in which some of the mothers I have been working with have shared their perspectives and engaged in dialogue with the audience.

To me doing research is about "hanging out" in the community building trust and rapport; it is about having a genuine interest and a deep belief that we have a lot to learn from the community. It is about designing research studies that allow us to learn about the richness of students', teachers', parents' mathematical knowledge and their cultural ways of thinking and being to then inform our further work. It is about documenting the brilliancy and creativity of people whose knowledge is often not recognized (in my context, working-class Mexican American communities) and to challenge the deficit discourse that is often used to describe some communities.

How do we approach the research we do with the communities in our work? Personally, I am not a great believer in researcher-designed "interventions." Get to know the community and work collaboratively with them, learn with and from them. I sometimes wonder about "well-designed" studies that check all the boxes from a research point of view: who are they for? Do they mostly perpetuate the cycle of researchers talking to researchers? In what ways does my work directly contribute to the community who has been sharing their time and expertise with me? If what I am doing is mostly to advance my career or to play the game of academia, is it worth it? What are we giving back to the community when we do our research?

The best moments in my work have been when students, teachers, or parents have challenged my ideas. Yes, I am aware of the power differential between researchers and participants, which is why for me it is very important to work towards an authentic two-way dialogue, a real sense of reciprocity. I close this reflection with a quote from a mother that captures much of my approach to research: *"[Marta] would never try to make us feel that she knew and we didn't; she was always there working with us so we never felt her having an advantage knowing more than us. She was with us"* (Magali, December 2020).

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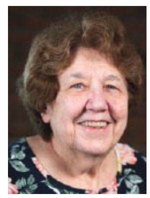

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ACKNOWLEDGEMENT OF REVIEWERS

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JOURNAL OF MATHEMATICS EDUCATION AT TEACHERS COLLEGE

CALL FOR PAPERS

This call for papers is an invitation to mathematics education professionals, especially Teachers College students, alumni, and associates, to submit articles describing research, experiments, projects, innovations, or practices in mathematics education. The journal features full reports (approximately 3500 to 4500 words) and short reports (approximately 500 to 1500 words). Full reports describe findings from specific research, experiments, projects, innovations, or practices that contribute to advancing scholarly knowledge in mathematics education. Short reports (previously "Notes from the field") provide examples, commentary, and/or dialogue about practices out in the field of mathematics education or mathematics teacher education; examples from classroom experience are encouraged. Although past issues of *JMETC* focused around a theme, authors are encouraged to submit articles related to any current topic in mathematics education, from which pertinent themes for future issues may be developed. Articles must not have been submitted to or accepted for publication elsewhere. All manuscripts must include an abstract (approximately 150 words in length) and keywords. Manuscripts should be composed in Microsoft Word and follow APA format. Guest editors will send submitted articles to the review panel and facilitate the blind peer-review process. Articles for consideration should be submitted online at jmetc.columbia.edu, and are reviewed on a rolling basis; however, to be considered for the Spring issue, articles should be received by **January 31, 2022**.

CALL FOR REVIEWERS

This call for reviewers is an invitation to mathematics educators with experience in reading or writing professional papers to join the review panel for future issues of *JMETC.* Reviewers are expected to complete assigned reviews within three weeks of receipt of the manuscript in order to expedite the publication process. Reviewers are responsible for editorial suggestions, fact and citations review, and identification of similar works that may be helpful to contributors whose submissions appear appropriate for publication. Neither authors' nor reviewers' names and affiliations will be shared with one another; however, reviewers' comments may be sent to contributors of manuscripts to guide revision of manuscripts (without identifying the reviewer). If you wish to be considered for review assignments, please register and indicate your willingness to serve as a reviewer on the journal's website: jmetc.columbia.edu.

CALL FOR EDITOR NOMINATIONS

Do you know someone who would be a good candidate to serve as a guest editor of a future issue of *JMETC?* Students in the Program in Mathematics Education at Teachers College are invited to nominate (self-nominations accepted) current doctoral students for this position. Being asked to serve as a guest editor is a testament to the high quality and standards of the student's work and research. In particular, nominations for a guest editor should be a current doctoral student whose scholarship is of the highest quality, whose writing skills are appropriate for editorial oversight, and whose dedication and responsibility will ensure timely publication of the journal issues. All nominations should be submitted to Ms. Juliana Fullon at jmf2213@tc.columbia.edu .